THE STATISTICAL DIRECT REACTION MODEL FOR THE (p, α) REACTION

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The model for the (p, α) reaction on heavy nuclei is developed. Starting from the three-body approach to the (p, α) reaction and the quasi-particle-phonon model for heavy deformed nuclei, the cross sections for the (p, α) reactions on ^{162}Dy, ^{166,168}Er and ^{176,178,180}Hf targets are calculated.

1. Formulation of the model

In the present paper the unified model of the (p, α) reaction on heavy nuclei is studied. The basic assumptions involved in the model are as follows:

1. The interaction of the fast protons ($E_p \geqslant 20$ MeV) with the heavy nuclei can be described in terms of the three body approach, where the interacting nuclei are: the proton α-particle and the (T − α) core.

2. The amplitude of the (p, α) reaction is described in the lowest perturbation order by the Born–Norman Series for the transition operator $U_{\beta\alpha}$.

3. The wave function of the final nucleus is described for the heavy nuclei in terms of the quasi-particle-phonon model [3].

In the three body approach a nuclear rearrangement reaction (p, α) can be represented schematically as

$$p+[\alpha+(T-\alpha)] \rightarrow \alpha+[p+(T-\alpha)]. \tag{1}$$

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Let us introduce the transition operators $U_{\beta\alpha}$ from the initial channel α to the final channel β [1]

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_{\alpha}^{(-1)} + V_{\beta} G V_{\alpha}, \tag{2}$$

where $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$, $V_{\delta}(\delta = \alpha, \beta)$ are the channel interactions, G_{γ} are the resolvents of the channel Hamiltonians and G the resolvent of the total Hamiltonian. As is shown in Ref. [1] the lowest order series for the transition operator (2) can be written as

$$U_{\beta\alpha} = G_0^{(-1)} \bar{\delta}_{\beta\alpha} + \sum_{\substack{\gamma \neq \alpha \\ \gamma \neq \beta}} t_{\gamma} + \sum_{\substack{\gamma \neq \beta \\ \delta \neq \gamma \\ \delta \neq \alpha}} t_{\gamma} G_0 t_{\delta}. \tag{3}$$

The transition operator $U_{\beta\alpha}$ is used to build-up the transition matrix $R_{\beta\alpha}$:

$$R_{\beta\alpha}^{m',m} = \langle \phi_{\beta m'} | U_{\beta\alpha} | \phi_{\alpha m} \rangle, \tag{4}$$

where the functions $\phi_{\alpha m}$ are eigenfunctions of the channel Hamiltonian h_{α}. According to Ref. [1] from formulae (3) and (4) we obtain:

$$R_{\beta\alpha}^{m',m} = \langle \phi_{\beta m'} | G_0^{(-1)} \bar{\delta}_{\beta\alpha} | \phi_{\alpha m} \rangle$$
$$+ \sum_{\substack{\gamma \neq \alpha \\ \gamma \neq \beta}} \langle \phi_{\beta m'} | t_{\gamma} | \phi_{\alpha m} \rangle + \sum_{\substack{\gamma \neq \beta, \delta \neq \alpha \\ \delta \neq \gamma}} \langle \phi_{\beta m'} | t_{\gamma} G_0 t_{\delta} | \phi_{\alpha m} \rangle + \ldots \tag{5}$$

In series (5) the third and next terms describe the mechanism involving at least three successive rearrangements. In our approach we shall consider only the first step of the p+T interaction and neglect other terms. The first term in formula (5) describes the heavy particle pick-up. This term determines the cross section for the production of α-particles with scattering angles greater than 90°. In our calculation we confine ourselves to small scattering angles and we shall neglect the first term of series (5).

