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# SPECTRUM LAW FOR LEPTONS AND QUARKS

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A new approach to lepton and quark spectra, proposed recently, is further developed, in particular for quarks. The existence of elementary black holes of lepton and quark types with spin 1/2 is implied.

In a recent paper, a new approach to lepton and quark spectra was proposed on the base of some difference equations for mass and charge [1]. The mass and electric charge of leptons and quarks were described as scalar „fields" $m(n)$ and $Q(n)$ defined on a discrete space of nonnegative integers $n = 0, 1, 2, \ldots$, and then they were conjectured to satisfy a coupled system of two difference equations of the second order,

$$[(\Delta+1)^2-\lambda^2]m(n) = \varepsilon Q^2(n) \tag{1}$$

and

$$[(\Delta+1)^2-1]Q(n) = 0, \tag{2}$$

where

$$\Delta f(n) = f(n+1)-f(n), \tag{3}$$

whilst $\lambda$ and $\varepsilon$ denoted universal constants, at least for leptons and quarks separately. So Eqs. (1) and (2) contained in principle four constant parameters, $\varepsilon$ and $\lambda$ for leptons and $\varepsilon$ and $\lambda$ for quarks, and, being difference equations of the second order, they implied in addition four integration constants to be determined from the physical boundary conditions imposed on $m(0)$, $\Delta m(0)$ and $Q(0)$, $\Delta Q(0)$. We were inclined, however, to consider the pure number $\lambda$ as a new fundamental constant of nature which is equal for leptons and quarks, while the mass parameters $\varepsilon$ for leptons and quarks, being mass scales, may be different.

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On the spectrum of leptons $l(n)$ and quarks $q(n)$ we imposed the physical boundary conditions, $l(0) = \nu_e$, $l(1) = e^-$ and $q(0) = u$, $q(1) = d$. Then, we obtained the spectra of leptons,

$$l(2n) \equiv \nu_n = \nu_e, \nu_\mu, \nu_\tau, \ldots,$$

$$l(2n+1) \equiv l_n^- = e^-, \mu^-, \tau^-, \ldots \; (n = 0, 1, 2, \ldots), \tag{4}$$

where

$$m(2n) = 0, \qquad Q(2n) = 0,$$

$$m(2n+1) = \left(m_e + \frac{\varepsilon}{\lambda^2 - 1}\right)\lambda^{2n} - \frac{\varepsilon}{\lambda^2 - 1}, \qquad Q(2n+1) = -1 \tag{5}$$

and for quarks,

$$q(2n) \equiv u_n = u, c, (t), \ldots$$

$$q(2n+1) \equiv d_n = d, s, b, \ldots \qquad (n = 0, 1, 2, \ldots), \tag{6}$$

where

$$m(2n) = \left(m_u + \tfrac{4}{9}\frac{\varepsilon}{\lambda^2 - 1}\right)\lambda^{2n} - \tfrac{4}{9}\frac{\varepsilon}{\lambda^2 - 1}, \qquad Q(2n) = \tfrac{2}{3},$$

$$m(2n+1) = \left(m_d + \tfrac{1}{9}\frac{\varepsilon}{\lambda^2 - 1}\right)\lambda^{2n} - \tfrac{1}{9}\frac{\varepsilon}{\lambda^2 - 1}, \qquad Q(2n+1) = -\tfrac{1}{3}. \tag{7}$$

In the case of leptons we determined the parameters $\lambda$ und $\varepsilon$ using the masses $m(3) \equiv m_\mu$ and $m(5) \equiv m_\tau = 1782^{+2}_{-7}$ MeV [2,3]. Then we got

$$\lambda = 3.993^{+0.004}_{-0.009} \simeq 4, \qquad \varepsilon = 97.51^{+0.01}_{-0.04}\ \text{MeV}. \tag{8}$$

The predicted masses for two next lepton pairs $(\nu_3, l_3^-)$ and $(\nu_4, l_4^-)$ were

$$m_{\nu_3} = 0, \qquad m_{l_3} = 28.5^{+0.2}_{-0.5}\ \text{GeV},$$

$$m_{\nu_4} = 0, \qquad m_{l_4} = 455^{+3}_{-10}\ \text{GeV}. \tag{9}$$

So, we can see that this theory predicts an *infinite sequence of lepton pairs* $(\nu_n, l_n^-)$ $(n = 0, 1, 2, \ldots)$ consisting of massless neutrinos $\nu_n$ and massive charged leptons $l_n^-$ with rapidly growing masses like $\exp(n \ln \lambda) = \exp(1.384n)$. The discovery of new leptons, in particular the charged lepton $l_3^-$ or its antiparticle with the mass $\simeq 28.5$ GeV, would be crucial for testing out theory.

In the case of quarks, two free parameters $\lambda$ and $\varepsilon$ (or only $\varepsilon$ if $\lambda$ is equal for leptons and quarks) cannot be determined with equal precision since at present the quark masses are not known well enough (and, moreover, the notion of quark mass itself is rather ambiguous). In any case, this theory predicts an *infinite sequence of quark pairs* $(u_n, d_n)$ $(n = 0, 1, 2, \ldots)$ with infinitely growing masses.

