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WEAK INTERACTIONS*

By R. J. N. Phillips

Rutherford Laboratory, Chilton, Didcot, Oxon**

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These lectures are an introduction to the standard weak-electromagnetic gauge model and its experimental tests, followed by six-quark phenomenology and some Grand Unification schemes.

1. Charged-current Fermi interaction

The study of nuclear and muon β-decays led to the *V-A* current-current interaction of Feynman, Gell-Mann, Marshak and Sudarshan [1].

$$\mathscr{L} = (G/\sqrt{2})J_\lambda J_\lambda^\dagger, \tag{1}$$

$$J_\lambda = i\bar{\nu}_e\gamma_\lambda(1+\gamma_5)e+i\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\mu+i\bar{u}\gamma_\lambda(1+\gamma_5)\,(d\cos\theta_C+s\sin\theta_C)$$
$$+i\bar{c}\gamma_\lambda(1+\gamma_5)\,(s\cos\theta_C-d\sin\theta_C), \tag{2}$$

$$G = 1.026\times 10^{-5}\, m_p^{-2}. \tag{3}$$

Cabibbo [2] suggested the third term of Eq. (2) to explain the relative weakness of hyperon β-decay; the fourth term was suggested by Bjorken and Glashow [3] for lepton-quark symmetry and later exploited by GIM [4].

Universalities. This Lagrangian has equal couplings for e and μ, confirmed by the $\pi \to e\nu/\mu\nu$ branching ratio. It also has the same overall coupling for leptons and quarks, consistent with the latest values [5] for u → d, s (in units of the μ → ν coupling):

$$|u \to d|^2 = 0.948\pm 0.005, \qquad |u \to s|^2 = 0.048\pm 0.005, \tag{4}$$

which sum to unity within the errors.

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** Address: Rutherford Laboratory, Chilton, Didcot, Oxfordshire OX11 OQX, England.

This framework successfully describes all low-energy CC (= charged current) phenomena; e.g. hyperon semileptonic decays [5, 6]. The Cabibbo angle corresponding to Eq. (4) is

$$\theta_C = 0.221 \pm 0.011. \tag{5}$$

Intermediate bosons can easily be introduced to mediate the weak interaction between currents, in analogy with electromagnetism. For the charged currents of Eq. (2) we need charged bosons $W^{\pm}$. A big W-mass could explain at once both the weakness and the short range (pointlike character) of CC interactions. Intermediate bosons are also desirable as a step toward gauge theories with their renormalizability [7].

So can we have a gauge theory with charged gauge bosons like these?

2. *Gauge theories*

(a) Electrodynamics: U(1) symmetry

Suppose we start with a set of spinor fields ψ, and ask for invariance of the Lagrangian under the group of *local* gauge transformations

$$\psi(x) \to \exp[i\theta(x)Q]\psi(x), \tag{6}$$

where Q is the charge operator. Then we must introduce a massless vector field $A_\mu(x)$ with the following interaction and transformation properties:

$$\mathscr{L}_{\rm int} = ie\bar{\psi}\gamma_\mu Q\psi A_\mu, \qquad A_\mu(x) \to A_\mu(x) + \frac{1}{e}\partial_\mu\theta(x). \tag{7}$$

The extra gauge term from Eq. (7) is needed to compensate the derivatives of $\theta(x)$ from the spinor kinetic energy $-\bar{\psi}\gamma_\mu\partial_\mu\psi$. Here ∂_μ denotes $\partial/\partial x_\mu$.

This is familiar electrodynamics. The minimal gauge-invariant coupling is generated by replacing the derivative ∂_μ by the covariant derivative D_μ:

$$D_\mu = \partial_\mu - ieA_\mu Q. \tag{8}$$

(b) Isospin: SU(2) symmetry

If our spinors have an internal degree of freedom like isospin, we can define isospin-dependent gauge transformations

$$\psi(x) \to \exp[i\vec{\theta}(x)\cdot\vec{T}]\psi(x), \tag{9}$$

where $\vec{T} = (T^1, T^2, T^3)$ are the generators of the isospin group operating on the spinors ψ. Local gauge invariance now requires an isospin triplet of massless gauge fields with

$$\mathscr{L}_{\rm int} = ig\bar{\psi}\gamma_\mu\vec{T}\cdot\vec{A}_\mu\psi + (gA^3,\ g^2A^2 \text{ terms}),$$

$$\vec{A}_\mu(x) \to \vec{A}_\mu(x) + \frac{1}{g}\partial_\mu\vec{\theta}(x), \qquad D_\mu = \partial_\mu - ig\vec{T}\cdot\vec{A}_\mu. \tag{10}$$

The spinor-gauge boson interaction has the form

$$ig\bar{\psi}\gamma[T^+A^- + T^-A^+ + T^3A^3]\psi,$$

where $T^\pm = (1/\sqrt{2})(T^1 \pm iT^2)$ are the isospin-raising and lowering operators. So here we have a theory with charged and neutral gauge fields $A^\pm$ and A^3: can they possibly represent the $W^\pm$ and photon?

Georgi and Glashow [8] said yes. First separate the (initially massless) spinors into left and right-handed components: $\psi_L = \frac{1}{2}(1+\gamma_5)\psi$, $\psi_R = \frac{1}{2}(1-\gamma_5)\psi$. Then assign the e^- and ν_e components to the following isospin triplets and singlets

$$\begin{pmatrix} X^+ \\ X^0 \cos\beta + \nu \sin\beta \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} X^+ \\ X^0 \\ e^- \end{pmatrix}_R, \quad (\nu)_R, \quad (X^0 \sin\beta - \nu\cos\beta)_L. \tag{11}$$

This achieves what we want. The photon A^3 couples via T^3, i.e. with equal strength to X_L^+ and X_R^+, and with opposite sign to e_L^- and e_R^-, giving net vector interactions as required. The photon decouples from all the X^0 and ν components because they have $T^3 = 0$. The $W^\pm$ couple the $T^3 = 0$ components to the $T^3 = \pm 1$ components within any triplet: this includes a $\nu_L \leftrightarrow e_L$ coupling (pure V–A) with relative strength $\sin\beta$ that can be adjusted to suit. The μ, ν_μ treatment is similar; for quarks an integral charge assignment is needed. (Note that "weak isospin" here is *not* what we use in strong interaction physics: e.g. leptons have it).

