CONSTITUENT QUARK MODEL AND PRODUCTION OF PARTICLES FROM NUCLEAR TARGETS*

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Assuming an independent particle production from constituent quarks, the A-dependence of the spectra of leading and produced particles outside of the target fragmentation region are discussed. The energy and momentum conservation constraints are estimated. The comparison with the data on nucleon and pion induced reactions shows good agreement with the model. Multiplicities in nucleus-nucleus interactions are also computed and compared with cosmic ray data.

1. Introduction

In this paper we continue the investigation of low-$p_\perp$ particle production from nuclear targets outlined in Ref. [1]. In this approach the mechanism of production explicitly involves the internal structure of hadrons. Assuming that the incident hadron, h, is composed of N_h constituents, c, it was shown in [1] that, in the limit of infinitely high energy, there is the following relation between the plateau heights of nuclear and hydrogen targets

$$\lambda_A = \frac{\bar{\nu}_{hA}}{\bar{\nu}_{cA}} \lambda_{\mathrm{H}}. \tag{1.1}$$

In this formula: λ_{H} is the density in rapidity distribution of particles produced in hadron h-hydrogen (H) collisions, and λ_A is the same density in hadron h-nucleus (A) collisions.

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The average number of inelastic non-diffractive collisions in h-A and c-A interactions is denoted by $\bar{\nu}_{hA}$ and $\nu_{cA}^{|}$, respectively:

$$\bar{\nu} = \frac{A\sigma_H}{\sigma_A}, \tag{1.2}$$

with σ_H (σ_A) being the inelastic nondiffractive cross section on hydrogen (nucleus A).

It was shown in Ref. [1] that λ_A of (1.1) is sensitive to the number of constituents N_h. A comparison [2] of the density in the central rapidity region for 200 GeV proton-nucleus interactions with λ_A computed from (1.1) indicates that $N_h = 3$ is favoured by the data. These results suggest an identification of these three constituents with the three constituent quarks and encourages one to pursue this approach in more details.

In the present paper we discuss (a) the finite energy corrections to (1.1) coming from energy and momentum conservation, (b) the phase space effects in the projectile fragmentation region, and (c) the data on proton-nucleus, pion-nucleus and nucleus-nucleus interactions.

Our conclusions are: (*i*) the finite energy corrections in the central region of rapidities are small above ~200 GeV and thus Eq. (1.1) approximates well the data in this region, (*ii*) the phase space effects in the projectile fragmentation region are important and, in fact, to a large extent, the shape of the spectra is determined by the height of the central plateau and these phase space effects. A discussion of the remaining part of the spectrum, the target fragmentation region, requires an essential extension of the model and is not considered in this paper.

2. *Constituent quark model and multiparticle production from nuclear targets*

Let us resume the arguments of Ref. [1] applied to the constituent quark model. It is assumed that: (*i*) a hadron is made of N_h constituent quarks which, independently from each other, produce particles during a collision, (*ii*) an inelastically interacting constituent quark of high energy produces, on the average, the same number of particles, independently of the target with which the interaction takes place. These assumptions are expected to be valid in the central region of rapidities provided the energy available for production is high enough. Under these conditions the particle production in hadron-nucleus and hadron--nucleon collisions looks as shown in Fig. 1. According to our assumption (*ii*) the number of produced particles in the central region is proportional to the number of constituent quarks which interacted with the target. As seen in Fig. 1 this number is, on the average, larger for heavy nucleus than for hydrogen target. It can be calculated using the formulae of Ref. [3] and, eventually, one obtains the formula (1.1) for the multiplicity in the central region [1].

There are several reason which limit the applicability of this simple picture. The energy and momentum conservation makes impossible a strict independence of production from different constituent quarks. Though this effect is weak in the central region at high energies, it becomes very important in the projectile fragmentation region [4]. Thus in this region the formula (1.1) cannot be valid and appropriate corrections must be introduced. In the present paper we estimate the corresponding phase space corrections. At the other end of the rapidity distribution, in the target fragmentation region, particle production is clearly target-dependent and consequently the assumption (*ii*) must be violated. To describe production in this region one needs an additional mechanism of production of slow (in the laboratory) particles [5]. This will not be discussed in the present paper.