Notice now that the mass formulae (7) imply the following relations for quark masses:

$$\frac{m_t - m_c}{m_c - m_u} = \frac{m_b - m_s}{m_s - m_d} = \frac{4m_s - m_c}{4m_d - m_u} = \lambda^2 \tag{10}$$

and

$$m_t - m_c\lambda^2 = 4(m_b - m_s\lambda^2) = m_c - m_u\lambda^2 = 4(m_s - m_d\lambda^2) = \tfrac{4}{9}\,\varepsilon. \tag{11}$$

If $m_c \gg m_u$ and $m_b \gg m_d$ we get from the first and second relations (10)

$$\frac{m_t}{m_c} \simeq \frac{m_b}{m_s - m_d} \simeq 1 + \lambda^2. \tag{12}$$

Then the first relation (11) gives us

$$m_c \simeq \tfrac{4}{9}\,\varepsilon. \tag{13}$$

If $\lambda$ is equal for leptons and quarks, then taking for the charmed quark c its effective constituent mass $m_c \simeq 1.65$ GeV deducted from the charmonium spectrum [4] we predict that

$$m_t \simeq (1+\lambda^2)m_c \simeq 28.0\,\text{GeV}. \tag{14}$$

The estimation for $\varepsilon$ is now

$$\varepsilon \simeq \tfrac{9}{4}\,m_c \simeq 3.71\,\text{GeV}. \tag{15}$$

Eventually, from other relations (11) we obtain

$$m_u \simeq 0, \qquad m_d \gtrsim 0, \qquad m_s \gtrsim \tfrac{1}{9}\,\varepsilon \simeq 0.413\,\text{GeV},$$

$$m_b \gtrsim \tfrac{1}{9}\,\varepsilon(1+\lambda^2) \simeq 6.99\,\text{GeV}. \tag{16}$$

Since the bound states $\varphi = s\bar{s}$, $\psi = c\bar{c}$, and $\Upsilon = b\bar{b}$ have masses $m_\varphi = 1.0$ GeV, $m_\psi = 3.1$ GeV and $m_\Upsilon = 9.4$ GeV, we can interprete the result (16) as an indication that the heavier is the quark the larger is the role played by the Coulomb-like attraction giving negative binding energy (in comparison with the confining attraction which gives a positive contribution to the bound state mass). The negative binding energy seems to be quite relevant already for $b\bar{b}$ and should be even more important for $t\bar{t}$ (in spite of the effective asymptotic freedom).

Notice also that if $m_u \ll (4/9)\varepsilon/(\lambda^2-1)$ and $m_d \ll (1/9)\varepsilon/(\lambda^2-1)$ the mass formulae (7) give the approximate relations

$$m(2n) \simeq 4m(2n+1) \qquad (n > 0) \tag{17}$$

i.e., in particular for $n = 1$ and $n = 2$,

$$m_c \simeq 4m_s, \qquad m_t \simeq 4m_b. \tag{18}$$

Relations (18), found already in Ref. [5] on a different ground, are consistent with the estimation (14) and (16). If $m_b$ were about 5 GeV, $m_t$ would be predicted about 20 GeV. In any case, the mass of the bound state $t\bar{t}$ might be about 36–40 GeV.

In our theory a fundamental question arises whether masses of leptons and quarks can really increase to infinity with growing $n$. At some critical $n_{cr}$ their Compton wave length becomes smaller than their Schwarzschild gravitational radius. Then they start to play the role of *elementary black holes* with spin 1/2. The condition for such a critical $n_{cr}$ is provided by the equality

$$\frac{\hbar}{m_{cr}c} = \frac{2Gm_{cr}}{c^2} \tag{19}$$

or

$$m_{cr} = \sqrt{\frac{\hbar c}{2G}} = 8.6345 \times 10^{18}\,\mathrm{GeV}, \tag{20}$$

where $G = 6.6720 \times 10^{-8}\,\mathrm{cm}^3\,\mathrm{g}\,\mathrm{sec}^{-2}$ is the gravitational constant, while $m_{cr} = m(2n_{cr}+1)$ for leptons and $m_{cr} = m(2n_{cr})$ for quarks are given by Eqs. (5) and (7). From Eq. (20), using the values (8) and (15), we get

$$n_{cr} = 17.5, \qquad n_{cr} \simeq 16.5 \tag{21}$$

for leptons and quarks, respectively. This estimation implies that there are only 18 charge leptons $l_n^-$ $(n = 0, 1, 2, ..., 17)$, 17 up-quarks $u_n$ and 17 down-quarks $d_n$ $(n = 0, 1, 2, ..., 16)$, not being black holes. Of course, none of the neutrinos $\nu_n$ are black holes. The estimated mass of the highest lepton and the highest quark not being a black hole is

$$m(35) = 1.96 \times 10^{18}\,\mathrm{GeV} \tag{22}$$

and

$$m(32) \simeq 1.93 \times 10^{18}\,\mathrm{GeV}, \tag{23}$$

respectively.

Notice finally that the number of quark flavours appearing beneath the estimated critical number $n = 16$ is $2(16+1) = 34$. Thus, it is approximately twice as large as the number 16 which is the perturbative upper bound for the number of quark flavours allowed by the requirement of the exact asymptotic freedom in quantum chromodynamics.

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## REFERENCES


[1] W. Królikowski, *Difference Equations for Mass and Charge*, Warsaw University preprint IFT/2/79, April 1979.

[2] W. Bacino et al., *Phys. Rev. Lett.* **41**, 13 (1978).

[3] For a recent review of the heavy lepton phenomenology cf. M. L. Perl, SLAC-PUB-2219 (1978).

[4] For a review cf. K. Gottfried, in Proc. 1977 Int. Symposium on Lepton and Photon Interactions at High Energies, Hamburg 1977, Ed. by F. Gutbrod, DESY, 1977.

[5] G. Preparata, TH-2599-CERN (1978) and references therein.