SU(2) is prodigal with extra spinor fields X^+, X^0, etc. but very economical in gauge fields. There is only one neutral current, coupling to the photon. But since 1973 we know there are weak neutral currents too, so SU(2) is not enough.

(c) SU(2)×U(1) symmetry

To get invariance under *both* SU(2) and an independent U(1) gauge group simultaneously, we must have both lots of gauge fields (call them $\vec{W}_\mu$ and B_μ now) coupled to the spinors by

$$\mathscr{L}_{int} = ig\bar{\psi}\gamma_\mu\vec{T}\cdot\vec{W}\psi + \tfrac{1}{2}\, ig'\bar{\psi}\gamma_\mu Y B_\mu\psi, \tag{12}$$

where T and Y are the corresponding isospin and hypercharge operators (g' and $\frac{1}{2}Y$ replace e and Q in the electrodynamic example).

Charged currents couple to $W^\pm$. We get the standard V–A CC model by assigning isodoublets

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \tag{13}$$

where d′, s′ are the Cabibbo-rotated forms from Eq. (2). Right-handed components are in isosinglets and decouple. The overall coupling strength at small Q^2 comes right provided

$$g^2/(8m_W^2) = G/\sqrt{2}\,. \tag{14}$$

The factor 1/8 on the left comes from 1/2 and $1/\sqrt{2}$ in the normalization of ψ_L and $T^{\pm}$, squared.

There are two neutral currents, coupled to W^3 and B. In general we expect the physical neutral bosons A and Z (eigenstates of the mass matrix) to be orthogonal linear combinations of W^3 and B, defined by a mixing angle θ:

$$A = W^3 \sin\theta + B\cos\theta, \qquad Z = W^3\cos\theta - B\sin\theta. \tag{15}$$

These couple to the corresponding linear combinations of neutral isospin and hypercharge currents

$$\mathscr{L}_{\text{int}} = J^A_\mu A_\mu + J^Z_\mu Z_\mu,$$

$$J^A_\mu = i\bar{\psi}\gamma_\mu[g\sin\theta T^3 + \tfrac{1}{2}g'\cos\theta Y]\psi,$$

$$J^Z_\mu = i\bar{\psi}\gamma_\mu[g\cos\theta T^3 - \tfrac{1}{2}g'\sin\theta Y]\psi. \tag{16}$$

We want J^A_μ to be the electromagnetic current $ie\bar{\psi}\gamma_\mu Q\psi$, with $Q = T^3 + \frac{1}{2}Y$ in analogy with the Gell-Mann-Nishijima formula, so we demand

$$g\sin\theta = g'\cos\theta = e. \tag{17}$$

This fixes the other current J^Z_μ that gives weak interactions: for massive Z^0 and $Q^2 \ll m_Z^2$ we get the effective interaction:

$$\mathscr{L}_{\text{eff}} = (G/\sqrt{2})\varrho J^{NC}_\mu J^{NC\dagger}_\mu,$$

$$\varrho = m_W^2/(m_Z^2\cos^2\theta), \qquad J^{NC}_\mu = 2i\bar{\psi}\gamma_\mu T^3\psi - 2\sin^2\theta J^A_\mu. \tag{18}$$

This is the Salam–Weinberg [9] neutral current (NC) model. The factor ϱ has been pulled out for convenience; it sets the relative NC/CC scale; later we shall see maybe $\varrho = 1$. θ gets called the Weinberg angle.

Notice J^{NC} has a *V-A* piece depending on T^3 and a pure *V* piece proportional to the electromagnetic current. It is instructive to spell out the NC couplings explicitly:

$$J^{NC} = i\bar{\nu}_L\gamma\nu_L + i\bar{e}_L\gamma e_L[-1+2\sin^2\theta] + i\bar{e}_R\gamma e_R[2\sin^2\theta]$$

$$+ i\bar{u}_L\gamma u_L[1-\tfrac{4}{3}\sin^2\theta] + i\bar{u}_R\gamma u_R[-\tfrac{4}{3}\sin^2\theta]$$

$$+ i\bar{d}_L\gamma d_L[-1+\tfrac{2}{3}\sin^2\theta] + i\bar{d}_R\gamma d_R[\tfrac{2}{3}\sin^2\theta]. \tag{19}$$

The ν_μ, μ, c, s couplings are identical to those for ν_e, e, u, d. Notice that the eight coefficients in Eq. (19) (including the zero coefficient for $\bar{\nu}_R\gamma\nu_R$) are all different. There is no Cabibbo--angle dependence; basically Z couples to the doublet components of Eq. (13), but θ_C drops out:

$$\bar{d}'_L\gamma d'_L + \bar{s}'_L\gamma s'_L = \bar{d}_L\gamma d_L + \bar{s}_L\gamma s_L. \tag{20}$$

There are no flavour-changing neutral currents either. They are present in each term on the left of Eq. (20) but cancel out; this is part of the GIM magic [4].

Summing up, the SU(2)×U(1) neutral currents have just two free parameters θ and ϱ; other parameters are tied down via Eqs. (14), (17), (18) by electromagnetic and CC data.

(d) $W^{\pm}$ and Z masses

Gauge models usually start with massless spinors (so that we can distinguish L, R components) and massless gauge bosons (for unbroken symmetry). Masses enter through interaction with Higgs scalar fields.

