QUARK FRAGMENTATION FUNCTIONS FROM A QUARK-PARTON MODEL OF MULTIPARTICLE PRODUCTION IN e^+e^- ANNIHILATION

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Qurk fragmentation functions are extracted from a Monte Carlo quark-parton model of multiparticle production in e^+e^- annihilation. Scale breaking due to transverse momenta and masses of quarks is taken into account. The model leads asymptotically to the retention of electric and baryonic charges of quarks.

The functions describing the fragmentation of quarks to hadrons play an important rôle in the framework of the quark-parton model [1]. This concerns in particular the deep inelastic lepton-nucleon scattering [1], large p_T processes [2] and the e^+e^- annihilation.

In this note we describe a Monte Carlo quark-parton model for multiparticle production in e^+e^- annihilation and we extract the quark fragmentation functions from the obtained results. We compare our results with the fragmentation functions found in the recent analysis of data by Field and Feynman [2].

Our calculations are done at finite energies, what enables us to estimate also the scale breaking effects caused by the transverse momenta and masses of quarks.

Our model is based on the assumption that the dynamics behind the multiparticle production in e^+e^- annihilation and in hadron-hadron collisions is basically the same. This assumption is substantiated by the experimental results [3] and by the theoretical considerations (e.g. [4] and references cited therein).

Guided by this assumption we use for the calculation of the multiparticle production in e^+e^- annihilation the same procedure as we did in our recently proposed model for multiproduction in hadron-hadron collisions [5].

In such an approach the e^+e^- collision is supposed to proceed in three stages (see e.g. [1, 5–8]). In the first one the e^+e^- annihilate to a quark-antiquark ($Q\bar{Q}$) pair. In the second one the rapidity gap between the Q and the $\bar{Q}$ is filled by $Q\bar{Q}$ pairs. In the third stage the

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neighbouring (in rapidity) $Q\bar{Q}$ pairs recombine to mesons and QQQ and $\overline{QQQ}$ triplets recombine to baryons and antibaryons.

We do not try to make a quantitative description of the whole process. Instead, following [5], we construct a model for the distribution of Q's and $\bar{Q}$'s just prior to the recombination and describe the process of recombination.

Basically we assume that the distribution of Q's and $\bar{Q}$'s is given by the cylindrical phase space modified in such a way that the Q and the $\bar{Q}$ produced in the first stage ($e^+e^- \to Q\bar{Q}$) keep a sizable part of their original momenta. In this way they play the rôle of "leading quarks" as the valence quarks (of the incoming hadrons) do in the hadron--hadron collisions [5].

Thus, the probability to find two "valence" partons with momenta $\vec{p}_1$, $\vec{p}_2$ and additional n quarks with $\vec{p}_i$, $i = 3, \ldots, n+2$ and n antiquarks with $\vec{p}_i$, $i = n+3, \ldots, 2n+2$ is given by the expression ($N = 2n+2$)

$$dP_N(\vec{p}_1, \ldots, \vec{p}_N) \sim G^{2n} W_{\mathrm{id}} \sqrt{|x_1|}\sqrt{|x_2|}\, dCPS(\vec{p}_1, \ldots, \vec{p}_N), \tag{1}$$

where G is the "coupling constant" regulating the average multiplicity, W_{id} is (roughly estimated) factor for identical particles. $\sqrt{|x_1|}$, $\sqrt{|x_2|}$ are the factors first used by Kuti and Weisskopf [9], giving higher probabilities to configurations where "valence" partons have large values of the longitudinal momentum fractions $x = 2p_{\mathrm{L}}/\sqrt{s}$ · $dCPS$ stands for the cylindrical phase space

$$dCPS(\vec{p}_1, \ldots, \vec{p}_N) = \exp\left(-\sum p_{\mathrm{T}i}^2/R^2\right)\delta\left(\sum \vec{p}_i\right)\delta\left(\sqrt{s} - \sum E_i\right)\Pi d^3p_i/2E_i,$$

where R is the parameter regulating the transverse momentum cut-off and $\sqrt{s}$ is the total c.m.s. energy of the e^+e^- annihilation.

The quantum numbers of Q's and $\bar{Q}$'s in Eq. (1) are (except of those of the leading "valence" quarks) selected at random. However, in assigning the quantum numbers to quarks the occurrence of strange ones is supressed by a phenomenological factor λ(λ = probability to create an $s\bar{s}$ pair divided by the probability to create a $u\bar{u}$ one).

The parameters G, R and λ are all fixed by the comparison with the data on multiparticle production in hadron-hadron collisions. In this respect they do not play the rôle of free parameters when calculating the multiparticle production in e^+e^- annihilation. The masses of quarks are (as earlier [5]) fixed at values $m_{\mathrm{u}} = m_{\mathrm{d}} = 0.01$ GeV/c^2, $m_{\mathrm{s}} = 0.16$ GeV/c^2.

