HARTREE–FOCK CALCULATIONS FOR EVEN-EVEN NUCLEI IN THE 2p-1f SHELL USING A VELOCITY DEPENDENT s-WAVE EFFECTIVE INTERACTION

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Hartree–Fock calculations have been performed for even-even nuclei in the 2p–1f shell. We use the method of Parikh and Svenne considering ^{40}Ca as an inert core. Such calculations are carried out using a velocity dependent effective potential of the s-wave interaction. Binding energies, quadrupole moments, energy gaps and pick up strengths are calculated and compared with the previous results of Parikh and Svenne and with the experimental data, whenever possible. Good agreement is obtained.

1. Introduction

In general, one may classify the Hartree–Fock (HF) calculations for finite nuclei into three different types according to the potential used. The first type of calculations is based on the use of an effective interaction which has no relation to the two-nucleon scattering data such as the Skyrme interaction [1] and the modified delta function interaction [2]. The second type makes use of non-singular momentum dependent interactions fitted directly to the phase shifts such as Reid soft core potential [3]. The third type of interactions is the realistic interactions such as Hamada–Johnston [4] and Yale–Shakin [5] potentials.

Recently, much effort has been made to relate the parameters of the effective forces to those of the realistic nucleon-nucleon interaction such as relating the parameters of the Skyrme interaction with the realistic reaction matrices which were derived via the local density approximation [6].

On the other hand, Dzhibuti and Mamasakhlisov [7] have retained the radial form and the parameters of the realistic potential on going over to the effective potential. A modified form of their potential has been applied successfully by Dzhibuti and Sallam [8] to the determination of the binding energies and root mean square radii of a large group of nuclei from ^{4}He to ^{208}Pb using the shell model technique. On the other hand, Parikh and Svenne

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(PS) [9], in an attempt to trace a similarity between the s–d shell nuclei and the p–f shell nuclei, have carried out HF calculations based on the Yale–Shakin potential. In their calculations they considered ^{40}Ca as an inert core and the HF variation was done over the extra-particles moving in the p–f shell.

In the present work, following (PS) method we apply the potential introduced by Dzhibuti and Sallam to calculate the binding energies, the quadrupole moments, the energy gaps and the pick-up strengths for a group of even-even nuclei in the p–f shell. Both (PS) as well as the present HF calculations do not include the Coulomb potential. Therefore, in our results for the binding energies, we take into consideration such an effect by adding the Coulomb energy of a uniform charge density corresponding to the particles outside the core.

The second section will be devoted to the method of calculation and to the description of the two-body potential. The results and the discussion are given in Section 3.

2. Method of calculations and description of the potential

A. HF formalism:

The main assumption of the HF theory is that the wave function of the intrinsic ground state of the nucleus is a single determinant of single-particle states $|\lambda\rangle$ which are the solution of the eigenvalue problem:

$$h|\lambda\rangle = e_\lambda|\lambda\rangle, \tag{1}$$

where

$$\langle j_1 m_1|h|j_2 m_2\rangle = \varepsilon_{j_1}\delta_{j_1 j_2}\delta_{m_1 m_2} + \sum_{\mu=1}^{M}\langle j_1 m_1, \mu|V|j_2 m_2, \mu\rangle_{\mathrm{A}}, \tag{2}$$

ε_j are the appropriate single-particle energies of ^{40}Ca. M is the number of particles outside the core ^{40}Ca, and μ are the occupied HF orbitals. The subscript A on the matrix element of the two-body potential V denotes antisymmetrization. The HF orbits $|\mu\rangle$ are expanded in a basis $|jm\rangle$ of eigenstates, in the p–f shell, of the spherical harmonic oscillator

$$|\mu\rangle = \sum C^{\mu}_{jm}|jm\rangle. \tag{3}$$

If the summation runs over j only, we get axially symmetric solutions. If the summation runs over both j and m, we get triaxial solutions. Since h itself depends on the eigenvectors $|\lambda\rangle$, the problem must be solved self-consistently, using an iteration procedure. The initial values of C^{μ}_{jm} are obtained from diagonalizing the auxiliary Hamiltonian

$$H = -\frac{\hbar^2}{2m}\nabla^2 + \tfrac{1}{2}mw^2r^2 + Dr^2Y_{20}(\theta) + Gr^2Y_{22}(\theta, \phi). \tag{4}$$