Scalar fields ϕ^i are introduced gauge-invariantly but with mutual interaction such that the lowest energy state does *not* have $\langle\phi^i\rangle = 0$ for all i; their vacuum expectation values (vev) break symmetry spontaneously. Suppose a particular field ϕ has nonzero vev $\langle\phi\rangle = \lambda$; we can pick a phase convention such that λ is real. Then to get back to the sort of fields we are accustomed to, we define a new field ϕ' with zero vev:

$$\phi = \phi' + \lambda. \tag{21}$$

Spinor masses enter via Yukawa interactions with ϕ, present in the original Lagrangian, that now turn into Yukawa couplings to ϕ' plus mass terms, e.g.

$$f\bar{\psi}\psi\phi = f\bar{\psi}\psi\phi' + f\lambda\bar{\psi}\psi. \tag{22}$$

Until we have some theory about the Yukawa couplings, their strengths f and hence the spinor mass matrix elements are arbitrary (see the lectures by Weyers [10]). Gauge boson masses enter through the covariant generalization of the ϕ kinetic energy term:

$$D_\mu\phi D_\mu\phi^* = D_\mu\phi' D_\mu\phi'^* + \lambda^2 \text{ (mass terms)}. \tag{23}$$

When the gauge bosons acquire masses, they break the original symmetry of the current--current interactions that they mediate.

The simplest nontrivial Higgs assignment is an isodoublet (ϕ^0, ϕ^-) with $T = \frac{1}{2}$, $Y = -1$. At least one doublet is needed for the fermion masses, since they require $\bar{\psi}_L\psi_R\phi$ terms and ψ_R are isosinglets. ϕ^- cannot have nonzero vev (it would make the photon massive), so we choose $\langle(\phi^0, \phi^-)\rangle = (\lambda, 0)$. Putting this in Eq. (23) gives the boson mass terms

$$\mathscr{L}_{\text{mass}} = -\frac{\lambda^2}{4}(g^2\vec{W}^2 + g'^2B^2 - 2gg'W^3B) = -\frac{\lambda^2}{4}(2g^2W^+W^- + (g^2+g'^2)Z^2), \tag{24}$$

$$\frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2+g'^2} = \cos^2\theta, \tag{25}$$

$$\varrho = 1. \tag{26}$$

Thus the W/Z mass ratio is constrained and $\varrho = 1$ (true for any number of Higgs doublets). No compelling reason is known for taking doublets only, but experiment agrees with this constraint. The latest determination is [11]

$$\varrho = 1.004 \pm 0.018. \tag{27}$$

3. *Tests of Salam-Weinberg model*

CC phenomena relate only to standard *V-A* and GIM models [1–4]. To test Salam–Weinberg (S–W) we need NC data, especially neutrino scattering. A handy formula for neutrino scattering from a pointlike fermion f is

$$\frac{d\sigma}{dxdy}(\nu f \to \nu f) = \frac{G^2 mE}{2\pi} x f(x) \left[g_{Lf}^2 + g_{Rf}^2(1-y)^2\right]\varrho^2 \tag{28}$$

where g_L, g_R are the fermion weak couplings, defined by the quantities in brackets in Eq. (19). Some authors normalize g_L, g_R differently: we shall stick to this normalization here. For a lepton target, put $m = m_l$, $f(x) = \delta(1-x)$. For a nucleon target, put $m = m_N$ and sum over quark distributions $f(x)$. Eq. (28) holds equally for $\bar{\nu}\bar{f} \to \bar{\nu}\bar{f}$. For $\bar{\nu}f \to \bar{\nu}f$ and $\nu\bar{f} \to \nu\bar{f}$, interchange suffices L, R. Here Q^2 = (momentum transfer)^2, ν = lab energy transfer, $x = Q^2/(2m\nu)$ and $y = \nu/E$ as usual. We put $\varrho = 1$ henceforth.

Baltay [12] summarized the tests up to August 1978. Most of the following comes from his paper, which should be consulted for further details, references and graphical comparisons with data (omitted here for brevity).

(a) $\nu_\mu - e^-$ scattering

There is information in principle in the y-dependence, but so far only the total cross section is measured

$$\sigma = \frac{G^2 m_e E_\nu}{2\pi}\left(1 - 4\sin^2\theta + \tfrac{16}{3}\sin^4\theta\right). \tag{29}$$

Experiment	Events	Background	$\sigma/10^{-42} E_\nu$ cm^2
Aachen–Padova	32	21	1.1 ± 0.6
Columbia–BNL	11	0.7	1.8 ± 0.8
Gargamelle 1979 [13]	9	0.5	$2.4^{+1.2}_{-0.9}$
World average			1.53 ± 0.44

Experiments finding only upper limits have been omitted. Using $G^2 m_e/2\pi = 4.30 \times 10^{-42}$ cm^2/GeV, the world average gives

$$\sin^2\theta = 0.23^{+0.12}_{-0.15}. \tag{30}$$

(b) $\bar{\nu}_\mu - e^-$ scattering

$$\sigma = \frac{G^2 m_e E_\nu}{2\pi} \left(\tfrac{1}{3} - \tfrac{4}{3}\sin^2\theta + \tfrac{16}{3}\sin^4\theta\right). \tag{31}$$

Experiment	Events	Background	$\sigma/10^{-42}\, E_\nu\, \mathrm{cm}^2$
Gargamelle (PS)	3	0.4	$1.0^{+2.1}_{-0.9}$
Aachen–Padova	17	7.4	2.2 ± 1.0
World average			1.8 ± 0.9

$$\sin^2\theta = 0.3^{+0.1}_{-0.3}. \tag{32}$$

(c) $\bar{\nu}_e - e^-$ scattering

Reactor antineutrinos are used. There is a CC contribution in addition to the NC one; the effect is to substitute $g_L + 2$ for g_L in Eq. (28), giving

$$\sigma = \frac{G^2 m_e E_\nu}{2\pi} \left(3 - 2\sin^2\theta + \tfrac{16}{3}\sin^4\theta\right). \tag{33}$$

The experimental number [14] and inferred θ are

$$\sigma = (5.7 \pm 1.2) \times 10^{-42}\ \mathrm{cm}^2\ E_\nu/\mathrm{GeV}, \tag{34}$$

$$\sin^2\theta = 0.29 \pm 0.05. \tag{35}$$

It is amusing that CC interactions alone (zero NC) also predict $\sigma = 5.7$, so this result is consistency with S–W rather than direct support. Fortunately the y-dependence for CC differs from full CC+NC, and the scanty statistics favour the latter.