After generating the sequence of partons with assigned quantum numbers and momenta, the program simulates the process of their recombination into hadrons. The "rules for recombination" [5] are such that only nearby (in rapidity) partons can recombine. In general from a given combination ($Q\bar{Q}$, QQQ, $\overline{QQQ}$) of recombined partons more than one type of hadron can be formed. In such a case the relative probabilities of different possibilities are given by the coefficients in the SU(6) hadronic wave functions. An example of the recombination is given in Fig. 1.

The resonances (a copious production of them is a typical feature of this type of models [5, 7, 8]) then decay according to experimental branching ratios.

The model, when applied to multiparticle production in pp collisions, gives a reasonable quantitative description of average multiplicities and inclusive spectra of produced hadrons in a wide range of energies [5]. We fix the parameters of the model at values $G = 1.47$, $\lambda = 0.38$ $R_T^2 = 0.20$ GeV$^2/c^2$ by comparison with data on multiproduction in

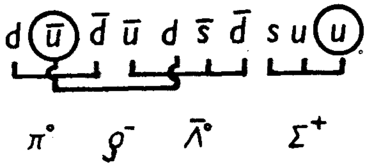

Fig. 1. An example of the quark recombination. The leading "valence" quarks are circled

hadron-hadron collisions at corresponding energies and then we calculate the multiparticle production in e^+e^- annihilation. The results and their comparison with data on e^+e^- will be published elsewhere, here we shall present only the u-quark fragmentation functions.

The procedure for obtaining these functions in our model is straightforward. One simply calculates in the above mentioned way, for example, the π^+ distributions for the case when the u and $\bar{u}$ play the rôle of the leading "valence" quarks (expressed by the $\sqrt{|x|}$ factors in the Eq. (1)). The density of π^+ on the side of the leading u quark in the Feynman variable $z = p_L(\pi^+)/p_{max}$ gives directly the fragmentation function $D_u^{\pi^+}(z)$. The other fragmentation functions are determined in a similar way.

Our results for the fragmentation functions of the u quark into pions and kaons are presented in Fig. 2. We do not make here a direct comparison with data on fragmentation functions. Instead of that we take here — as a standard reference — the recent work by Field and Feynman [2]. Fig. 3 shows our prediction for the fragmentation of the u-quark into protons.

A few comments are in order. The fragmentation functions were calculated for various c.m.s. energies of the e^+e^- annihilation. Since these energies are not yet asymptotical, the functions $D_q^h(z)$ do depend on energy and the results show strong scale breaking effects mainly in the low z region.

The fact is easy to understand. While at asymptotical energies one expects the fragmentation function behaviour of the form $D(z) \sim 1/z$ at finite energies one gets $D(z) \sim 1/(z^2 + m_T^2/P^2)^{1/2}$.

However, our results approach surprisingly well the Field and Feynman curves (for the fragmentation into pions). Having in mind, that the latters summarize well the currently available experimental information and that all the parameters of our model were fixed by data on multiparticle production in hadronic collisions, such an agreement is definitely nontrivial.

For the fragmentation into kaons our results differ from those of Field and Feynman in the region of small z. However, in their analysis only the large z region was fixed by the data, whereas the situation in the small z region is experimentally less clear.

A noticeable result is the large probability of the fragmentation of the u-quark to proton (for large z it is about as large as that for the fragmentation of the u quark to π^+).