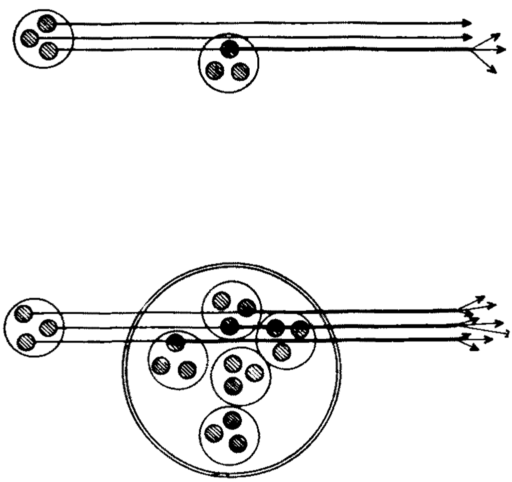

Fig. 1. Nucleon-nucleon and nucleon-nucleus collisions in the constituent quark model

The projectile fragmentation region was discussed already in Ref. [6]. It was pointed out that the A-dependence of the composition of particles produced in this region and, in particular, the meson/baryon ratio is a sensitive test of the constituent quark model. This test is satisfied by the low energy data [6].

In the present paper we attempt to estimate the A-dependence of the *shape* of the rapidity spectrum of produced and leading particles, assuming that the main factor which determines the *shape* of the spectrum in the projectile fragmentation region is the constraint of energy and momentum conservation. The comparison with the data, presented in Section 3 and 4, shows that indeed this seems to be the case. We also find that these phase space corrections are appreciably different for particles of different masses. This may affect the simple predictions of Ref. [6].

3. Single-particle spectra in nucleon-induced reactions

To obtain the spectrum of produced particles we applied the formula (1.1) for the rapidity density in the central region and corrected it for the effects of energy and momentum conservation using the uncorrelated cluster production model as described in the Appendix.

The ratio

$$R_A(y) = \frac{1}{\sigma_A}\frac{d\sigma_A}{dy}\bigg/\frac{1}{\sigma_{\rm H}}\frac{d\sigma_{\rm H}}{dy} = \frac{dn_A}{dy}\bigg/\frac{dn_{\rm H}}{dy}, \tag{3.1}$$

resulting from these calculations is shown in Fig. 2 for $A = 72.4$ and incident energy 200 GeV. This is compared with the emulsion data [7] of 200 GeV incident protons[1]. The theoretical uncertainty seen in Fig. 2 is caused by uncertainties in the values of the cluster masses and transverse momenta. They are explained in detail in the Appendix.

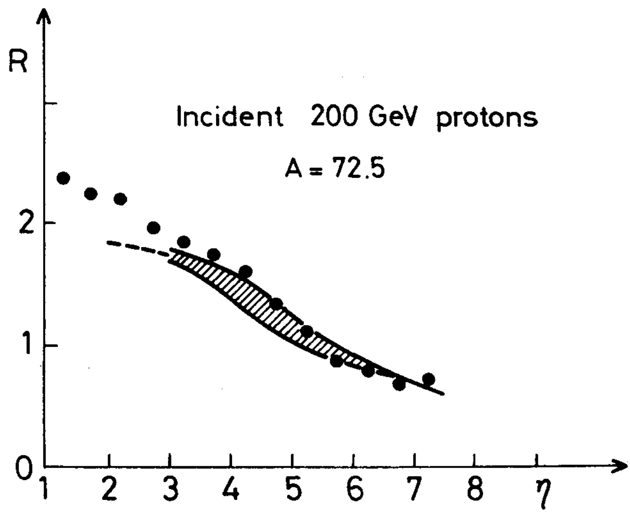

Fig. 2. Multiplication of particles in proton emulsion interactions at 200 GeV. Data from Ref. [7]. The largest errors are at the largest rapidities: ~ 15%

The corrections implied by the decay of clusters are neglected. They might be important for the largest rapidities.

