SU(8) SYMMETRY AND THE BARYON MASS SPECTRUM

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Mass relations among the charmed and uncharmed baryons predicted in the quark models are derived in the framework of SU(8) symmetry. Higher order effects and spin triplet mass breaking interactions are studied.

1. *Introduction*

SU(4) charm symmetry scheme proposed to study resonances predicts a spectrum of large number of charmed hadrons. The recent experiments [10] have provided evidence for their existence. In view of these, it is interesting to study the properties of charmed particles in the framework of higher symmetries. This study of properties will provide the way for the validity of higher symmetries. Mass relations among these particles have been obtained by several authors, using various techniques. In this paper we discuss the mass relations among the charmed and uncharmed baryons in the framework of SU(8) symmetry.

2. *Mass spectrum*

Quark models [1, 2] do not give the well known Gell-Mann–Okubo octet mass sum rule and equal spacing rule for decimet without additional assumptions. To obtain Gell-Mann–Okubo mass sum rule, one has to assume the two body interactions [1] of s and u quarks to obey:

$$D_{su} = \tfrac{1}{2}(D_{ss}+D_{uu}), \tag{1}$$

where D_{ij} is the two body interaction energy between the i and j quark in triplet state. Extension of this assumption to

$$D_{ij} = \tfrac{1}{2}(D_{ii}+D_{jj}), \quad i, j = \mathrm{u, d, s, c} \tag{2}$$

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leads to further relations among the charmed baryons like equal spacing rule for 20-plet. This assumption has to be extended to all the four quarks in order to get relations among the masses of charmed baryons.

For general mass breaking (i. e. up to all orders), we can at most have the following three distinct baryon contractions [3]

$$B^{XYZ'}B_{XYZ}M^{Z}_{Z'}, \quad B^{XY'Z'}B_{XYZ}M^{X}_{X'}M^{Z}_{Z'}, \quad \text{and} \quad B^{X'Y'Z'}B_{XYZ}M^{X}_{X'}M^{Y}_{Y'}M^{Z}_{Z'}. \tag{3}$$

These correspond to $\underline{63}$, $\underline{1232}$ and $\underline{13104}$ representations of SU(8), present in the direct product:

$$\underline{120}^*\otimes\underline{120} = \underline{1}\oplus\underline{63}\oplus\underline{1232}\oplus\underline{13104}. \tag{4}$$

With the assumption of $\underline{63}$ dominance, we observe that SU(8) symmetry, when mass breaking operator is assumed to transform like (15, 1) component of 63, predicts equal spacing rule for $3/2^+$ baryons, but fails to give good results for $1/2^+$ baryons. We get:

$$\Sigma - \mathrm{N} = \Xi - \Sigma = \Xi_1 - \Sigma_1 = \Omega_1 - \Xi_1 = \Omega_2 - \Xi_2, \tag{5}$$

$$\underset{\substack{1115\\(\mathrm{MeV})}}{\Lambda} = \underset{\substack{1192\\(\mathrm{MeV})}}{\Sigma}, \quad \underset{\substack{2.26\\(\mathrm{GeV})}}{\Lambda'} = \underset{\substack{2.50\\(\mathrm{GeV})}}{\Sigma_1}, \quad \Xi'_1 = \Xi_1. \tag{6}$$

Thus states mixed due to SU(2) and SU(3) breaking are predicted to be mass degenerate and mixing angles cannot be fixed. In order to improve the above situation we consider the second order effects. The second order mass breaking Hamiltonian (H_2) is assumed to transform like (20″, 1) and (84, 1) components of 1232 representation. At SU(4) sub-level we take:

$$a_1 T^{33}_{33} + a_2 T^{44}_{44} + a_3 T^{34}_{34} \tag{7}$$

components of 20″ and 84 representations. We observe that (84, 1) component does not contribute for $1/2^+$ baryons. H_2 with the first order breaking then yields following sum rules:

$$\Omega_1 - \Xi_1 = \Xi_1 - \Sigma_1, \tag{8a}$$

$$2(\Sigma_1 - \Sigma) = 2\Xi_2 + \Xi_1 - 3\Xi'_1, \tag{8b}$$

$$(\Omega_2 - \Xi_2) + (\Xi - \Sigma) = 2(\Xi_1 - \Sigma), \tag{8c}$$

$$\Xi + \mathrm{N} = 2\Sigma, \tag{8d}$$

$$\Xi_2 + \mathrm{N} = 2\Sigma_1, \tag{8e}$$

$$\Lambda = \Sigma, \ \Lambda'_1 = \Sigma_1, \tag{8f}$$

$$m_{\Xi_1', \Xi_1} = \frac{\sqrt{3}}{2}(\Xi'_1 - \Xi_1). \tag{8g}$$

Relations (8a) to (8c) have been obtained in quark models. Relations (8d) and (8e) are collapsed form of Gell-Mann–Okubo mass sum rule and its analog (20). This is because of the relation (8f) which shows that $\Lambda-\Sigma^0$ and $\Lambda_1-\Sigma_1^+$ mixings are electromagnetic in origin. When the electromagnetic breaking is included, these states no longer remain degenerate. We then get modified Gell-Mann–Okubo mass sum rule and its analog

$$2(P+\Xi^0)-3\Lambda-\Sigma^0 = 2(\Sigma^0-\Sigma^-), \tag{9}$$

$$2(P+\Xi_2^{++})-3\Lambda_1'^{+}-\Sigma_1^+ = 2(\Sigma_1^+-\Sigma_1^0), \tag{10}$$

$$m_{\Lambda\Sigma^0} = \frac{\sqrt{3}}{2}(\Lambda-\Sigma^0), \qquad m_{\Lambda_1'^+\Sigma_1^+} = \frac{\sqrt{3}}{2}(\Lambda_1'^{+}-\Sigma_1^+). \tag{11}$$