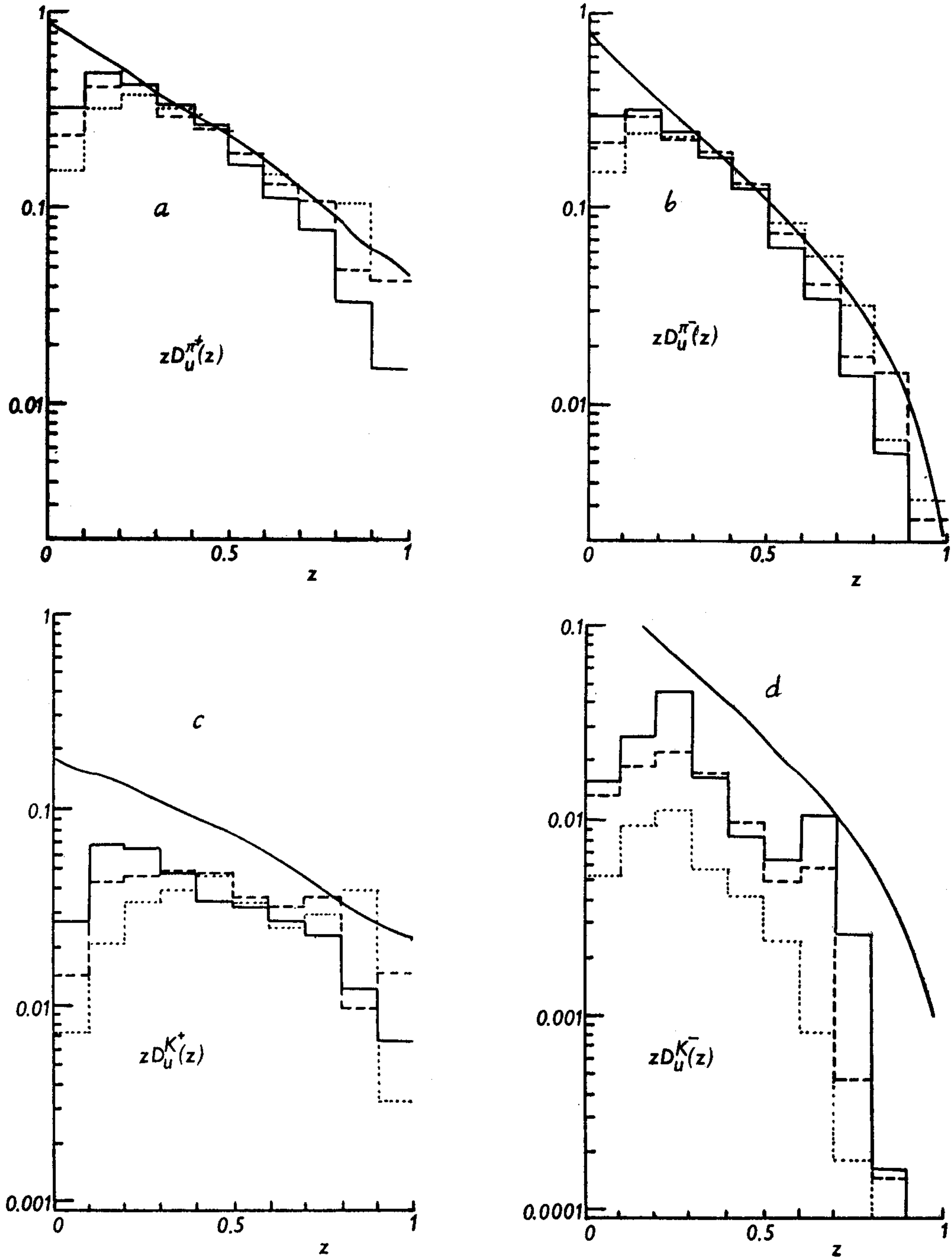

Fig. 2. Fragmentation functions calculated in the model (histograms) for $\sqrt{s} = 3$ GeV (dotted), 4.8 GeV (dashed), 7.4 GeV (solid). The curves represent the Field and Feynman calculation [2]: a. — u quark into π^+, b. — u quark into π^-, c. — u quark into K^+, d. — u quark into K^-

This feature can be easily understood. Suppose the leading quarks is moving right and has the largest rapidity. Then there are four possibilities for the configurations of partons at the end of the rapidity plot: QQu, $Q\bar{Q}$u, $\bar{Q}$Qu and $\bar{Q}\bar{Q}$u. In the first case we obtain

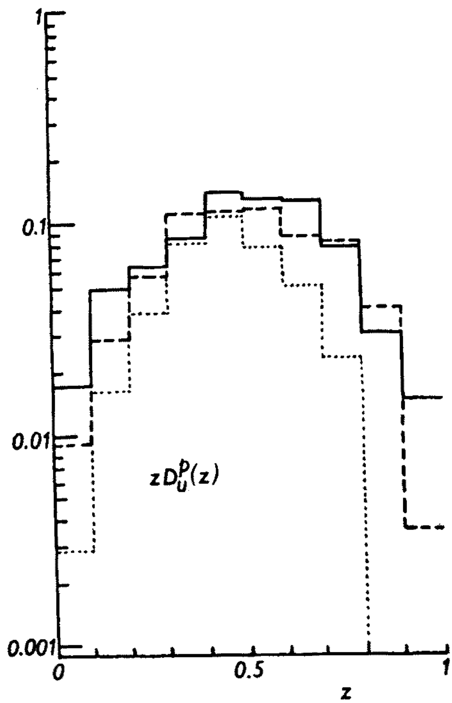

Fig. 3. The model predictions for the u-quark fragmentation into protons. The energies are the same as in Fig. 2

by recombination a fast baryon or a baryonic resonance. After the decay of the resonance the resulting baryon still keeps a large momentum fraction. In the other three cases the recombination leads to a pseudoscalar or vector meson production. However, the decays of vector mesons (like $\varrho \to \pi\pi$) lead to two stable mesons with small momentum fractions. Since according to SU(6) weights vector mesons are three times more frequent than "direct" pseudoscalar mesons, the fragmentation of the quark (for a large z) to a stable meson is about as frequent as the fragmentation to a baryon.

At high energies our model leads to the full retention of both electric and baryonic charges of quarks. This is connected with the fact that, apart from leading quarks, the quantum numbers of partons are assigned statistically and that both mesons and baryons can be produced (the details of the argument will be published elsewhere).

Concluding, we would like to stress again that the presented fragmentation functions were obtained in a model which gives satisfactory results also in multiparticle and low mass dimuon production [10] in hadronic collisions and whose parameters were completely fixed by the hadronic data. The fact that fragmentation functions calculated in this way are very close to those obtained by Field and Feynman [2] from a detailed analysis of eN, νN and e^+e^- data is, in our opinion, most encouraging.

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