(d) Inclusive $\nu N \to \nu X$ scattering: $I = 0$ targets

This measures isospin averaged couplings (ignoring s, $\bar{s}$, c, $\bar{c}$ sea):

$$\frac{d\sigma^{\nu N}}{dx\,dy} = \frac{G^2 m_N E}{\pi} x\{\mathrm{VAL}(x)\,[\bar{g}_L^2 + \bar{g}_R^2(1-y)^2] + \mathrm{SEA}(x)\,[\bar{g}_L^2 + \bar{g}_R^2]\,(1+(1-y)^2)\},$$

$$\bar{g}_L^2 = \tfrac{1}{2}(g_{Lu}^2 + g_{Ld}^2), \quad \bar{g}_R^2 = \tfrac{1}{2}(g_{Ru}^2 + g_{Rd}^2), \tag{36}$$

where $\mathrm{VAL}(x) = \frac{1}{2}(u + d - \bar{u} - \bar{d})$ and $\mathrm{SEA}(x) = \frac{1}{2}(\bar{u} + \bar{d})$. For $\bar{\nu}N$ scattering interchange L, R. $\bar{g}_L^2$ and $\bar{g}_R^2$ can be separated either by comparing νN, $\bar{\nu}N$ or by studying y-dependence. In the valence-only approximation we get simply

$$R^\nu = \frac{\sigma^{NC}(\nu N)}{\sigma^{CC}(\nu N)} = \frac{\bar{g}_L^2 + \frac{1}{3}\bar{g}_R^2}{4} = \frac{1 - 2\sin^2\theta + \frac{40}{27}\sin^4\theta}{4},$$

$$R^{\bar{\nu}} = \frac{\sigma^{NC}(\bar{\nu} N)}{\sigma^{CC}(\bar{\nu} N)} = \frac{\bar{g}_L^2 + 3\bar{g}_R^2}{4} = \frac{1 - 2\sin^2\theta + \frac{40}{9}\sin^4\theta}{4}. \tag{37}$$

In practice we cannot safely ignore the sea, and the formulae get more complicated. Baltay [12] gives the following weighted means of world data, and the corresponding θ-values

$$R^{\nu} = 0.29 \pm 0.01 \quad : \quad \sin^2\theta = 0.24 \pm 0.02$$

$$R^{\bar{\nu}} = 0.35 \pm 0.03 \quad : \quad \sin^2\theta = 0.3 \pm 0.1 \tag{38}$$

(e) Elastic scattering $\nu(\bar{\nu}) + \mathrm{p} \to \nu(\bar{\nu}) + \mathrm{p}$

This is *coherent* scattering from all constituent quarks so Eq. (28) does not apply. The general form of the proton vertex is

$$(p|J_{\mu}^{\mathrm{NC}}|p) \sim g_1(Q^2)\gamma_{\mu}\gamma_5 + f_1(Q^2)\gamma_{\mu} + f_2(Q^2)\sigma_{\mu\nu}q_{\nu}/(2m_{\mathrm{p}}). \tag{39}$$

The relevant pieces of J_{μ}^{NC} (see Eq. (18)) are $i(\bar{u}_{\mathrm{L}}\gamma_{\mu}u_{\mathrm{L}} - \bar{d}_{\mathrm{L}}\gamma_{\mu}d_{\mathrm{L}})$ and $2\sin^2\theta J_{\mu}^{\mathrm{A}}$. The matrix element of the first can be got from the neutron β-decay vertex by isospin rotation. The matrix element of the second is given by the proton electromagnetic form factors. Hence theory predicts g_1, f_1 and f_2. Harvard–Penn–BNL have the best measurements so far [15].

	HPB (1978)	S$-$W ($\sin^2\theta = 0.22$)
g_1	$= 0.6^{+0.1}_{-0.2}$	0.63
f_1	$= 0.5^{+0.3}_{-0.5}$	0.06
$f_1 + f_2$	$= 0.9 \pm 0.2$	1.12

The errors on f_1 and f_2 are strongly correlated: their sum has a smaller error than either alone.

(f) Semi-inclusive $\nu\mathrm{N} \to \nu\pi\mathrm{X}$, $\bar{\nu}\mathrm{N} \to \bar{\nu}\pi\mathrm{X}$

In the quark-parton picture the recoiling quark q takes all the lab energy ν of the virtual Z-exchange, finally fragmenting into a hadron H with probability $D_{\mathrm{q}}^{\mathrm{H}}(z)$ depending on the energy fraction $z = E_{\mathrm{H}}/\nu$:

$$\frac{d\sigma}{dxdydz}(\nu\mathrm{N} \to \nu\mathrm{HX}) = \frac{G^2 m_{\mathrm{N}} E}{2\pi} \sum_{\mathrm{q}} xq(x)\,[g_{\mathrm{Lq}}^2 + g_{\mathrm{Rq}}^2(1-y)^2] D_{\mathrm{q}}^{\mathrm{H}}(z), \tag{40}$$

where q denotes both quarks and antiquarks. For $\bar{\nu}$ beam interchange L, R.

