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ON THE MULTIPLICITY OF SECONDARY PARTICLES IN THE INTERACTIONS OF HIGH-ENERGY PROTONS AND PIONS WITH NUCLEI

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An analysis of the interactions of protons and pions at different primary energies (from a few GeV to some thousands of GeV) with emulsion nuclei was made. Experimental data were compared with the predictions of several models, especially of a simple model of non-interacting fireballs (NFB).

1. *Introduction*

The comparison of experimental results concerning the interactions of high-energy hadrons with nuclei with the predictions of different models is still of great interest to physicists ([1] and earlier works cited e. g. in [2, 3]). Very often the predictions of models are compared with experimental dependences: R vs E_0 and R vs $\bar{\nu}$, where E_0 — primary energy, $\bar{\nu}$ — mean number of collisions inside the nucleus, $R = \langle n \rangle_A / \langle n \rangle_H$ — mean normalized multiplicity, i. e. the ratio of the multiplicity in the interaction of the hadron with the nucleus with mass number A to the multiplicity in the hadron-proton interaction at the same energy.

In this work we shall examine only those models in which the hadron-nucleus interaction is interpreted as a sequence of approximately independent collisions (interactions) with single nucleons inside the nucleus. A review of those models may be found e.g. in the paper by Zalewski [4]. One of the oldest models of such a type was introduced many years ago in the investigations of jets produced by cosmic particles in nuclear emulsion. It was a model of particle production through fireballs, which are shortlived and interact weakly with nuclear matter. This model was proposed in the Cracow Emulsion Group by M. Mięsowicz (see [5], where earlier works on this subject are also cited) and it was called the model of non-interacting fireballs (NFB). The comparison of experimental data with

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calculations of the NFB model, based on very simple assumptions on the number of collisions and the inelasticity coefficient, was made previously in [2].

In this work we shall discuss the experimental dependences R vs E_0 and R vs $\bar{\nu}$ for primary protons and pions taking into account the low energy secondary particles, which are not considered in many experimental works. We shall compare them with predictions of the modifications of the Energy Flux Cascade Model [7, 15], the model of excited states of nucleons [8], the NFB model and the parton model (Brodsky et al. [9]).

2. *The mean number of collisions inside the nucleus*

Unfortunately the mean number $\bar{\nu}$ of collisions of a hadron inside the nucleus cannot be measured directly. The authors who deal with targets of different mass numbers A usually use for calculating $\bar{\nu}$ the formula:

$$\bar{\nu} = A \frac{\sigma_{hp}}{\sigma_{hA}}, \tag{1}$$

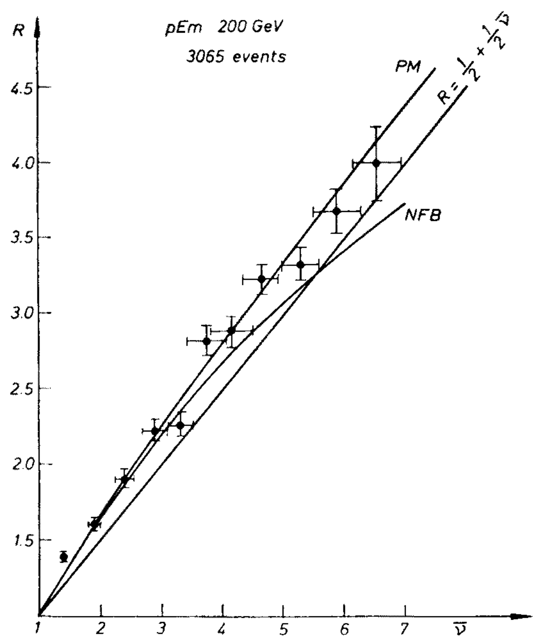

Fig. 1. The R vs $\bar{\nu}$ dependence for pEm interactions at 200 GeV

where σ_{hp} and σ_{hA} are inelastic (and incoherent) cross-sections of hadrons for interactions with protons and nuclei with mass number A.