The A-dependence of the spectrum is shown in more detail in Fig. 3, where $R_A(y)$ is plotted versus A on the log-log scale for different values of y at incident energies 200 and 1000 GeV. One sees that for y close to the central rapidity region, R_A follows approximately the formula

$$R_A(y) = A^{\varepsilon(y)}. \tag{3.2}$$

In the projectile fragmentation region, however, this is no longer valid and therefore a two--parameter description seems to be more appropriate.

Since the data are traditionally fitted by the formula (3.2) we attempted to summarize our results in this way. To this end we determine the parameter $\varepsilon_{\text{eff}}(y)$ by minimalizing the integral

$$\int_0^{u_{\max}} du\{\ln R_A(y) - \varepsilon_{\text{eff}}(y)u\}^2, \tag{3.3}$$

[1] $A = 72.4$ corresponds to the "average emulsion-nucleus" in proton-emulsion interactions, and $A = 74$ for pion-emulsion interactions [8]. Note also that the theoretical curve refers to rapidities, y, the experiment gives pseudorapidities, η. Their approximate equality is discussed e.g. in Ref. [2].

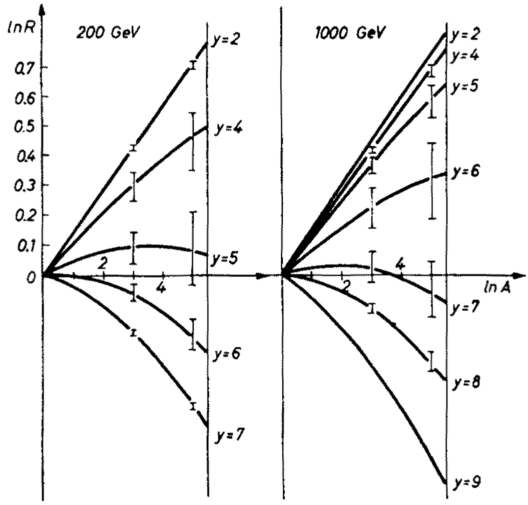

Fig. 3. A-dependence of the pion spectrum at fixed rapidities. The bars represent the uncertainties of our calculations

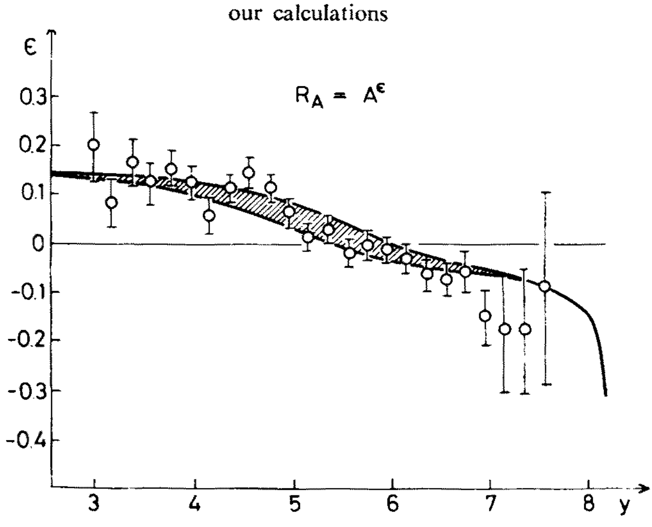

Fig. 4. A-dependence of particle multiplication for 300 GeV neutron-nucleus interactions. Data from Ref. [9]