The constants D and G are chosen appropriately to give the desired solution whether it is axial prolate or oblate, or triaxial. The total energy E_{HF} is given by:

$$E_{\mathrm{HF}} = \tfrac{1}{2}\sum_{\lambda,\lambda'}\{e_j C^{\lambda}_{jm}C^{\lambda'}_{jm} + \varepsilon_\lambda\delta_{\lambda\lambda'}\}. \tag{5}$$

B. Two-body potential:

The effective potential used takes the following form [8]

$$V_{\text{eff}}(r) = \tfrac{1}{2}\{V_{\text{real}}(r)\exp[-\vec{a}\cdot\vec{\nabla}]+\exp[\vec{a}\cdot\vec{\nabla}]V_{\text{real}}(r)\}_{\vec{a}\to\vec{r}}$$

$$+\eta(A)\frac{\hbar^2}{M}\{\delta(r)\nabla^2+\nabla^2\delta(r)\}, \tag{6}$$

where $V_{\text{real}}(r)$ is the initial realistic potential parametrized in accordance with the two-nucleon problem in vacuum and takes the form:

$$V_{\text{real}}(r) = [a_\tau(\vec{\tau}_1\cdot\vec{\tau}_2)+a_{\sigma\tau}(\vec{\sigma}_1\cdot\vec{\sigma}_2)(\vec{\tau}_1\cdot\vec{\tau}_2)]e^{-r^2/r_c^2},$$

$$a_\tau = 2.096\ \text{MeV}, \quad a_{\sigma\tau} = 7.767\ \text{MeV}, \quad r_c = 2.18\ \text{fermi}. \tag{7}$$

The second term of equation (6) contains an additional parameter η which represents phenomenologically the multiparticle effects.

3. Results and discussion

To carry out HF calculations we have to determine the oscillator constant $\alpha\left(=\sqrt{\frac{m\omega}{\hbar}}\right)$ which may be chosen to fit the experimental root mean square radius of the nucleus R

$$R = \sqrt{\tfrac{3}{5}}\, r_0 A^{1/3}, \tag{8}$$

where r_0 is given by [10]

$$r_0 = 1.123+2.352A^{-2/3}-2.07A^{-4/3}+O(A^{-2}), \tag{9}$$

and

$$R = (\langle r^2\rangle/Z)^{\frac{1}{2}}, \tag{10}$$

where $\langle r^2\rangle$ is given by [11]

$$\langle r^2\rangle = \frac{1}{\alpha^2}[60+\tfrac{9}{2}M_{\text{p}}]. \tag{11}$$

M_{p} is the number of protons outside the inert core. The values of R and α are calculated using equations (8) to (11). The values of η (A) are obtained from the values given in Ref. [8]. Table I gives the appropriate values of η, R and α. The single particle energies ε_j which appear in equation (5) are taken from Ref. [9]. The results of the HF calculations with the velocity-dependent effective potential of s-wave interaction are presented in Table II. The values of the binding energies are given relative to ^{40}Ca as a core. Since the Coulomb effect is not taken into consideration in the HF solution, a Coulomb correction for the binding energy has been included by adding the Coulomb energy of a uniform