Our target was emulsion. We used quite another method of calculating $\bar{\nu}$, which allowed us to reach much higher values of $\bar{\nu}$ than it is possible by means of formula (1). For pEm interactions we can reach $\bar{\nu} \approx 7$.

In our previous work [3] we concluded that the number of grey tracks N_g (mainly protons with energies from 30 to 380 MeV) produced in hadron-nucleus interactions can be used as a good measure of the number of collisions inside the nucleus. We found the $\bar{\nu}$ vs N_g dependence for pEm [3] and πEm [11] interactions and knowing the experimental dependence R vs N_g we could find the R vs $\bar{\nu}$ dependences. Mean number of collisions, presented in Fig. 1 and Fig. 2, are found by this method[1].

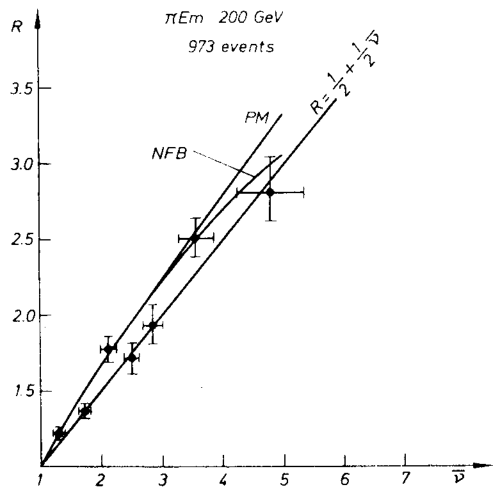

Fig. 2. The R vs $\bar{\nu}$ dependence for μEm interactions at 200 GeV

3. *Emulsion experimental data*

This work deals mainly with the data on multiplicities in the interactions of protons and pions with the nuclei of emulsion in the wide intervals of primary energies (from 6.2 up to ~3000 GeV for protons and from 7.5 to 200 GeV for pions). These data (Figs. 3 and 4) come from the works [12] and from the works cited by us previously in [2, 3, 10]. We show also in Figs. 3 and 4 the data obtained by electronic techniques (Busza et al. [13]) for pEm and πEm. Elementary multiplicities $\langle n \rangle_H$ which are used for calculating R (for pions and protons) are taken from the works [14] and from the works cited in [10].

It is well known that in emulsion experiments, although all charged particles are measured, only relativistic secondary particles (with $\beta > 0.7$) are included in the multiplicity n_s. Hence one cannot compare the values of $R_s = \langle n_s \rangle / \langle n \rangle_H$ with theoretical predictions concerning R (not R_s), as many authors do. The corrections for the missing slow particles which are not included in $\langle n_s \rangle$ should be introduced into the emulsion experimental data.

[1] It should be noticed that the curves R vs $\bar{\nu}$ calculated by means of identical calculations connected with a certain model may be different when using the formula (1) than when calculating $\bar{\nu}$ from the number of grey tracks (see [3]), because in these two methods the mean values $\bar{\nu}$ may be connected with different distributions of ν.