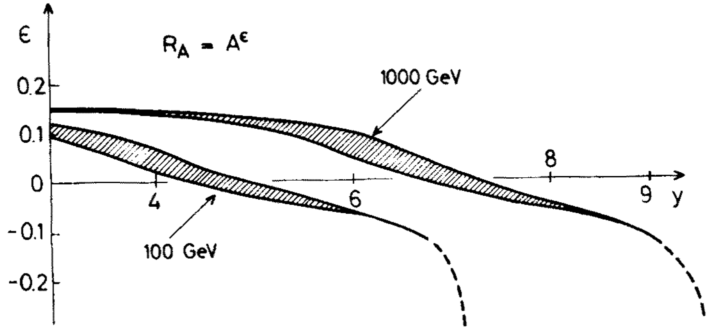

Fig. 5. A-dependence of particle multiplication for 100 and 1000 GeV nucleon-nucleus collisions

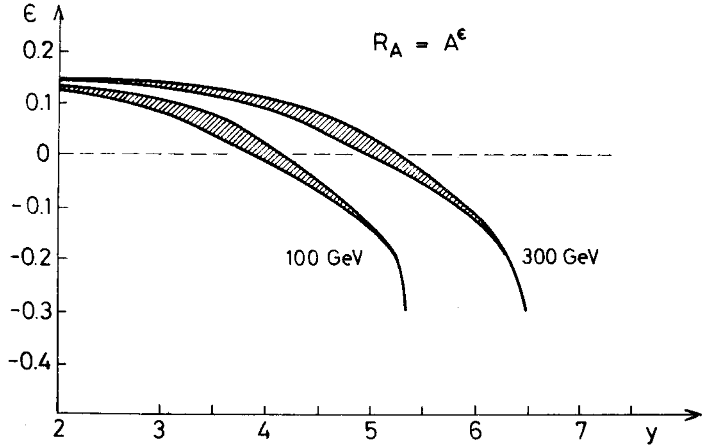

Fig. 6. A-dependence of nucleon spectrum in 100 and 300 GeV nucleon-nucleus collisions

where $u = \ln A$ and $u_{max} = \ln 238$. The minimalization gives

$$\varepsilon_{\rm eff} = \frac{3}{u_{max}^3} \int_0^{u_{max}} du\, u \ln R_A(y). \tag{3.4}$$

In the Fig. 4 $\varepsilon_{\rm eff}(y)$ is plotted versus y for incident energy 300 GeV and compared with the data of Ref. [9]. One sees quite a good agreement of the model with the data.

The predictions for the energies 100 and 1000 GeV are shown in Fig. 5. $\varepsilon_{\rm eff}(y)$ estimated for the nucleon spectrum is shown in Fig. 6 for Laboratory energies 100 and 300 GeV.

4. Single-particle spectra in pion-induced reactions

Calculations of the single-particle spectra for incident pions follow exactly the steps discussed in Section 3 and in the Appendix. There is, however, a simplifying feature: the A dependences of the relative (to hydrogen) spectra of the produced and the leading particles are this time identical, because the masses of the leading particles are the same as the masses of the lightest produced particles. In Fig. 7 we compare our $R_A(y)$ at $A = 74$ calculated for central and large rapidities[1], with the experimental data of Ref. [10]. One sees a fair agreement. The apparent discrepancy in the forward part of the spectrum might be perhaps due to the internal structure of the projectile, as suggested in Ref. [6]. In Fig. 8 we give our predictions for the exponent ε of Eq. (3.2) for 100 GeV and 300 GeV pion-nucleus interactions which, we hope, can be compared with some of the forthcoming experiments.

