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ON TWO NUCLEON SOLITARY WAVE EXCHANGE POTENTIALS: ADDENDA

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Simplifications in previously derived nucleon-nucleon solitary exchange potentials are obtained for potentials arising from field theories with self interaction currents of the form $\lambda_1\phi^{2q+1}+\lambda_2\phi^{4q+1}$.

In several recent publications [1–3] nucleon-nucleon potentials have been derived from quantum field theories with intrinsically nonlinear, solitary wave excitations. These solitary wave exchange potentials (SWEP) resemble phenomenological boson exchange potentials such as the Reid potential [4] but have the advantage that only a few free parameters appear in the potentials. Thus, the low energy scattering data can be described with very few parameters. Furthermore, using the SWEP derived in [3], the binding energy of ^{4}He has been calculated to be $\simeq$ 12–13 MeV [5]. In this note we extend the results of [3] so that explicit forms of the potential are obtained for self interacting meson currents of the form

$$j = \lambda_1\phi^{2q+1}+\lambda_2\phi^{4q+1}, \tag{1}$$

where λ_1 and λ_2 are arbitrary constants and $q \neq 0, -1/2, -1$.

From [3], Eq. (3.2), the general expression for the solitary wave exchange potential for nucleon-nucleon scattering in the nonrelativistic approximation is

$$V^{\text{SWEP}}_{\lambda_1\lambda_2 q}(r) = (2\pi)^{-3}(g/2M)^2\sigma_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}\vec{\tau}_1\cdot\vec{\tau}_2\sum_{n=0}^{\infty}\Gamma(2qn+2)\,(2qn+1)^{2qn-2}[\zeta_q/V^q]^{2n}$$
$$\times[C_n^{(1/2q)}(\xi_q)]^2\int e^{i\vec{k}\cdot\vec{r}}d^3k[\vec{k}^2+m_{qn}^2]^{-qn-1}, \tag{2}$$

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where a misprint in the original form has been corrected. The improvement consists of an evaluation of the integral for general q. One has

$$\int e^{i\vec{k}\cdot\vec{r}}d^3k[\vec{k}^2+m_{qn}^2]^{-qn-1}$$
$$= 4\pi^{3/2}2^{-qn-1/2}(m_{qn})^{-2qn+1}(m_{qn}r)^{qn-1/2}K_{qn-1/2}(m_{qn}r)/\Gamma(qn+1), \quad (3)$$

where

$$\mathrm{Re}\,(qn+1/2) > -1/2, \quad (4)$$

$$|\arg m_{qn}r| < \pi/2, \quad (5)$$

where $K_{qn-1/2}(m_{qn}r)$ is a Bessell function of the second kind of imaginary argument [6]. Using the first two identities following Eq. (3.4) of [3] (it should be remarked that the first of these is valid only when the operators $\vec{\sigma}_1\cdot\vec{\nabla}\vec{\sigma}_2\cdot\vec{\nabla}$ are followed by functions of r) and the results of Eqs (3)–(5) the solitary wave exchange potential becomes

$$V^{\mathrm{SWEP}}_{\lambda_1\lambda_{2q}} = \vec{\tau}_1\cdot\vec{\tau}_2(g/2M)^2\frac{(\pi/2)^{1/2}}{(6\pi^2)}\sum_{n=0}^{\infty}\frac{\Gamma(2qn+2)(2qn+1)2^{qn-2}(\zeta_q/V^q)^{2n}[C_n^{(1/2q)}(\xi_q)]^2}{\Gamma(qn+1)2^{qn}(m_{qn})^{2qn-3}}$$
$$\times\{\vec{\sigma}_1\cdot\vec{\sigma}_2[x_{qn}^{qn-1/2}K_{qn-5/2}(x_{qn})-3x_{qn}^{qn-3/2}K_{qn-3/2}(x_{qn})]+S_{12}x_{qn}^{qn-1/2}K_{qn-5/2}(x_{qn})\}, \quad (6)$$

where

$$x_{qn} = m_{qn}r = (2qn+1)mr. \quad (7)$$

At this point we note that Eqs (3.4), (4.1a), (4.1b) and (4.2) of [3] contain factors ζ_q/r^q, λ_1/r^q and λ_2/r^q. The correct expressions are ζ_q/V^q, λ_1/V^q and λ_2/V^{2q}. In addition, there should be + between e^{-3x}/x and 75/4 of Eq. (4.3).

With the general expression for $V^{\mathrm{SWEP}}_{\lambda_1\lambda_{2q}}$ of Eq. (6), valid for $r > 0$, and the repulsive delta function potential at the origin [1], it is possible to find a wide range of possible potentials, including "super" soft core [5]. A case of particular interest occurs for $q = 3/2$, for which, for $r > 0$, the leading Yukawa term ($n = 0$) is modified, for $n = 1$, by an attractive term which has only a logarithmic singularity as $r \to 0$. This is to be contrasted to the $q = 1$ case [3] where an additional Yukawa repulsion occurs for $n = 1$. Thus, the $q = 3/2$ case, for which the corresponding field theory predicts a reasonable mass relation for spin zero mesons [7], can also be expected to lead to better binding energies for systems such as ^{4}He. Work on this problem is in progress.

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