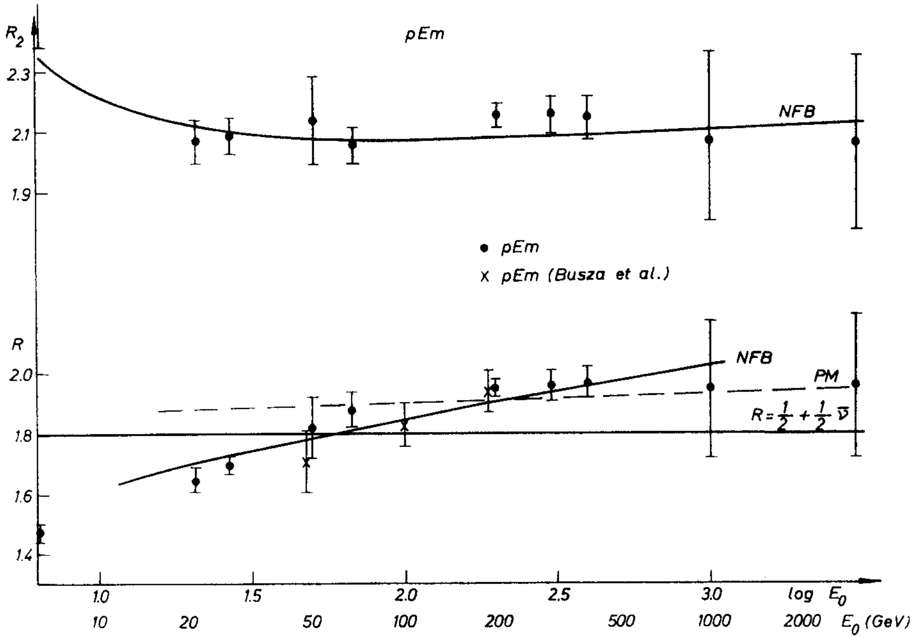

Fig. 3. The R vs E_0 and R_2 vs E_0 dependences for pEm interactions

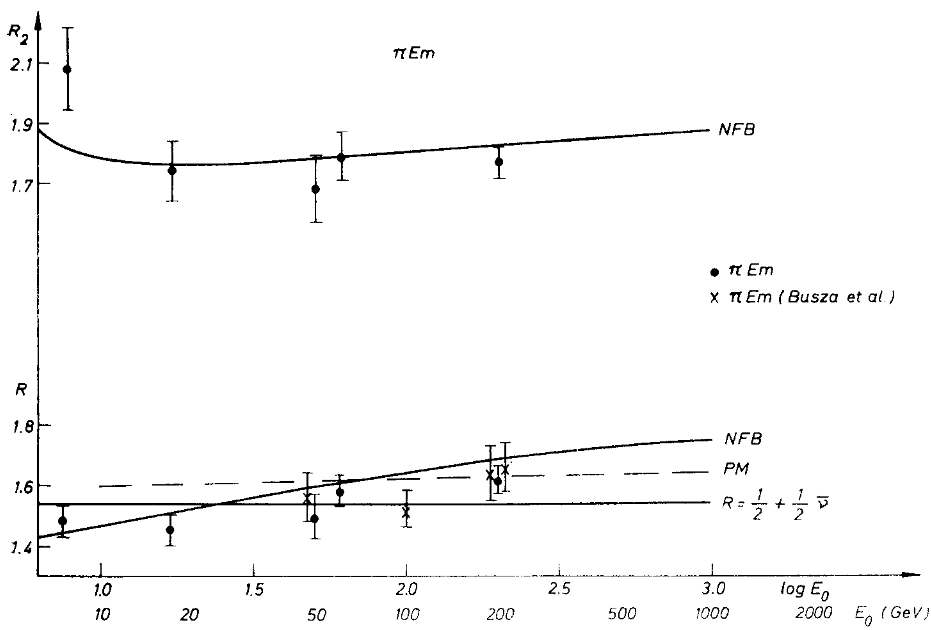

Fig. 4. The R vs E_0 and R_2 vs E_0 dependences for μEm interactions

The total mean multiplicity of charged particles in the interactions of hadrons with the nucleus of a mass number A is

$$\langle n\rangle_A = \langle n_s\rangle_A+\bar{\nu}_A(n_\pi+l_p n_p) = \langle n_s\rangle_A+\delta, \tag{2}$$

where $\bar{\nu}_A$ is the mean number of collisions in the target nucleus, l_p is the fraction of protons among the nucleons of this nucleus, n_p and n_π are the mean numbers of slow ($\beta < 0.7$) protons and pions in hadron-proton interactions[2].

