LETTERS TO THE EDITOR

NOTE ON THERMODYNAMICS OF COSMOLOGICAL EVENT HORIZONS

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We discuss thermodynamical quantities related with cosmological event horizons and show that they possess some uncomfortable properties.

Recently Hawking [1-3] proposed to apply thermodynamical notions to describe stationary black holes. It turns out that one can assign to a stationary black hole a temperature and entropy and relate them to the surface gravity and area of the black hole event horizon. Gibbons and Hawking [4] extended that close connection between event horizons and thermodynamics, found in the case of black holes, to cosmological models with a positive (repulsive) cosmological constant. They analyzed de Sitter model and Schwarzschild-de Sitter space-time and have shown that an observer in these models will have an event horizon whose area can be interpreted as the entropy or lack of information of the observer about the regions which he cannot see. With the cosmological event horizon one can associate a surface gravity which appears in extended first law of event horizons in a manner similar to that in which temperature appears in the first law of thermodynamics.

Here we would like to concentrate on the Schwarzschild -de Sitter model and show that the entropy connected with the cosmological event horizon possesses some uncomfortable properties. We take the metric of the Schwarzschild-de Sitter space-time in its standard form

$$ds^2 = \left(1-\frac{2M}{r}-\tfrac{1}{3}\Lambda r^2\right)dt^2 - \frac{dr^2}{1-\frac{2M}{r}-\frac{1}{3}\Lambda r^2} - r^2(d\theta^2+\sin^2\theta d\varphi^2); \tag{1}$$

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therefore, positions of horizons are given by positive roots of the cubic equation

$$3r-6M-\Lambda r^3 = 0. \tag{2}$$

From observations it is known that $\Lambda < 10^{-54}\ \text{cm}^{-2}$ so we can treat the Λ term as a perturbation. Following notation used by Gibbons and Hawking let r_+ and r_{++} denote correspondingly the black hole and cosmological event horizons, then keeping only the leading terms in Λ we get

$$r_+ = 2M+\tfrac{1}{3}(2M)^3, \tag{3}$$

$$r_{++} = \sqrt{\frac{3}{\Lambda}}-M. \tag{4}$$

The area of the black hole horizon A_{BH} and the cosmological event horizon A_{C} are therefore

$$A_{\text{BH}} = 4\pi(2M)^2\left(1+\frac{2\Lambda}{3}(2M)^2\right), \tag{5}$$

$$A_{\text{C}} = 4\pi\sqrt{\frac{3}{\Lambda}}\left(\sqrt{\frac{3}{\Lambda}}-2M\right). \tag{6}$$

One can define black hole and cosmological surface gravities κ_{BH} and κ_{C} by

$$K_{a;b}k^b = \kappa k_a \tag{7}$$

on the horizon, where k_a is the unique time-like Killing vector which is null on both horizons. In our case the surface gravities are

$$\kappa_{\text{BH}} = \frac{1}{4M}\left(1-\frac{\Lambda}{3}13M^2\right), \tag{8}$$

$$\kappa_{\text{C}} = \sqrt{\frac{\Lambda}{3}}\left(1-2M\sqrt{\frac{\Lambda}{3}}\right). \tag{9}$$

Area and surface gravity of the horizon are related with entropy and temperature by simple relations

$$S = \frac{1}{32\pi}A, \qquad T = 4\kappa, \tag{10}$$

where we adopt units in which $G = c = \hbar = 8\pi k = 1$. The entropy of the black hole is interpreted as a measure of lack of information on state of matter enclosed by the event horizon. According to Gibbons and Hawking, entropy related with the cosmological horizon measures lack of information of the observer about the regions which he cannot see.

It was noticed by Kundt [5] that even in adiabatic collapse of spherically symmetric distribution of matter entropy of the configuration increases by a factor $\sim 10^{18}$ when black hole forms. Bekenstein [6] argues that this gain in entropy of black hole could be understood in the framework of relation between thermodynamics and information theory. On the other hand Hawking [7, 8] has shown that even a static black hole is a source of thermal radiation of temperature given by (10) implying that its entropy is proportional to the area of the horizon and establishing its real thermodynamical meaning.

Let us first notice that entropy and temperature of the cosmological horizon does not satisfy the third law of thermodynamics since when $\Lambda \to 0$, $T \to 0$ but $S \to \infty$. It was noticed by Davies [9] that extreme Kerr–Newman black holes also violate the third law of thermodynamics but in this case the zero point entropy is finite and it has clear physical meaning being a measure of number of different microscopic configurations which could form the same extreme Kerr–Newman black hole. How one should interpret violation of the third law of thermodynamics in the cosmological case is not clear. Even the de Sitter model has this property and since in this case in the limit $\Lambda \to 0$, $S \to \infty$ and one recovers the complete Minkowski space. This seems to contradict the interpretation of entropy as a measure of lack of information of the observer about the regions which he cannot see. The stationary observer in the Minkowski space can see all of it.

Here one should add a word of caution. In general, the procedure of taking limits on certain number of parameters which appear in the metric does not lead to a unique space–time. However using the standard de Sitter line element in the spherical coordinates and taking the limit $\Lambda \to 0$, without changing coordinates, leads globally to Minkowski space.

Calculating temperature and entropy for the future null boundary $\mathscr{I}^+$ of the Minkowski space we get $\kappa = 0$ so temperature is zero but surface area is infinite so from (10) it follows that the entropy is infinite too. According to the standard interpretation [10] $\mathscr{I}^+$ is the absolute horizon for all physical observers and there is nothing left beyond.

It is clear from these examples that the interpretation of the surface area of the observers horizon as a measure of lack of information about the region which he cannot see is questionable. It is also doubtful if the thermodynamical quantities could be meaningfully related with cosmological horizons.

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