TABLE I

Values of $\eta(A)$, R and α for different nuclei

Nucleus	$\eta(A)$ (fm^3)	R (fm)	α (fm^{-1})
^{42}Ca	3.8	3.51	0.494
^{44}Ca	3.7	3.551	0.488
^{46}Ca	3.6	3.59	0.482
^{48}Ca	3.5	3.629	0.477
^{44}Ti	3.7	3.551	0.499
^{46}Ti	3.6	5.59	0.493
^{48}Ti	3.5	3.629	0.488
^{50}Ti	3.517	3.667	0.483
^{48}Cr	3.5	3.629	0.497
^{50}Cr	3.517	3.667	0.492
^{52}Cr	3.525	3.704	0.487
^{54}Cr	3.55	3.74	0.482
^{52}Fe	3.525	3.704	0.494
^{54}Fe	3.55	3.74	0.489
^{56}Fe	3.566	3.776	0.484
^{58}Fe	3.583	3.81	0.48
^{56}Ni	3.566	3.776	0.49
^{58}Ni	3.583	3.81	0.486
^{60}Ni	3.6	3.844	0.482
^{62}Ni	3.617	3.877	0.478
^{60}Zn	3.6	3.844	0.487
^{62}Zn	3.617	3.877	0.483
^{64}Zn	3.633	3.91	0.478
^{68}Zn	3.667	3.973	0.471
^{64}Ge	3.633	3.91	0.483
^{70}Ge	3.683	4.004	0.471
^{72}Ge	3.7	4.034	0.468

charge density [12] corresponding to the particles outside the core. The quadrupole moments Q_{20} and Q_{22} are calculated using the operator form

$$\hat{Q}_{\lambda,\mu} = \left(\frac{4}{2\lambda+1}\right)^{\frac{1}{2}} (\alpha r)^{\lambda} Y_{\lambda,\mu}(\theta, \phi).$$

The energy gaps are also listed in the same table. Inspection of Table II shows that the nuclei ^{42}Ca, ^{50}Ti, ^{58}Fe, ^{62}Zn, ^{68}Zn, ^{64}Ge and ^{70}Ge have the lowest energy solution with axial oblate shape, the nuclei ^{46}Ca, ^{44}Ti, ^{46}Ti, ^{48}Ti, ^{50}Cr, ^{52}Cr, ^{52}Fe, ^{62}Ni, ^{60}Zn, ^{64}Zn have the lowest energy solution with axial prolate shape. ^{60}Ni has the lowest energy solution with triaxial shape. All these results agree with PS results. On the other hand, PS have obtained two different solutions for ^{44}Ca, namely prolate and triaxial, but they are nearly degenerate. However, in our case, the difference between these two solutions is appreciable which favours a triaxial shape. For the nuclei ^{54}Cr, ^{56}Ni and ^{56}Fe we get

Calculated properties of the even-even nuclei*

Nucleus	Type	$(-E_{HF})_{PW}$	$(-E_{HF})_{PS}$	$(-E_{HF})_{PWS}$	$(BE)_{EXP}$	$(Q_{20})_{PW}$	$(Q_{22})_{PW}$	$(Q_{20})_{PS}$	$(G_n)_{PW}$	$(G_n)_{PS}$	$(G_p)_{PW}$	$(G_p)_{PS}$
^{42}Ca	0	17.191	17.168	17.191	19.835	−0.246		−0.262	0.043	0.005		
	P	17.154	17.113	17.154		0.136		0.228	0.001	0.06		
^{44}Ca	O	35.119		35.119	38.898	−0.127			0.128			
	P	35.058	35.104	35.058		0.108		0.391	0.169	0.10		
	T	35.189		35.198		−0.147	0.152		0.264			
	T	35.298	35.130	35.298		0.154	0.221	0.342	0.331	0.20		
^{46}Ca	P	53.593	53.581	53.593	56.719	0.284		0.344	0.157	0.20		
	P	53.521		53.521		0.120			0.001			
^{48}Ca	SPh.	72.959	72.568	12.959	73.940	0.0		0.0	1.993	1.94		
^{44}Ti	O	24.565	23.594	23.885	33.531	−0.482		−0.524	0.533	0.53	0.533	0.53
	P	25.531	25.322	24.851		0.717		0.867	1.065	2.34	1.216	2.38
^{46}Ti	O	46.122	44.167	45.452	56.139	−0.572		−0.630	0.079	0.36	0.476	0.88
	P	48.286	46.115	47.616		0.977		1.068	1.254	0.88	1.550	2.19
^{48}Ti	O	67.835	64.470	67.165	76.642	−0.628		−0.680	0.017	0.02	0.584	0.49
	P	69.0	65.920	68.33		0.910		1.006	0.476	0.49	0.817	1.86
	P		85.597					1.298		0.30		1.53
^{50}Ti	O	89.574	85.289	88.914	95.733	−0.593		−0.679	1.382	1.32	0.565	1.07
	P	88.967		88.307		1.251			0.018		1.178	
	T	89.566	85.366	88.906		0.327	0.490	0.165	1.335	1.36	0.558	1.36
	T		56.543					0.692		2.24		2.24
^{48}Cr	O	57.292	52.687	54.622	69.664	−0.640		−0.757	0.45	0.46	0.420	0.43
	P	61.167	55.876	58.497		1.201		1.333	1.969	1.17	1.978	1.15
	T	61.170		58.50		1.201	0.013		1.966		1.972	
^{50}Cr	O	82.0	75.323	79.35	92.986	−0.667		−0.911	0.133	0.20	0.317	0.36
	P	82.260	79.009	79.61		1.456		1.643	0.341	0.43	1.583	2.0
	P	87.892		85.242		1.237			0.662		2.041	
^{52}Cr	O	106.609	99.043	103.989	114.291	−0.572		−1.272	1.694	2.36	0.525	0.32
	O	106.480		103.86		−0.535			1.372		0.032	
	P	109.245	102.198	106.625		1.517		1.629	1.461	1.27	2.293	1.74