In Ref. [11] it was reported that the dependence of the total relative multiplicities, R_A, on the average number of collisions $\bar{\nu}$ is "universal" in the sense that it does not depend on the incident particle. Our model, which is applicable to $y \geqslant y_{\rm central}$, predicts that this "universality" can only have an integral, not a differential, character. In other words our $R_A(y)$ for fixed $\bar{\nu}$, shown in Fig. 9, is different for protons and pions. This is well borne

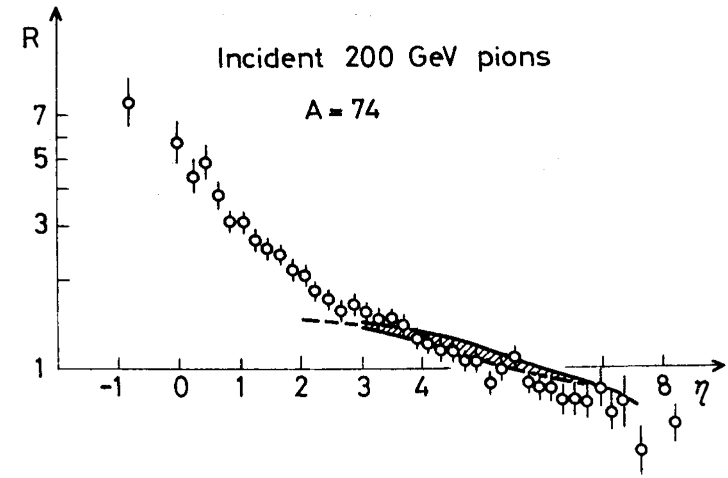

Fig. 7. Multiplication of particles in pion emulsion interactions at 200 GeV. Data from Ref. [10]

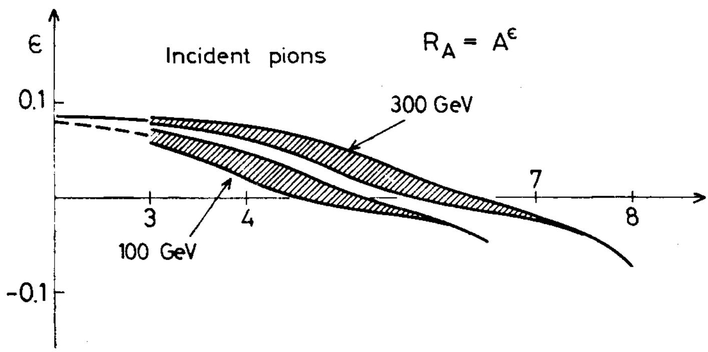

Fig. 8. A-dependence of particle multiplication for 100 and 300 GeV pion-nucleus collisions

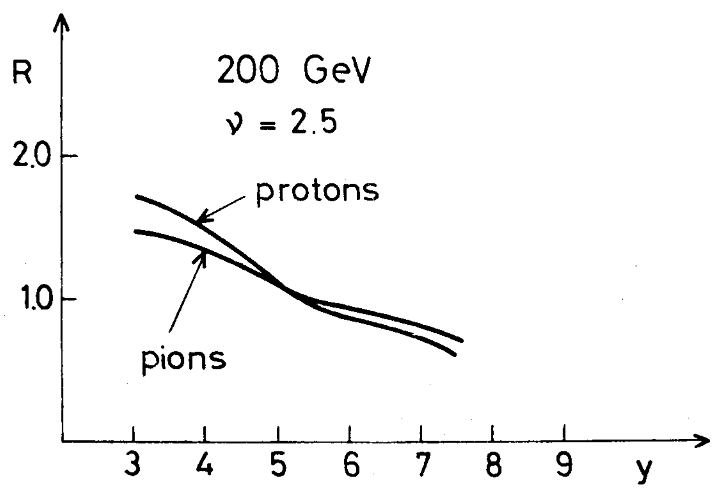

Fig. 9. Particle multiplication in 200 GeV nucleon-nucleus and pion-nucleus collisions at fixed $\bar{\nu} = 2.5$

out by experiment as shown in Ref. [2] for the central region of rapidities. On the other hand, in our model, an integration over y brings multiplicities quite close to each other as Fig. 10 shows. There, the integrals

$$R_A(y > 2.25) = \int_{2.25}^{\infty} dy \frac{dn_A}{dy} \left(\int_{2.25}^{\infty} dy \frac{dn_H}{dy} \right)^{-1},$$

as functions of $\bar{\nu}$ for protons and pions, differ only by 5% at the most. By extending the integral to smaller rapidities one presumably attains "universality". This, however, is out-