Let us consider the (p, α) reaction on heavy, doubly even deformed nuclei. Here the whole spectroscopic information is provided by the wave function of the final nucleus. Considering formula (5) and the results of Ref. [2], we can describe the angular distribution of α-particles as:

$$\frac{d\sigma}{d\Omega}(E_f) = A g_0^2 \frac{p_{\alpha}}{p_{\mathrm{p}}} |M_{\mathrm{p}\alpha}|^2 \sum_{K} \sum_{lj} (2j+1)^{(-1)} S_{jl}(E_f^{jk}),$$

where

$$A = \tfrac{3}{2} \frac{\hbar^2}{\mu r_0^2} \theta_0^2 G^2(\mathrm{T} \to (\mathrm{T}-\alpha)+\alpha) G^2(\mathrm{p}+\alpha \to \mathrm{p}+\alpha),$$

$$g_0^2 = \frac{m_{\mathrm{T}'}^2}{m_{\mathrm{p}} m_{\mathrm{T}-\alpha}} \left(-2 \frac{Q_{\mathrm{p}} m_{\mathrm{p}} m_{\mathrm{T}-\alpha}}{m_{\mathrm{T}'}} \right)^{1/2} \tag{6}$$

all abreviations being the same as in Ref. [2].

The wave function of the non-rotational state with angular momentum projection on the nucleus symmetry axis K and parity π of the odd A nucleus has the form [3]:

$$\psi_n(K^\pi) = \frac{1}{\sqrt{2}} \sum_\sigma \left\{ \sum_s c_s^n \alpha_{s\sigma}^\dagger + \sum_g D_g^n (\alpha^\dagger Q^\dagger)_g \right\}. \tag{7}$$

Then for the spectroscopic factor S_{jl} we get:

$$S_{jl} = |\sum_s a_{lj}^{sk} u_s c_s^n|^2, \tag{8}$$

where u_s is the Bogolubov transformation factor and a_{lj}^{sk} describes the transformation from the spherical to deformed basis.

To determine the cross section for the (p, α) reaction we use the averaged weighted spectroscopic factors with the weight [3]

$$\varrho(E_f - \eta) = \frac{1}{2\pi} \frac{\Delta}{(E_f - \eta)^2 + (\Delta/2)^2}. \tag{9}$$

Considering formulae (6)–(9), the cross section for the (p, α) reaction has the form

$$\frac{d^2\sigma}{d\Omega d\eta} = \sum_f \varrho(\eta - E_f) \frac{d\sigma}{d\Omega}(E_f), \tag{10}$$

where the summation obeys all the states of the final nucleus which can be created by the captured proton. Substitution of formula (6) to (10) gives

$$\frac{d^2\sigma}{d\Omega d\eta} = A \sum_f F(E_f) \varrho(E_f - \eta) \sum_{kj} (2j+1)^{(-1)} \left| \sum_s a_{lj}^{sk} u_s c_s^f \right|^2, \tag{11}$$

where

$$F(E_f) = \frac{p_\alpha}{p_p} |M_{p\alpha}|^2 g_0^2. \tag{12}$$

The term $d^2\sigma/d\Omega d\eta$ can be written as a contour integral around the poles in the complex plane η. After some rearrangements we get

$$\frac{d^2\sigma}{d\Omega d\eta} = \frac{A}{\pi} \sum_{kj} (2j+1) \left\{ \sum_s (a_{lj}^{sk} u_s)^2 \operatorname{Im} \frac{F\left(\eta + i\frac{\Delta}{2}\right)}{\mathscr{F}_s\left(\eta + i\frac{\Delta}{2}\right)} + \sum_{s>s'} a_{lj}^{sk} a_{lj}^{s'k} u_s u_{s'} \operatorname{Im} \frac{F\left(\eta + i\frac{\Delta}{2}\right) \theta_s\left(s', \eta + i\frac{\Delta}{2}\right)}{\theta\left(\eta + i\frac{\Delta}{2}\right)} \right\}. \tag{13}$$