TABLE II (continued)

Nucleus	Type	$(-E_{HF})_{PW}$	$(-E_{HF})_{PS}$	$(-E_{HF})_{PWS}$	$(BE)_{EXP}$	$(Q_{20})_{PW}$	$(E_{22})_{PW}$	$(Q_{20})_{PS}$	$(G_n)_{PW}$	$(G_n)_{PS}$	$(G_p)_{PW}$	$(G_p)_{PS}$
^{54}Cr	O	131.603	120.952	129.003	131.953	−0.807		−1.016	0.940	0.49	0.426	0.60
	O	125.844		123.244		−0.916			0.235	0.288	0.288	
	P	131.273	123.846	128.673		1.188		1.304	0.744	0.98	1.544	1.61
	P		90.899					1.951		0.67		0.70
^{52}Fe	O	95.924	84.004	90.024	105.642	−0.818		−1.294	0.178	0.22	0.184	0.27
	O		86.704					−1.414		0.37		0.44
	P	98.548		92.648		1.813			0.526		0.555	
^{54}Fe	O	123.698	114.488	117.848	129.704	−0.926		−1.629	1.615	2.95	0.09	0.14
	P	120.053	116.76	123.203		1.536		1.835	2.30	1.92	0.387	0.41
	P	123.196		117.346		1.797			1.403		0.525	
	T	129.057	139.975	123.207		1.536	0.007	1.734	2.30	0.77	0.386	0.39
^{56}Fe	O	150.793	138.385	145.993	150.206	−0.183		−1.482	0.603	0.35	0.821	0.49
	P	150.794	137.152	144.994		1.156		0.659	0.602	0.67	0.821	1.73
	T	151.216		145.416		1.213	0.166		0.597		0.792	
^{58}Fe	O	173.589	162.789	166,839	167.890	−1.158		−1.468	0.537	0.23	0.26	0.42
^{56}Ni	O	141.007	131.202	130.707	141.954	−1.308		−1.865	2.08	3.28	2.155	3.28
	P	143.57	131.44	133.27		1.824		1.974	2.763	2.53	2.61	2.46
	T	141.029	157.025	130.729		0.66	1.143	1.978	2.089	0.78	2.165	1.55
^{58}Ni	O	166.732	157.025	156.512	164.406	−1.074		−1.821	0.843	0.25	1.925	2.83
	O	170.918		160.698		−1.021			0.47		1.484	
	P	170.044	156.972	159.824		1.878		1.970	0.108	0.38	2.523	1.58
	T	170.756		160.536		0.651	0.954		0.367		1.434	
	T	167.015		156.795		0.646	1.005		0.875		1.765	
^{60}Ni	T	196.331	183.891	186.201	184.792	0.925	0.853	1.714	0.631	1.15	0.293	0.97
^{62}Ni	O	223.592	210.827	213.542	203.213	−1.408		−1.723	2.51	1.434	1.89	1.89
	P	223.927	210.407	213.877		1.66		1.692	2.379	2.12	1.06	1.16
^{60}Zn	P	186.193		170.353		1.899			0.194		0.314	
^{62}Zn	O	216.711	201.115	200.431	136.431	−1.441		−1.752	0.353	0.50	0.043	0.34
^{64}Zn	P	246.345	230.637	230.755	217.117	21.34		1.759	1.621	2.31	0.485	0.60