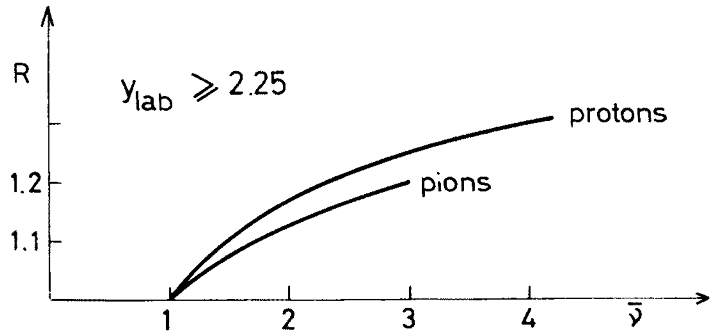

Fig. 10. Integrated particle multiplication ($y \geqslant 2.25$) in 200 GeV nucleon-nucleus and pion-nucleus collisions plotted versus average number of collisions $\bar{\nu}$

side of the region of applicability of our model. Note however that, since we expect the relation $R_A = \lambda_A/\lambda_H$ to be exact in the limit of infinitely high energies, we predict that the "universality" reported in [11] is a finite energy phenomenon and will disappear with increasing energies, unless the hadron-nucleon cross sections for different projectiles become the same in this limit.

5. *Nucleus-nucleus collisions*

As shown in Ref. [1], the constituent quark model gives also a formula for the plateau heights in nucleus-nucleus collisions. It reads

$$\lambda_{AB} = \frac{\bar{\nu}_{AB}}{\bar{\nu}_{cA}\bar{\nu}_{cB}} \lambda_H. \tag{5.1}$$

The data on particle production in nucleus-nucleus interactions at high energies is still very scarce and the highest energies are, so far, being measured in emulsion (energy/nucleon $\simeq$ 20 GeV) [12], where the total relative multiplicities are available. To compare the formula (5.1) with these data one has to integrate the spectrum over the whole rapidity region i.e. also in the target fragmentation region where our model is not applicable. Nevertheless we attempted the following approximate comparison. We observe that in the nucleon--nucleus interaction the total average multiplicity relative to hydrogen is approximately equal to λ_A/λ_H, given by Eq. (1.1) for A's up to $\sim$ 70. This is seen in Fig. 11. This relation

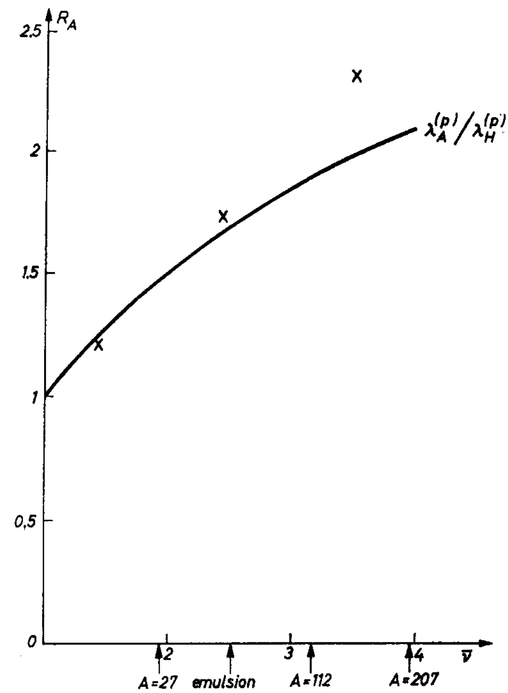

Fig. 11. Comparison of integrated and central particle multiplications in nucleon-nucleus collisions. Data from Ref. [11]