The nice thing here is that D_{q}^{π} weights u, d terms differently, unlike the isospin averages of Eq. (38). Charge conjugation and charge-symmetry give

$$D_{\mathrm{u}}^{\pi+} = D_{\mathrm{d}}^{\pi-} = D_{\bar{\mathrm{u}}}^{\pi-} = D_{\bar{\mathrm{d}}}^{\pi+} \text{ (favoured)},$$

$$D_{\mathrm{u}}^{\pi-} = D_{\mathrm{d}}^{\pi+} = D_{\bar{\mathrm{u}}}^{\pi+} = D_{\bar{\mathrm{d}}}^{\pi-} \text{ (unfavoured)}. \tag{41}$$

In the favoured cases the quark is a valence component of the final pion and D is much bigger. These fragmentation functions can be determined from CC π-production. Then if as a zeroth approximation we neglect antiquarks and the unfavoured fragmentation channels, the process $\nu N \to \nu\pi^+ X$ depends only on g_{Lu}^2 and g_{Ru}^2, while $\bar{\nu}N \to \bar{\nu}\pi^- X$ depends on g_{Ld}^2 and g_{Rd}^2. In practice one keeps all terms, but these examples illustrate the idea.

Baltay [12] assembled this table of single-pion data and the corresponding S–W prediction for $\sin^2\theta = 0.25$. It includes also low energy *exclusive* single pion production, treated by Adler theory [16].

Experiment	$\sigma(NC)/\sigma(CC)$	Result	S–W
CIR	$\nu\pi^0X/\mu^-\pi^0X$	0.42 ± 0.14	$0.48\pm$
Aachen–Padova			
Gargamelle	$\bar{\nu}\pi^0X/\mu^+\pi^0X$	0.92 ± 0.14	$0.60\pm$
	$\nu n\pi^+/\mu^-p\pi^+$	0.13 ± 0.06	$0.07\pm$
ANL (BC)	$\nu p\pi^0/\mu^-p\pi^+$	0.40 ± 0.22	$0.17\pm$
	$\nu p\pi^-/\mu^-p\pi^+$	0.12 ± 0.04	$0.07\pm$
	$(\nu p\pi^0+\nu n\pi^0)/\mu^-p\pi^0$	0.90 ± 0.16	$0.84\pm$
	$(\bar{\nu}p\pi^0+\bar{\nu}n\pi^0)/\mu^+n\pi^0$	1.14 ± 0.22	$1.20\pm$
Gargamelle	$\nu p\pi^0/\mu^-p\pi^0$	0.56 ± 0.10	0.42 ± 0.13
	$\nu n\pi^0/\mu^-p\pi^0$	0.34 ± 0.09	
	$\nu p\pi^-/\mu^-p\pi^0$	0.45 ± 0.13	0.28 ± 0.08
	$\nu n\pi^+/\mu^-p\pi^0$	0.34 ± 0.07	

(g) Deuteron breakup $\bar{\nu}D \to \bar{\nu}np$

Measurements have been reported very recently [14] for NC deuteron breakup using $\bar{\nu}_e$ from a reactor. Because of the very low energy, there is only a $^3S_1 \to {}^1S_0$ transition at the nuclear vertex (final $^3S_1$ wave functions have zero overlap on the orthogonal deuteron bound state) so purely axial coupling is selected. The S–W prediction is therefore unique, independent of $\sin^2\theta$. The predicted event rate for the experimental conditions is 50 ± 3 per day, in reasonable agreement with the observed rate 38 ± 9 per day.

(h) Parity violation in atomic physics

When two atomic levels of opposite parity lie close, the e-nuclear NC interaction can mix them to give parity-violating effects.

One such effect is an optical rotation at particular wavelengths in Bi vapour. Results so far are conflicting; only one group agrees with S–W expectations [12].

Group	Wavelength (Å)	Rotation ($\times10^8$)	W–S
Seattle	8757	-0.5 ± 1.7	$-(10–18)$
Oxford	6480	$-5\ \pm1.6$	$-(13–23)$
Novosibirsk	6480	$-19\ \pm5$	$-(13–23)$

Another effect is circular dichroism (asymmetric absorption cross sections for photons of helicity ± 1). A Berkeley group has just reported an effect for 2927 Å in Thallium [17]

$$[\sigma(+)-\sigma(-)]/[\sigma(+)+\sigma(-)] = (5.2\pm 2.4)\times 10^{-3} \tag{42}$$

consistent with $(2.3\pm 0.9)\times 10^{-3}$ expected for S–W with $\sin^2\theta = 0.25$.

(i) Polarized electron inclusive $eN \to eX$

The effective e-quark interaction at low Q^2 is

$$\begin{aligned}\mathscr{L}_{\text{eff}} = {} & \bar{e}_L\gamma_\mu e_L \cdot \bar{q}_L\gamma_\mu q_L\left[-\frac{e^2}{Q^2}\varepsilon_q+\sqrt{2}\,Gg_{Le}g_{Lq}\right] \\ & +\bar{e}_L\gamma_\mu e_L \cdot \bar{q}_R\gamma_\mu q_R\left[-\frac{e^2}{Q^2}\varepsilon_q+\sqrt{2}\,Gg_{Le}g_{Rq}\right] \\ & +\bar{e}_R\gamma_\mu e_R \cdot \bar{q}_L\gamma_\mu q_L\left[-\frac{e^2}{Q^2}\varepsilon_q+\sqrt{2}\,Gg_{Re}g_{Lq}\right] \\ & +\bar{e}_R\gamma_\mu e_R \cdot \bar{q}_R\gamma_\mu q_R\left[-\frac{e^2}{Q^2}\varepsilon_q+\sqrt{2}\,Gg_{Re}g_{Rq}\right]\end{aligned} \tag{43}$$

for each quark q, where ε_q is the quark charge. Let us denote the four bracketed coefficients in Eq. (43) by A_{LL}, A_{LR}, A_{RL}, A_{RR} for brevity. Then in analogy to Eq. (28) the cross-section for left-handed e–N scattering is

$$\frac{d\sigma}{dxdy}(L) = \frac{Em_N}{4\pi}\sum_q xq(x)\,[A_{LL}^2+A_{LR}^2(1-y)^2]. \tag{44}$$