^{68}Zn	O	303.532		288.172	253.194	−0.862			2.109		0.100	
	O	298.882	286.543	283.522		−0.914		0.994	1.990	2.51	0.813	0.45
	P	300.411	285.529	285.051		0.641		0.718	0.766	0.81	0.377	0.77
^{64}Ge	O	237.074	219.166	214.624		−1.512		−1.807	0.589	0.43	0.712	0.53
^{70}Ge	O	328.13	311.35	306.17	268.502	−1.144		−1.034	2.626	2.64	0.913	0.02
^{72}Ge	Sph.		342.099		286.885			0.0				1.16
	O	357.7		335.89		−0.449					0.155	
	O	351.757		329.947		−0.384					0.172	
	P	357.753		335.942		0.458					0.144	
	P	351.846		330.036		0.439					0.310	

* O stands for oblate, P for prolate and T for triaxial solutions. $(-E_{HF})_{PW}$ are the HF energies calculated in the present work, $(-E_{HF})_{PS}$ are the HF energies calculated by PS, and $(-E_{HF})_{PWS}$ are the HF energies calculated in the present work after correction for the Coulomb effect. $(BE)_{EXP}$ are the experimental binding energies [14] relative to ^{40}Ca. All energies are given in MeV. The intrinsic quadrupole moments Q_{20} and Q_{22} are given in units of barns. G_n and G_p are the neutron and proton gaps in MeV.