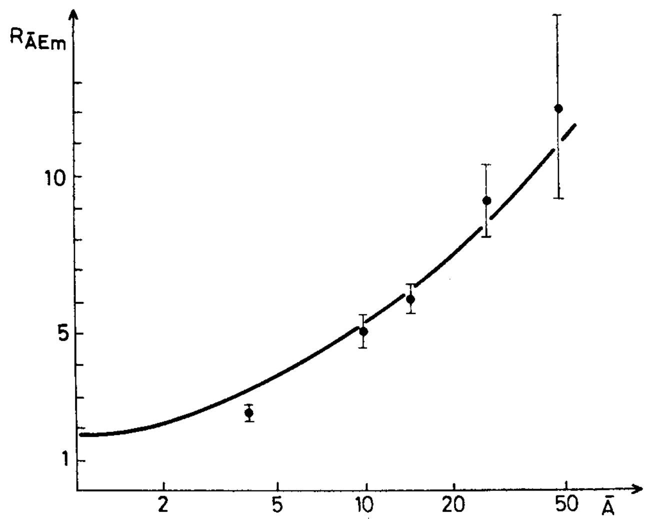

Fig. 12. A-dependence of particle multiplication in nucleus-emulsion interactions. Data from Ref. [12]

follows from a compensation between the surplus of particles in the target fragmentation region and energy-momentum conservation constraints which tend to decrease the number of produced particles. Since this relation is exact in the limit of infinite energies, we thought it reasonable to extrapolate it to nucleus-emulsion interactions for not too heavy incident nuclei. The comparison of the $\bar{\lambda}_{AB}/\lambda_{\rm H}$ calculated from Eq. (5.1) and averaged over a mixture of nuclei in emulsion with the data of Ref. [12] is given in Fig. 12. One sees a very reasonable agreement.

6. *Conclusions*

We find that the hypothesis of particle production by independent constituent quarks is supported by the data from nuclear targets. In particular,

(*i*) The multiplication in the central rapidity region is correctly explained by the assumption [1] that particle production in this region by a single constituent quark is independent of the target.

(*ii*) The phase-space corrections are unimportant in the central region above 200 GeV of the incident energy but largely determine the spectrum in the projectile fragmentation region.

(*iii*) The model also agrees with the cosmic ray data on nucleus-nucleus interactions.

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APPENDIX

Phase-space corrections to A-dependence of single-particle spectra

As explained in Section 2, the simple formula (1.1) suggested by the constituent quark model should be corrected for effects of energy and momentum conservation. In this Section we shall describe an attempt to estimate these corrections.

The effects of energy and momentum conservation are not uniquely determined because they depend sensitively on dynamics of the collision. It is therefore necessary to specify the matrix element of the process. Our idea was to minimize the dynamical assumptions as far as possible and at the same time to take into account some general features characteristic of particle production phenomena at high energies. To this end we have chosen the description of particle production by a cluster model with clusters distributed according to the longitudinal phase-space (with two leading clusters present). For high incident energies the rapidity distribution of clusters is then given by [13, 14]

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \lambda(1-x)^{\lambda}, \text{ produced clusters,} \tag{A1}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dy} = \frac{x}{1-x}\lambda(1-x)^{\lambda}, \text{ leading clusters,} \tag{A2}$$

where y is the rapidity of the cluster and

$$x = \frac{\varepsilon + k_{\parallel}}{E+p} \tag{A3}$$

is the Feynman scaling variable of the cluster. Here $(\varepsilon, k_{\parallel})$ are cluster energy and longitudinal momentum, (E, p) are energy and momentum of the incident particle. Finally, λ is the parameter giving the density of clusters in the central rapidity region (without energy-momentum conservation constraints).

The A-dependence enters in the Eqs (A1) and (A2) through the parameter λ which is a function of A. In the constituent quark model of Ref. [1] it is given by Eq. (1.1).

Several features of Eqs (A1) and (A2) are to be noticed:

(a) we have

$$x = \frac{\varepsilon + k_{\parallel}}{E+p} = \frac{m_t e^y}{E+p}, \tag{A4}$$

where m_t is the transverse mass of the cluster. Thus the mass and transverse motion of the clusters are important for determining the rapidity spectrum.