From equations (8g) and (11) we get the mixing angles:

$$\theta_{\Lambda\Sigma^0} = \theta_{\Lambda_1'^+\Sigma_1^+} = \theta_{\Xi_1'\Xi_1} = 30°, \tag{12}$$

which are very large, while these mixing angles are expected to be of the order of 1° [4]. Other mixing angles (due to SU(3) breaking) may be greater than $\theta_{\Lambda\Sigma^0}$ by an order of magnitude.

To remove these discrepancies we can take 13104 representation into consideration, but it would introduce more parameters and thus the predictive power of SU(8) symmetry would be lost. 13104 can be neglected if the general mass breaking Hamiltonian is assumed to be of the current⊗current form [5], which transforms like

$$\underline{63}\otimes\underline{63} = \underline{1}\oplus 63_S\oplus 63_A\oplus 720+945\oplus 945^*\oplus 1232. \tag{13}$$

Then representations common in the two direct products (4) and (13) will contribute to the mass splitting terms of the baryons. Natural candidates then are 1, 63 and 1232. Hence 13104 is neglected.

When second order effects are included for $3/2^+$ baryons, equal spacing rule is not obtained, rather discrepancies from this rule are related in the following manner:

$$(\Omega-\Delta) = 3(\Xi^*-\Sigma^*), \tag{14}$$

$$(\Omega_2^*-\Xi_2^*)+(\Omega-\Xi^*) = 2(\Xi_1^*-\Sigma_1^*), \tag{15}$$

$$(\Omega_1^*+\Sigma_1^*-2\Xi_1^*) = (\Omega+\Sigma^*-2\Xi^*), \tag{16}$$

$$(\Omega_2^*+\Sigma^*-2\Xi_1^*) = (\Xi_2^*+\Delta-2\Sigma_1^*), \tag{17}$$

$$(\Omega_3-\Omega) = 3(\Omega_2^*-\Omega_1^*). \tag{18}$$

The mass breaking operator has been taken so far to be spin singlet in SU(2) (spin) structure of SU(8). In SU(6) magnetic interaction transforming as (8, 3) component of 35 was added in the electromagnetic mass splitting [6] to remove the discrepancies among the electromagnetic mass relations. Magnetic moment interactions, which appear between the quarks [6], have also been included in the charm quark model. In SU(6) symmetry, breaking differences (8) between spin triplet and spin singlet interactions were found to be of the order of SU(3) breaking. On the basis of these assumptions, we include the spin triplet interaction transforming as (15, 3) component of 63, in first order breaking we then obtain relations (8a) to (8c). Gell-Mann–Okubo mass sum rule and its charmed analog

$$2(N+\Xi) = 3\Lambda+\Sigma, \tag{19}$$

$$2(N+\Xi_2) = 3\Lambda'+\Sigma_1. \tag{20}$$

In addition we get also

$$2(\Xi_1-\Sigma_1) = \Xi-N, \tag{21}$$

which has been obtained in quark–diquark model [7]. Also the mixing angles turn out to be small. Equal spacing rule for 3/2 isobars is not disturbed.

The electromagnetic mass breaking among the 1/2 and 3/2 baryons can also be treated in the same way. Recently a charmed antibaryon state [9, 10] $\bar{\Lambda}_1'^{+}$ has been observed at 2.26 GeV, which decays to $\Lambda^-\pi^-\pi^-\pi^+$. Another state at 2.5 GeV is also observed which decays to $\bar{\Lambda}_1'^{+}/\pi^+$. This state can either be $\bar{\Sigma}_1^{*0}$ ($3/2^+$) or $\bar{\Sigma}_1^0$ ($1/2^+$). We give the mass values for each case as shown in the Table.

TABLE

Masses of charmed baryons $1/2^+$

Isomultiples and the particle label		Equal spacing rule
	Σ_1	2.50 (GeV)
B(6)	Ξ_1	2.69 (GeV)
	Ω_1	2.88 (GeV)
B(3)	Ξ_2	3.70 (GeV)
	Ω_2	3.96 (GeV)
B(3)*	Λ_1'	2.257 (GeV)
	Ξ_1'	2.50 (GeV)

Masses of charmed baryons $3/2^+$

Isomultiples and the particle label		Equal spacing rule
	Σ_1^*	2.49 (GeV) input
D(6)	Ξ_1^*	2.655 (GeV)
	Ω_1^*	2.81 (GeV)
D(3)	Ξ_2^*	3.768 (GeV)
	Ω_2^*	3.931 (GeV)
D(1)	Ω_3^*	5.019 (GeV)

3. *Conclusion*

We have studied the higher order effects by including contributions from 20″ and 84 representations. We have successfully extended the usual SU(6) to SU(8) symmetry, thus incorporating a new quark called the charmed quark c. We see that mass relations obtained under SU(8) symmetry are well satisfied.

Editorial note. This article was proofread by the editors only, not by the authors.

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