TABLE III

Neutron (a) and proton (b) pick-up strengths*

Nucleus	Type	PW				PS			
		$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
(a)									
^{42}Ca	O	2.0		0	0	2.0	0	0	0
	P	1.984	0.015	0.001	0.001	1.980	0.012	0.006	0.002
^{44}Ca	O	3.995	0.005	0.002	0.0				
	P	3.983	0.004	0.013	0.0	3.948	0.034	0.018	0.002
	Tl	3.952	0.023	0.025	0.0				
	T2	3.982	0.001	0.017	0.0				
^{46}Ca	P1	5.987	0.007	0.006	0.0	5.960	0.024	0.018	0.0
	P2	5.958	0.032	0.010	0.0				
^{48}Ca	Sph.	0.0	0	0	0	0.0	0	0	0
^{44}Ti	O	2.0	0	0	0	2.0	0	0	0
	P	1.594	0.303	0.054	0.049	1.404	0.331	0.174	0.089
^{46}Ti	O	3.912	0	0.088	0	3.824	0	0.176	0
	P	3.360	0.435	0.139	0.067	3.247	0.422	0.229	0.102
^{48}Ti	O	5.643	0.278	0.079	0	5.527	0.251	0.224	0.0
	P	5.469	0.372	0.114	0.045	5.278	0.383	0.253	0.087
^{50}Ti	O	7.474	0.442	0.056	0.028	7.154	0.463	0.332	0.053
	P	5.745	0.999	0.766	0.490	5.789	1.106	0.644	0.480
	T	7.483	0.432	0.056	0.029	7.360	0.256	0.344	0.040
^{48}Cr	O	3.912	0	0.088	0	3.775	0	0.224	0
	P	3.229	0.536	0.159	0.076	3.094	0.539	0.256	0.110
	T	3.228	0.536	0.160	0.077	2.692	2.660	0.454	0.192
^{50}Cr	O	5.725	0.196	0.079	0	5.546	0.190	0.263	0
	P1	3.665	0.838	0.963	0.533	3.006	0.908	1.091	0.494
	P2	5.190	0.563	0.178	0.069				
^{52}Cr	O1	7.687	0.260	0.038	0.015				
	O2	7.587	0.362	0.041	0.010				
	P	5.575	0.857	1.028	0.540	5.436	0.838	1.239	0.485
^{54}Cr	O1	7.660	2.038	0.286	0.016	7.358	0.592	2.014	0.041
	O2	7.363	0.593	2.007	0.037				
	P	7.715	0.821	1.002	0.462	7.505	0.698	1.386	0.406
^{52}Fe	O1	5.510	0.350	0.140	0	4.345	0.842	0.814	0
	O2					3.484	1.711	0.240	0.566
	P	3.469	0.826	1.154	0.551	3.282	1.041	1.108	0.567
^{54}Fe	O	6.972	0.780	0.173	0.075	4.617	1.724	1.077	0.583
	P1	5.591	0.894	0.941	0.574	5.194	0.958	1.251	0.559
	P2	5.387	0.972	1.275	0.546				
	T	5.387	0.792	1.275	0.46				
^{56}Fe	O	7.713	0.800	0.963	0.495	6.295	1.236	2.169	0.300
	P	7.713	0.801	0.991	0.495	7.402	0.370	2.154	0.075
	T	7.396	0.921	1.174	0.509	7.586	0.774	1.292	0.340
^{58}Fe	O	7.528	2.066	3.276	0.056				
^{56}Ni	O	6.120	1.178	0.499	0.283	4.453	1.699	1.263	0.585
	P	5.413	0.801	1.215	0.570	5.156	0.937	1.364	0.545
	T	6.089	1.193	0.510	0.208				

TABLE III (continued)

Nucleus	Type	PW				PS			
		$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
^{58}Ni	O1	7.024	0.839	2.030	0.107	5.677	1.572	2.522	0.498
	O2	7.591	1.853	0.517	0.039				
	P	5.654	1.887	1.847	0.612	5.528	1.881	1.929	0.665
	T1	7.382	1.885	0.635	0.098	5.386	1.742	2.204	0.668
	T2	6.793	1.010	2.043	0.154				
^{60}Ni	T	7.570	1.810	2.317	0.303				
^{62}Ni	O	7.735	2.251	2.994	1.022	7.465	2.212	3.145	1.177
	P	7.858	2.115	5.451	0.575	7.876	2.172	3.276	0.682
(b)									
^{44}Ti	O	2.0	0	0	0	2.0	0	0	0
	P	1.543	0.342	0.062	0.053	1.379	0.361	0.174	0.088
^{46}Ti	O	2.0	0	0	0	2.0	0	0	0
	P	1.471	0.402	0.064	0.062	1.322	0.425	0.152	0.098
^{48}Ti	O	2.0	0	0	0	2.0	0	0	0
	P	1.647	0.292	0.035	0.026	1.488	0.343	0.181	0.067
^{50}Ti	O	2.0	0	0	0	2.0	0	0	0
	P	1.522	0.392	0.044	0.043	1.207	0.592	0.885	0.117
	T	1.999	0.001	0.0	0.0	1.835	0.02	0.163	0.0
^{48}Cr	O	3.912	0	0.088	0	3.775	0	0.224	0
	P	3.191	0.75	0.158	0.077	3.054	0.582	0.256	0.109
	T	3.190	0.574	0.159	0.077	2.691	0.661	0.455	0.192
^{50}Cr	O	3.520	0	0.080	0	3.538	0	0.463	0
	P1	3.367	0.502	0.093	0.0367	2.871	0.768	0.236	0.123
	P2	3.032	0.661	0.212	0.095				
^{52}Cr	O1	3.890	0.101	0.010	0				
	O2	3.943	0	0.057	0				
	P	3.192	0.609	0.143	0.056	2.955	0.743	0.210	0.091
^{54}Cr	O1	3.795	0	0.205	0	3.542	0	0.457	0
	O2	3.940	0	0.060	0				
	P	3.471	0.383	0.111	0.034	3.148	0.546	0.228	0.078
^{52}Fe	O1	5.464	0.398	0.137	0.	4.275	0.911	0.811	0
	O2					3.415	1.806	0.236	0.545
	P	3.454	0.871	1.109	0.544	3.268	1.094	1.087	0.552
^{54}Fe	O	5.341	0.462	0.197	0	3.369	1.797	0.264	0.568
	P1	5.092	0.655	0.196	0.057	3.219	1.038	1.190	0.549
	P2	3.510	0.828	1 103	0.559				
	T	5.091	0.655	0.197	0.057				
^{56}Fe	O	5.490	0.373	0.114	0.023	4.315	0.874	0.812	0
	P	5.490	0.373	0.114	0.023	5.331	0.185	0.411	0 074
	T	5.437	0.401	0.131	0.027	3.622	0.774	1.260	0.340
^{58}Fe	O	5.374	0.428	0.198	0				
^{56}Ni	O	6.047	1.257	0.496	0.200	4.401	1.775	1.258	0.568
	P	5.396	0.850	1.189	0.565	5.131	0.988	1.347	0.535
	T	6.016	1.272	0.991	0.205				
^{58}Ni	O1	7.225	0.653	0.086	0.036	4.835	1.638	1.062	0.463
	O2	6.612	0.923	0.361	0.104				