Since the function $F(\eta)$ has no poles in the complex plane η, formula (13) assumes the form

$$\frac{d^2\sigma}{d\Omega d\eta} = \frac{A}{\pi} F(\eta) \sum_{kj} (2j+1)^{(-1)} \left\{ \sum_{s} (a_{lj}^{sk} u_s)^2 \operatorname{Im} \left[\mathscr{F}_s \left(\eta + i \frac{\Delta}{2} \right) \right]^{-1} \right.$$

$$\left. + 2 \sum_{s>s'} a_{lj}^{sk} a_{lj}^{s'k} u_s u_{s'} \operatorname{Im} \left[\frac{\theta_s \left(s', \eta + i \frac{\Delta}{2} \right)}{\theta \left(\eta + i \frac{\Delta}{2} \right)} \right] \right\}, \tag{14}$$

where

$$\mathscr{F}_s(\eta) = \theta / \theta_s \tag{15}$$

and θ is the determinant of the equation

$$\sum_{s'} \left\{ (\varepsilon(s') - \eta_i) \delta_{ss'} - \sum_{g} \frac{\Gamma_{sg} \Gamma_{s'g}}{p_g - \eta_i} \right\} c_{s'}^i = 0. \tag{16}$$

In formulae (15) and (16) ε denotes the energy of one-quasi-particle state, and $p(g) = \varepsilon + \omega^{(\lambda,\mu)}$ is the energy of the quasi-particle-plus-phonon state. The superscript (λ, μ) denotes the multipolarity of the phonon state g. The function Γ_{sg} defines the interaction of the quasi--particle state with the phonon one g. In numerical calculations we use the single particle energies and wave functions of the axial symmetric Saxon–Woods potential. The potential parameters, pairing and multipole-multipole interaction constants and the phonon number have been taken from Ref. [3].

2. *Cross sections for the* (p, α) *reaction on the heavy, deformed nuclei* ^{162}Dy, ^{166,168}Er *and* ^{176,178,180}Hf

In our calculation we chose the well deformed targets ^{162}Dy, ^{166,168}Er and ^{176,178,180}Hf which span the neutron number region of $N = 96$–108. Moreover all these targets have positive separation energy for α particles. As shown in Ref. [3] the phonon--quasi-particle model describes well the weighted one-quasi-particle strength functions for the low and high excitation energy.

It occurs that the structure of the nuclear states at intermediate and high excitation energies is mainly defined by fragmentation, i. e. the distribution of the strength of one--quasiparticle state over many nuclear levels. The general regularities of fragmentation of the single particle states in deformed nuclei can be summarized as follows:

1. The form of the distribution strongly differs from that of the Breit–Wigner one. As a rule in addition to the main maximum there appear several additional maxima.

2. The shape of the distribution function is mainly defined by the position of the single-particle state with respect to the Fermi level.

3. For single-particle states lying near the Fermi surface the distribution maximum is shifted by 0.5 to 1.5 MeV towards low energies with respect to $\varepsilon(s)$ [3].

In Figs. 1–6 the theoretical calculations of the cross sections for the (p, α) reaction on the ^{162}Dy, ^{166,168}Er and ^{176,178,180}Hf nuclei are presented. As a rule, in all α-particle energy spectra the quasi-discrete structure is preserved up to the binding energy of the protons in the final nuclei. To interpret the observed structure of α particle spectra we must

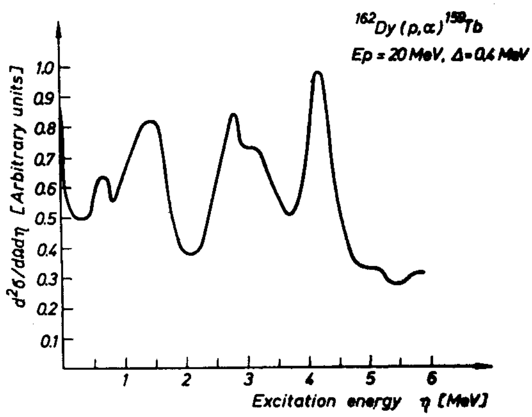

Fig. 1. Theoretical differential cross section for the ^{162}Dy(p, α) ^{159}Tb reaction. The proton energy *Ep* is equal to 20 MeV. The cross section is calculated for the scattering angle θ equal to 0°. The energy interval of averaging, Δ, is equal to 0.4 MeV