(For right-handed electrons interchange L, R). Hence the predicted asymmetry is

$$\frac{R-L}{R+L} = \frac{\sqrt{2}\,GQ^2}{e^2}\,\frac{\sum xq(x)\varepsilon_q[g_{Le}g_{Lq}-g_{Re}g_{Rq}+(1-y)^2(g_{Le}g_{Rq}-g_{Re}g_{Lq})]}{\sum xq(x)\varepsilon_q^2(1+(1-y)^2)}. \tag{45}$$

Note that the asymmetry increases with Q^2, and there is extra information in the y-dependence.

A spectacular SLAC experiment succeeded in measuring the asymmetry to accuracy 10^{-5}. The final result [18] from this experiment is

$$\frac{R-L}{R+L} = Q^2\left\{a_1+a_2\,\frac{1-(1-y)^2}{1+(1-y)^2}\right\},$$

$$a_1 = (-9.7\pm 2.6)\times 10^{-5}, \quad a_2 = (4.9\pm 8.1)\times 10^{-5}, \quad \sin^2\theta = 0.224\pm 0.020. \tag{46}$$

Although we have used parton language here, Wolfenstein [19] argues that the result for $\sin^2\theta$ in fact depends on general principles and is rather insensitive to parton assumptions.

This effect depends on interference with the purely vector electromagnetic current, and cannot be accommodated by any ad hoc S, T, P interaction model (that can fake the other V, A weak effects).

(j) Model-independent analyses

In a general V, A framework there are relatively few independent coupling constants. An interesting approach is to try to determine these couplings from data, to see if the solution is unique and how well it agrees with S–W.

In the ν-quark sector there are just four independent couplings $g_{Lu}, g_{Ru}, g_{Ld}, g_{Rd}$ (defining $g_{L\nu} = 1$, $g_{R\nu} = 0$). The first analysis by Sehgal [20] used inclusive plus semi--inclusive data to determine the squares of the couplings, leaving a 4-fold ambiguity (one couplings is zero and the overall sign is inaccessible). Hung and Sakurai [21] used elastic νp, $\bar{\nu}$p data to reduce to two solutions. Others showed the $\nu N \to \nu \pi N$ data [22] or simply better elastic data [23] made the solution unique — and in line with S–W. Here for example is the final solution of Abbott and Barnett [22].

	Abbott-Barnett	S−W $\sin^2\theta = 0.22$
$\frac{1}{2} g_{Lu} =$	0.35 ± 0.07	0.35
$\frac{1}{2} g_{Ru} =$	-0.19 ± 0.06	−0.15
$\frac{1}{2} g_{Ld} =$	-0.40 ± 0.07	−0.43
$\frac{1}{2} g_{Rd} =$	0.0 ± 0.11	0.07

The new $\bar{\nu}D \to \bar{\nu}np$ breakup data [14] also resolve the solution ambiguity, but were not available at the time.

In the ν-e sector, ν_μ and $\bar{\nu}_\mu$ scattering determine g_{Le}^2 and g_{Re}^2 leaving a sign ambiguity: one solution is like S–W. In principle $\bar{\nu}$–e scattering could resolve the ambiguity via NC–CC interference but in practice it does not. However, using factorization ν–q and ν–e determine the e–q sector, and the polarized electron data allow only the S–W-type solution for electron couplings [12].

	Model-independent solution	S−W
$g_{Ve} = \frac{1}{2}(g_{Le}+g_{Re}) =$	0.0 ± 0.1	$-\frac{1}{2}+2\sin^2\theta$
$g_{Ae} = \frac{1}{2}(g_{Le}-g_{Re}) =$	-0.55 ± 0.1	$-\frac{1}{2}$

(k) World-data fitting

We have seen that a wide range of data agree with the S–W model, with compatible θ-values. Brave souls have now begun to analyse all data at once, to get optimum fits

[11, 24]. Such a fit gave the ϱ-parameter quoted in Eq. (27) above; the same authors [11] quote the best fit for $\sin^2\theta$ (assuming $\varrho = 1$) as

$$\sin^2\theta = 0.228 \pm 0.008\ (\pm 0.008), \tag{47}$$

where the final parentheses enclose an estimate of systematic error. Reference [24] quotes a compatible value 0.234 ± 0.013 including systematic uncertainties.

(l) Future tests

S–W predicts a forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$.

$$\frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-) = \frac{\alpha^2}{4s}\left[F_1(1+\cos^2\theta)+2F_2\cos\theta\right], \tag{48}$$

where F_2 comes from VV, AA interference and does not violate parity. (For full formulae see e.g. Ref. [25].) For $s \ll m_Z^2$ the asymmetry is

$$\frac{\sigma(0)-\sigma(\pi)}{\sigma(0)+\sigma(\pi)} = \frac{F_2}{F_1} \simeq \frac{s}{8(s-m_Z^2)\sin^2\theta\cos^2\theta}. \tag{49}$$

We can look forward to interesting results from PETRA and PEP; e.g. the predicted asymmetry is -0.09 at $\sqrt{s} = 30$ for $\sin^2\theta = 0.23$. There are further fancier asymmetries with polarized $e^\pm$ beams [25].