The values of n_p and n_π were calculated for primary protons by Calucci et al. [15] on the basis of the experimental data from bubble chambers at hundreds GeV and from ISR. They are: $n_p = 0.48$, $n_\pi = 0.14$. We performed kinematical calculations for primary protons [3] and pions and we obtained a weak dependence of n_p on the primary energy: from 0.53 at $E_0 = 50$ GeV to 0.46 at $E_0 = 10000$ GeV in very good agreement with the results of Calucci et al.

$\bar{\nu}$, l_p and δ are of course different at different A. The values:

$$R = \frac{\langle n\rangle}{\langle n\rangle_H} = \frac{\langle n_s\rangle+\delta}{\langle n\rangle_H} = R_s+\frac{\delta}{\langle n\rangle_H} \tag{3}$$

can be compared with theoretical predictions.

For pEm interactions: $l_p = 0.455$, $\bar{\nu} = 2.59$ and $\delta = 0.93$, for πEm interactions: $l_p = 0.455$, $\bar{\nu} = 2.08$, $\delta = 0.75$ ($\bar{\nu}$ for pEm and πEm interactions used by us was calculated theoretically by M. Błeszyński [16]).

The correction for slow particles can also be made in another manner, without knowledge of $\bar{\nu}$. Namely we can subtract the number of slow particles from the elementary multiplicity in the denominator of the formula (3). Then we have:

$$R = \frac{\langle n_s\rangle}{\langle n\rangle_H-(n_p+n_\pi)} \approx \frac{\langle n_s\rangle}{\langle n\rangle_H-0.62}. \tag{4}$$

We stated that both methods of calculating R give the same results within the limits of errors and in this work we use formula (4). Formulas (2) and (3) can be used when assuming that we are dealing with subsequent independent interactions with nucleons inside the nucleus. The agreement of results obtained by means of formulas (3) and (4) indicates the correctness of such an assumption.

4. The R vs $\bar{\nu}$ dependence

In Figs. 1 and 2 the experimental dependences R vs $\bar{\nu}$ for pEm and πEm interactions at 200 GeV are presented. The values of $\bar{\nu}$ were obtained from the numbers of grey tracks by the method described in our work [3].

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[2] For simplicity in the following parts of this work the indices "A" used in formula (2) will be omitted and we shall write: $\langle n\rangle$, $\langle n_s\rangle$, $\bar{\nu}$.

For proton and pion interactions the experimental data can be approximated by the linear dependence $R = \alpha+\beta\bar{\nu}$, where $\alpha = 0.64\pm0.04$ and $\beta = 0.53\pm0.03$ for protons, and $\alpha = 0.53\pm0.07$ and $\beta = 0.51\pm0.04$ for pions.

The line $R = \frac{1}{2}+\frac{1}{2}\bar{\nu}$, which results from the model of excited states of nucleons [8] and from modifications of Gottfried's EFC model [7, 15], agrees very well with experimental data for πEm interactions but for pEm interactions it is difficult to consider the agreement good although the line is close to experimental data.

The curves PM are in very good agreement with the data for pEm and in a satisfactory agreement with the data for πEm interactions. These curves were calculated by us by means of the formula of Brodsky et al. [9] which results from the parton model of hadron-nucleus interactions, in which the wee partons of the projectile interact with the wee partons of different nucleons during the passage of the projectile through the nucleus. Brodsky et al. compared their curves R vs $\bar{\nu}$ with the data of Busza et al. [13] (different targets, E_0 from 50 to 200 GeV, data for primary protons and pions collected in the same figure) and also stated a good agreement, of course only for $\bar{\nu} \lesssim 4$ (this corresponds to the interaction p-U)[3].

Now we would like to say more about the model of non-interacting fireballs (NFB). Predictions of this model (NFB curves in Figs. 1 and 2) are also in sufficiently good agreement with experimental data.