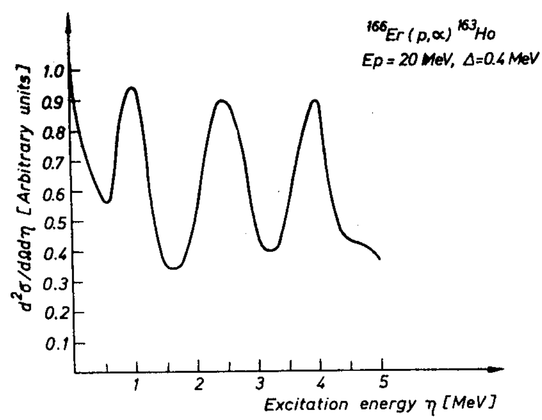

Fig. 2. Theoretical differential cross section for the ^{166}Er(p, α) ^{163}Ho reaction. For other information see caption to Fig. 1

recognize that because of the quasi-particle-phonon interaction the observed maxima cannot be linked to the one quasi-particle state. Each maximum included contributions from many non-rotational states. Even for the single particle states lying near the Fermi level the maximum of the distribution strength does not coincide with $\varepsilon(s_0)$. Moreover the form of the distribution strongly differs from the Breit–Wigner one. It seems that

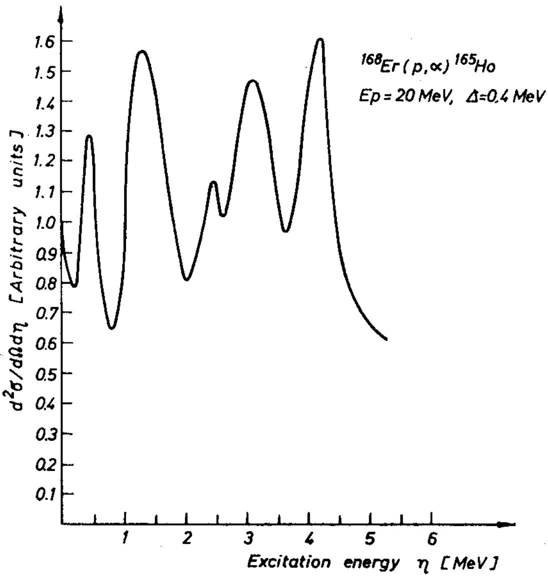

Fig. 3. Theoretical differential cross section for the ^{168}Er(p, α) ^{165}Ho reaction. For other information see caption to Fig. 1

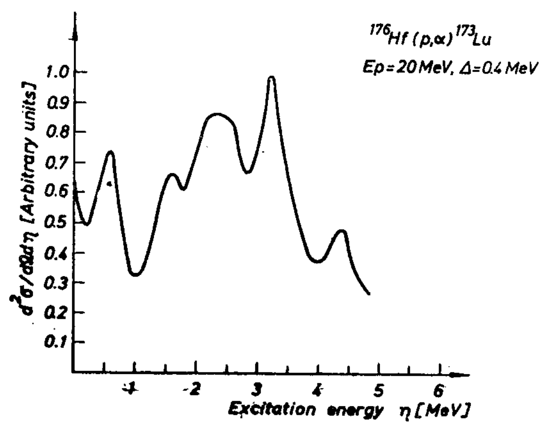

Fig. 4. Theoretical differential cross section for the ^{176}Hf(p, α) ^{173}Lu reaction. For other information see caption to Fig. 1

these facts affect substantially the interpretation of the cross section calculation for the direct nuclear reaction which populates high excitation states. It is generally accepted that the strength distribution of the single particle state at intermediate and high excitation energies has approximately the Breit–Wigner form with a center which coincides with the single-particle state. The width of this distribution is considered to be either constant or a smooth function of the excitation energy. The fragmentation of a given state is usually assumed to be independent of its quantum characteristics irrespective of whether it is a particle or hole state [4, 5]. The calculations performed in Ref. [3] have shown that the