There are asymmetries in deep inelastic $e^\pm$ or $\mu^\pm$ scattering of the general form

$$[\sigma(\mu^+)-\sigma(\mu^-)]/[\sigma(\mu^+)+\sigma(\mu^-)] \sim Q^2/(Q^2+m_Z^2) \tag{50}$$

that are the crossed-channel analogues of Eq. (49). The coefficient of the asymmetry depends on $\sin^2\theta$ via the quark couplings and distributions. The reader can actually construct the formula from Eq. (44), remembering that $g_{L,R} \to -g_{R,L}$ when $e^- \to e^+$.

The most dramatic test will be to observe the gauge bosons themselves, at a future $\bar{p}p$ or e^+e^- collider. For $\sin^2\theta = 0.23 \pm 0.01$ the predicted masses are

$$m_W = 77.8 \pm 1.7\ \text{GeV}, \qquad m_Z = 88.6 \pm 1.4\ \text{GeV}. \tag{51}$$

4. *Natural flavour-conservation in NC*

Experimentally $s \to d$ couplings are absent through order $G\alpha$. Hence not only the direct NC Z-couplings (of order G) must be suppressed but also second-order contributions like that in Fig. 1 (these have quadratically divergent loop integrals presumed cut off at m_W and are of order $G^2 m_W^2 \sim G\alpha$). The GIM magic [4] not only cancels the Z coupling exactly, as we already saw, but also manages to suppress the second-order terms too: it is easy to check that in the limit $m_u = m_c$ they cancel exactly. We should also worry about NC effects induced by Higgs scalars. In general Higgs scalars too could induce NC effects.

The s → d suppression is so remarkably strong and its explanation so remarkably neat that one is tempted to extrapolate. We guess that all flavour-changing NC are similarly suppressed and that the GIM mechanism embodies a general principle. How can this be?

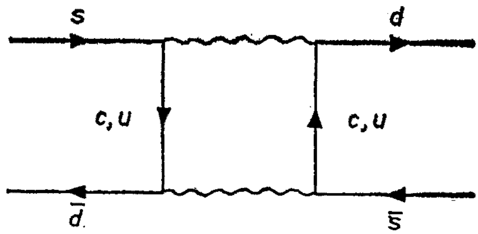

Fig. 1. Example of second-order neutral current

Glashow and Weinberg [26] showed that flavour conservation happens "naturally" in the SU(2)×U(1) framework provided

(*i*) All quarks with the same charge and helicity have the same T and T^3, i.e. belong to the same kind of multiplet.

(*ii*) Quarks of a given charge get their mass either through the couplings of precisely one Higgs field or through an SU(2) invariant mass term, but not both. (We have not discussed the latter mechanism, but it is a possibility).

Take an example. Suppose the right-handed quarks are in two singlets and one doublet violating condition (i):

$$[\mathrm{u}]_\mathrm{R}, \quad [\mathrm{d}']_\mathrm{R}, \quad \begin{bmatrix} \mathrm{c} \\ \mathrm{s}' \end{bmatrix}_\mathrm{R}. \tag{52}$$

Here $\mathrm{d}' = \mathrm{d}\cos\beta + \mathrm{s}\sin\beta$, $\mathrm{s}' = \mathrm{s}\cos\beta - \mathrm{d}\sin\beta$ includes some permissible mixing. Working out the $\mathrm{d}' \to \mathrm{d}'$ and $\mathrm{s}' \to \mathrm{s}'$ neutral currents from Eq. (18) we find a flavour--changing piece

$$J^{\mathrm{FC}} = \sin\beta\cos\beta(\bar{\mathrm{s}}_\mathrm{R}\gamma \mathrm{d}_\mathrm{R} + \bar{\mathrm{d}}_\mathrm{R}\gamma \mathrm{s}_\mathrm{R}) \tag{53}$$

that does not vanish in general. We can make it vanish by choosing $\beta = 0$ or $\pi/2$ *but this is unnatural.* We want a cancellation that works *for any mixing angle* like GIM.
Moral: any new quarks should be put into the same classes as before, left-handed doublets and right-handed singlets, e.g.

$$\begin{pmatrix} \mathrm{t} \\ \mathrm{b} \end{pmatrix}_\mathrm{L}, \quad (\mathrm{b})_\mathrm{R}, \quad (\mathrm{t})_\mathrm{R}. \tag{54}$$

5. *Triangle anomalies*

There are classes of graphs with divergent loop integrals that break some Ward identities and make difficulties for renormalization (see field theory textbooks or Ref. [27]). Figure 2 shows an example, a fermion loop with one axial and two vector current vertices.

diverging like $\int d^4p/p^3$. We would like the divergent contributions from different fermions to cancel out; only the leading term in the integral diverges, so the masses do not matter for this cancellation.

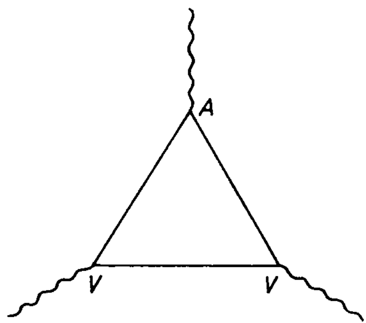

Fig. 2. Example of triangle graph with anomaly

When the fermions have internal symmetry, the divergent contributions from each fermion multiplet have the factor

$$\text{trace}\,[(O^a O^b + O^b O^a) O^c], \tag{55}$$

where O^a, O^b, O^c are the current couplings at the V, V, A vertices. In $SU(2) \times U(1)$ models, the O^i are linear combinations of T^i and Y, so all divergences can be classified in terms of T^i and Y: $\text{tr}[(T^aT^b + T^bT^a)T^c]$ etc. Upon inspection, all these traces vanish (i.e. the internal symmetry cancels the divergences) except for two, namely

$$\text{tr}\,[(T^aT^b + T^bT^a)Y], \tag{56}$$

$$\text{tr}\,[YYY]. \tag{57}$$

In the first case Eq. (56) only doublets (i.e. left-handed fermions) contribute; apart from the T^i factors that are the same for all, each lepton doublet contributes $\sum Y = -2$ and each quark doublet contributes $\sum Y = +2/3$ multiplied by 3 for colour. Hence

$$\sum Y = -2N_\mathrm{L} + 2N_\mathrm{Q} \tag{58}$$

and the coefficient vanishes if there are equal numbers of lepton and quark doublets.