(b) Both produced and leading cluster spectra depend on λ_A (and thus on A) in exactly the same way. Possible differences in rapidity distributions can thus arise only from differences in masses and transverse motion.

(c) In the constituent quark model λ_A increases with A [1]. Consequently, the rapidity density of clusters should increase with A in the central region ($x \approx 0$) and should decrease with increasing A in the projectile fragmentation region ($x \gtrsim 0.5$). This is qualitatively in agreement with data.

Our problem is to construct the functions $\frac{dn}{dy} = \frac{1}{\sigma}\frac{d\sigma}{dy}$ and compare them with the data. This is not straightforward because of the transverse motion and the cluster decay spectrum. Indeed the relation (A4) contains the transverse mass of the emitted cluster m_t which is unknown and which can (and does) vary with $y_{\rm lab}$. This leads to ambiguities which we tried to estimate using the following prescription:

(*i*) We assume that the distribution of m_t is for all $y_{\rm lab}$ given by

$$\varrho(m_t)dm_t = e^{-am_t}m_t dm_t, \text{ for } \mu \leqslant m_t \leqslant M, \tag{A5}$$

and $\varrho(m_t) = 0$ otherwise, where μ is the minimal possible mass ($\mu = 0.14$ for pion spectrum und $\mu = 0.94$ for proton spectrum), M is the maximal possible transverse mass at a given $y_{\rm lab}$

$$M = 2E_{\rm lab}e^{-y_{\rm lab}}, \tag{A6}$$

and a is a parameter independent of $y_{\rm lab}$. The parameter a is not exactly known and is the major source of the resulting ambiguities. We took $1 \leqslant a \leqslant 2$, corresponding to 1 GeV $\leqslant \overline{m}_t \leqslant$ 2 GeV in the central rapidity region. These two limiting values give us the limits for our predictions. The best value of a (from the analysis of hydrogen data) is close to $m_t \approx 1.4$ [15].

(*ii*) For any $y_{\rm lab}$ we can now determine the average value of m_t from the distribution (A5)

$$m_t(y_{\rm lab}) = \int\limits_{\mu}^{M} dm_t m_t^2 e^{-am_t} / \int\limits_{\mu}^{M} dm_t m_t e^{-am_t}. \tag{A7}$$

m_t is a function of $y_{\rm lab}$, because M is, according to Eq. (A6).

(*iii*) This value of m_t is used to calculate x from Eq. (A4) and then the spectra according to Eqs (A1) and (A2).

In the actual calculations we took into account that, for a given target, λ is not fixed, as indicated by KNO scaling [16]. For KNO function we took [17]

$$\psi(z) = \frac{\pi}{2} z e^{-\frac{\pi z^2}{4}}, \tag{A8}$$

which fits quite well the multiplicity spectrum. (Other possible choices of $\psi(z)$ would influence the spectra only very close to $x = 1$, where the diffractive phenomena influence the results anyway.) Thus finally the spectrum at a given A is

$$\frac{1}{\sigma_A} \frac{d\sigma_A}{dy} = \int \frac{d\lambda}{\lambda_A} \psi\left(\frac{\lambda}{\lambda_A}\right) \lambda(1-x)^{\lambda}, \tag{A9}$$

where the parameter λ_A is related to $\lambda_{\rm H}$ by the Eq. (1.1). In practical calculation we have used the approximation

$$\lambda_A = A^{\varepsilon} \lambda_{\rm H}, \tag{A10}$$

and $\lambda_{\rm H}$ was taken $= 1$, according to analysis of hydrogen data [16]. ε was estimated from Eq. (1.1) to be:

$$\begin{aligned} &\varepsilon = 0.15 \text{ for nucleon-nucleon collisions, and} \\ &\varepsilon = 0.09 \text{ for pion-nucleus collisions.} \end{aligned} \tag{A11}$$

These values were used for calculation of the curves shown in Figs 4 and 7.

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