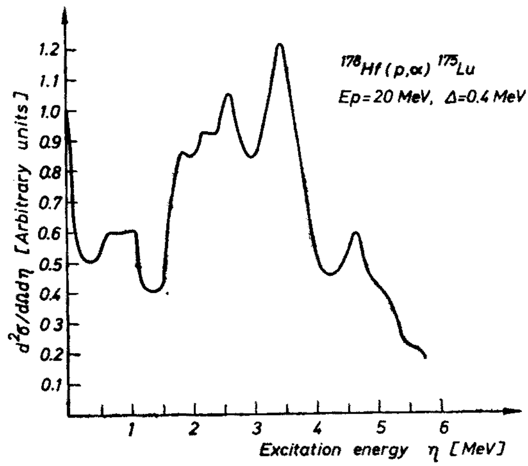

Fig. 5. Theoretical differential crosss section for the ^{178}Hf(p, α) ^{175}Lu reaction. For other information see caption to Fig. 1

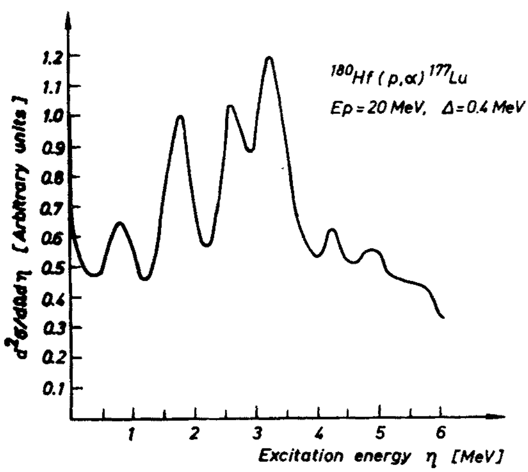

Fig. 6. Theoretical differential cross section for the ^{180}Hf(p, α) ^{177}Lu reaction. For other information see caption to Fig. 1

fragmentation of the state strength is highly complicated. The fragmentation essentially depends on the position and quantum numbers of the one-quasi-particle states.

In the description [3] of the strength function the energy interval of averaging, Δ, is a free parameter. In terms of the quasi-particle–phonon model the accuracy of the calculations of strength function is limited by the approximate description of fragmentation of the single particle states and approximate description of one--particle states in the Woods–Saxon potential. As is shown in Ref. [3], $\Delta = 0.4$ MeV is a reasonable value of energy averaging interval for heavy deformed nuclei which takes into account the theoretical limitations of considering the quasiparticle phonon model.

3. *Discussion and conclusions*

The cross sections calculated in Section 2 can be regarded as a proposal for future experimental measurement for the (p, α) reactions. As far as we know, no experimental data are available concerning the (p, α) reactions on the targets used by us. Our calculations offer the possibility of studying the (p, α) reaction in a broad excitation energy region. The whole α-particle spectrum is described by the same mechanism in which the α-particle serves as a spectator which yields the information on simple states generated by proton in the final nuclei. Complication of the nuclear states with increasing excitation energy is the reason for the α-particle spectrum being quasi-discrete. Our approach is quite different from the widely used simple method in which complication of the states is described by an unperturbed one-quasi-particle scheme smeared by Breit–Wigner formula [4, 5]. It must be recognized that in our calculation we do not intend to describe the correct position and strength of each local maximum of the cross sections measured with an accuracy of e. g. 10–20 keV. The proposed model of the (p, α) reaction gives an averaged (weighted with function $\varrho(\eta - E_f)$) description of an envelope of micro-structure consisting of thousands excited states which can be occupied by the proton. These envelopes shown in Figs. 1–6 are calculated assuming $\Delta = 0.4$ MeV. All maxima of the cross sections are generated by the particle states. This means that comparison of the calculated and experimental α-particle spectra can exclude other mechanisms in which hole states are generated. These hole states can be excited in the (p, α) reaction when the sequential mechanism is assumed, for example the two step (d, t) (t, α) pick-up.

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