TABLE III (continued)

Nucleus	Type	PW				PS			
		$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
	P	3.892	0.824	1.207	0.570	5.209	1.75	0.943	0.671
	T1	6.277	1.110	0.463	0.149	4.922	1.260	1.160	0.664
	T2	6.973	0.825	0.142	0.059				
^{60}Ni	T	6.756	0.887	0.306					
^{62}Ni	O	6.551	1.052	0.299	0.098	5.230	1.849	0.594	0.376
	P	5.556	0.663	1.285	0.496	5.376	1.204	0.758	0.666

* The various solutions for each nucleus are listed in the same order as in Table II.

different solutions than those obtained by PS. For the nuclei ^{48}Cr, ^{58}Ni, ^{54}Fe we get a degenerate solution while PS calculations favour one type of such solutions.

The agreement of the calculated binding energies with the experimental values, as a whole, is quite good. For the nuclei ^{42}Ca to ^{52}Fe, the binding energies are less than the experimental values but still they are in better agreement than those found by PS. For the heavier nuclei, the agreement is better. Addition of the Coulomb effects brings the calculated binding energies nearer to the experimental ones as can be seen from column (5) of the same table.

Levinson [13] gives a rough extrapolation of the gap between occupied and unoccupied levels as $G-80/A$ MeV. For the range of nuclei studied here, A ranges from 40 to 70, therefore G ranges from 2 MeV down to 1.1 MeV. The gaps corresponding to the lowest energy solutions, show in most cases approximate agreement with this rule.

Table III gives the pick up strengths for neutrons and protons which are calculated by taking the sum $\sum_{jm} |C^{\nu}_{jm}|^2$ over all occupied orbitals. The table, therefore, shows the fractional occupation of each spherical single-particle orbital. In general these numbers are in over-all agreement with experiment except in a few cases (e.g. ^{50}Ti, ^{52}Cr, ^{54}Fe, ^{56}Ni) where the largest component in the neutron wave function is not $(f_{7/2})^8$ as one might expect on a shell model basis. These results are in agreement with those of PS.

In conclusion, the values of the binding energies using this velocity dependent effective potential of s-wave interaction, following the same variational principle as PS, are in better agreement with experiment even without adding Coulomb effects. The addition of such a correction brings these values nearer to the experimental values especially for heavier nuclei. Since HF calculations are based on a variational principle one may hope that enlarging the basis of linear space of wave functions will improve the calculated values. The addition of a pairing force might improve the results too.

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