5. *Model of non-interacting fireballs* (*NFB*)

At present this model can be formulated as follows [6]:

1. The primary nucleon traverses the nucleus interacting with nucleons of this nucleus in subsequent collisions. In these interactions only clusters (fireballs) are produced and the nucleon loses the fraction K of its energy in each interaction (K — inelasticity coefficient).

2. Clusters produced in the forward hemisphere (in the nucleon-nucleon CM system) do not interact in the nucleus and decay outside it.

3. Clusters produced in the backward hemisphere decay mostly inside the nucleus. The products of these decays may interact in the nucleus and then they interact mostly elastically. They produce only a small number of additional particles but they can shift the maximum of the pseudorapidity distribution of secondary particles in the direction of large angles.

For comparison with experimental data the values of the normalized multiplicity R were calculated for different E_0 and different $\bar{\nu}$ on the basis of the assumptions of the NFB model[4]. The calculations were performed much more accurately than those in work [2]. In the present work we have taken into account the distributions of ν in the "mean"

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[3] Another model of particle production in the hadron-nucleus interactions which also takes into account the structure of the hadron ("constituent quark model") was proposed by Białas et al. [19]. The predictions of this model deal with the central part of the rapidity distribution in the hadron-nucleus interaction.

[4] Although the NFB model was proposed for primary protons, we performed the calculations also for primary pions, treating the primary pion as a "leading" particle.

nucleus of emulsion[5] at different N_g and the distribution of the inelasticity coefficient K. Namely we assumed that the distribution of K in the CM-system of colliding particles is rectangular with $\langle K \rangle = 0.5$ (this seems to be reasonable, although the experimental information about the distribution of K is spare [17]).

Multiplicities in proton and pion interactions with the nucleus at the primary energy E_0 for ν collisions could be calculated as a sum of elementary multiplicities $\langle n \rangle_H$, assuming that in each following interaction the energy is $(1-K)$ times smaller than in the preceding interaction and that

$$\langle n \rangle_H = a + b \cdot \log E + c(\log E)^2. \tag{5}$$

The coefficients in formula (5) have been found by fitting the experimental data concerning pp and πp interactions in the bubble chamber and ISR (for pp at the energies $4 \leqslant E \leqslant 2000$ GeV and for πp at the energies $4 \leqslant E \leqslant 360$ GeV).

In our calculations the fact that the mean multiplicity in pn (πn) interactions is a little smaller than in pp (πp) interactions at the same energy has also been taken into account.

The agreement of the curve R vs $\bar{\nu}$ calculated on the basis of the NFB model with experimental points for pEm interactions (Fig. 1) is good for $\bar{\nu} < 6$, for larger $\bar{\nu}$ the NFB curve lies slightly below the experimental points.

If one assumes the validity of the NFB model it may be supposed that the decay products of slow intermediate states (fireballs, clusters) which are decaying inside the nucleus can interact inside the nucleus. This effect has not been taken into account in our calculations and this might explain the disagreement between the NFB curve and experimental points for large $\bar{\nu}$. However, if it was desired to take into account this effect it would be necessary to make some assumptions concerning the physical properties of the intermediate states produced in elementary interactions.

For πEm interactions (Fig. 2) the agreement of the NFB curve with experimental points can be accepted as sufficient.

6. *The R vs E_0 dependence*

In Figs. 3 and 4 the experimental values of the normalized multiplicity for pEm interactions (from 6.2 up to ~3000 GeV) and for πEm interactions (for $7.5 \leqslant E_0 \leqslant 200$ GeV) are presented. The total number of events for pEm interactions available to us was ~25000 (e.g. 3948 events at 200 GeV). For πEm interactions we had a smaller number of jets: only ~9000 (e.g. 1892 events at 200 GeV). The points obtained by electronic

[5] We would like to explain what we have in mind when we say: "mean" nucleus of emulsion. Namely the calculations of Błeszyński [16, 20] concerning the distributions of the number of collisions ν of the primary particle in the emulsion which is a mixture of some elements have been made in the following manner: at first the distributions of the number of collisions for all constituents of the emulsion were calculated and afterwards these distributions have been added with different weights which are connected with the chemical composition of the emulsion and with the cross-sections for the interaction of the primary particle with the nucleus of each constituent of the emulsion. In a similar manner one can evaluate the average mass number for the emulsion: $\langle A \rangle_{Em} = 72$.

techniques for pEm and πEm interactions (Busza et al. [13]) are also shown. The good agreement of emulsion points to those of Busza et al. is seen.

In Figs. 3 and 4 the curves R vs E_0 derived from some of the models mentioned above are also drawn. Very good agreement with experimental points is seen for the parton model (PM) and also for the NFB model[6], but if it was wished to decide for which model the agreement is better, the experimental data for much higher energies ($\gtrsim$ 1000 GeV) with large statistics ($\sim$4000 events or so for one experimental point) would be necessary.

7. *The normalized multiplicity of "created" particles*

In some theoretical works (e.g. Białas and Czyż [18]) the authors are interested not in the total multiplicity of charged particles $\langle n \rangle$ but only in the multiplicity of particles which have been "created" in the interactions. Let us denote this multiplicity by $\langle n_2 \rangle$. Strictly speaking, $\langle n_2 \rangle$ which will be used in what follows means the average multiplicity of charged particles minus the number of charged particles which participated in interactions.

For p(π)-nucleus interactions:

$$\langle n_2 \rangle = \langle n \rangle - (1+\bar{v} l_p) = \langle n_s \rangle - [1+\{(1-n_p) l_p - n_\pi\} \bar{v}] = \langle n_s \rangle - (1+\delta_2). \qquad (6)$$

For emulsion: $\delta_2 = 0.25$ for pEm and $\delta_2 = 0.20$ for πEm. The normalized multiplicity of "created" particles

$$R_2 = \frac{\langle n_2 \rangle}{\langle n_2 \rangle_H} = \frac{\langle n_s \rangle - (1+\delta_2)}{\langle n \rangle_H - 2}. \qquad (7)$$

In Figs 3 and 4 the experimental values of normalized multiplicities of secondary particles "created" in pEm and πEm interactions (R_2) calculated by means of formula (7) are also presented. It appears that just R_2 (not R) does not depend on the energy and this independence is valid in a very wide interval from a few GeV up to some thousands of GeV. This property is also predicted by the NFB model because the curves R_2 which are almost straight lines resulting from NFB calculations agree quite well with the experimental points.

The independence of the normalized multiplicity of "created" particles R_2 of the primary energy may, from the physical point of view, be of great significance, as R_2 may be the universal parameter of the multiplication of particles in p(π)-nucleus interactions. Even if it is not so, at any rate R_2, because of its independence of primary energy, is a convenient quantity which permits us to gather experimental data obtained at different energies and compare them with each other.

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[6] In calculations of R vs E_0 curves for the NFB model the fact that the cross section σ_{in} changes with the diminishing of energy in subsequent collisions of the primary particle in the nucleus has been taken into account. This correction is rather small for pEm interactions and it can be neglected for πEm interactions.

8. *Conclusions*

The analysis of interactions of protons at primary energies from 6.2 to ~3000 GeV and pions at primary energies from 7.5 GeV to 200 GeV with nuclei of emulsion was performed. The dependences of the average normalized multiplicity of secondary charged particles R on the primary energy and on the mean number of collisions in the nucleus for pEm and for πEm interactions can be described with sufficient accuracy by means of a simple model of non-interacting fireballs (NFB) which was introduced many years ago in investigations of cosmic ray jets (Mięsowicz [5]). The above mentioned experimental dependences are also well described by the parton model in the version of Brodsky et al. [9]. The independence of normalized multiplicity of charged particles "created" in interaction (R_2) of the primary energy in a wide interval from a few GeV to some thousands of GeV was stated.

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