In the latter case Eq. (57) the book-keeping gets more complicated; both doublets and singlets now contribute but the singlets are right-handed with opposite-sign spatial coupling at the axial vertex. Keeping track of ν, e, u, d-type fermions separately, we find

$$\sum Y^3 = -N_\mathrm{L}^\nu - N_\mathrm{L}^\mathrm{e} + 8N_\mathrm{R}^\mathrm{e} + \tfrac{1}{9}N_\mathrm{L}^\mathrm{u} + \tfrac{1}{9}N_\mathrm{L}^\mathrm{d} - \tfrac{64}{9}N_\mathrm{R}^\mathrm{u} + \tfrac{8}{9}N_\mathrm{R}^\mathrm{d} \tag{59}$$

which again vanishes if we have symmetry between leptons and quarks.

Beside the VVA case illustrated above, there are AAA anomalies that cancel in the same way. There are also quadrangle anomalies ($AVVV$ and $AAAV$) and even pentagon anomalies (through contact terms introduced to correct the quadrangle anomalies) but they all cancel too.

Moral: make sure quarks and leptons appear symmetrically.

6. *More gauge fields*

One way to go beyond S–W is to add more symmetry, more gauge fields. Take for instance $SU(2)\times U(1)\times U(1)'$, adding a new hypercharge Y' that we can choose not to appear in the electric charge [28]. This gives charged $W^{\pm}$ fields as before plus three neutral fields W^0, B, B′ that mix to yield the photon and two boosons Z, Z′ coupled to corresponding neutral currents:

$$\mathscr{L}_{\rm int} = A_\mu J^A_\mu + Z_\mu(J^{SW}_\mu \cos\phi + J'_\mu \sin\phi) + Z'_\mu(J'_\mu \cos\phi - J^{SW}_\mu \sin\phi). \tag{60}$$

Thus the new hypercurrent J'_μ mixes with the old S–W current (defined by J^Z_μ of Eq. (16)). If $m_Z \ll m_{Z'}$, the Z_μ-interaction dominates at low Q^2 and we have effectively a single weak neutral current $J^{SW}_\mu \cos\phi + J'_\mu \sin\phi$, different from the S–W model.

Until summer 1978 this kind of approach was being urged to repair supposed defects in S–W, but now we must be careful instead to preserve all the successes. If for example we choose $Y' = 0$ for all leptons, the new current J'_μ will not appear in any lepton-lepton or lepton-hadron context: however the strength of J^{SW}_μ is affected, and we must arbitrarily impose

$$\cos^2\phi/m^2_Z + \sin^2\phi/m^2_{Z'} = \cos^2\theta/m^2_W \tag{61}$$

to recover the S–W NC successes.

Another scheme $SU(2)_L\times SU(2)_R\times U(1)$ introduces a new isospin $\vec{T}'$ instead [29]. There are three neutral gauge field W^0_L, W^0_R, B but also two charged gauge fields $W^\pm_L$, $W^\pm_R$ coupling to left and right-handed fermions respectively. We can now have a parity-conserving basic symmetry, and yet have the standard *V–A* CC interaction at small Q^2, by arranging $m_{W_R} \gg m_{W_L}$; i.e. parity violation comes from spontaneous symmetry breaking, an aesthetically attractive idea.

There are many other possibilities for extended flavour symmetry, but the data do not yet require any of them and no general survey is attempted here.

7. *More quarks and leptons*

(a) Six-quark six-lepton model

The discovery of the τ-lepton and presumed b-quark (charge-$\frac{1}{3}$, constituent of upsilon mesons) plus theoretical pressures of Sections 4 and 5 suggest that the scheme of Eqs. (1)–(2) be enlarged to at least six leptons and six quarks:

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L \begin{pmatrix}u\\ d'\end{pmatrix}_L \begin{pmatrix}c\\ s'\end{pmatrix}_L \begin{pmatrix}t\\ b'\end{pmatrix}_L. \tag{62}$$

The right-handed components are supposed to be singlets and d′, s′, b′ denote the most general mixings [30]

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{bmatrix} c_1 & s_1c_3 & s_1s_3 \\ -s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ -s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{bmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{63}$$

in terms of three mixing angles θ_i and one *CP*-violating phase δ, with $s_i = \sin\theta_i$, $c_i = \cos\theta_i$.

The parameterization of Eq. (63) is based on Euler angles (apart from δ) and suffers the associated pathologies. For example, if θ_1 and δ are small, big θ_2 does not necessarily imply big mixing anywhere; if $\theta_2+\theta_3$ is small, all the off-diagonal terms are small regardless of θ_2 by itself. So be wary.

In the limit $\theta_2, \theta_3 \to 0$, θ_1 is the original Cabibbo angle.

Note that CP conservation is compulsory in the 4-quark model; there is no room for a complex phase in the general 2×2 unitary mixing matrix.

(b) Limits on mixing angles

The analysis of Shrock and Wang [5] determines the mixing matrix elements for the $d \leftrightarrow u$ and $s \leftrightarrow u$ transitions, from $0^+ \to 0^+$ nuclear β-decays and semileptonic hyperon decays:

$$|d \leftrightarrow u| = |c_1| = 0.9737 \pm 0.0025, \qquad |s \leftrightarrow u| = |s_1c_3| = 0.219 \pm 0.011, \tag{64}$$

$$\theta_1 = 0.230 \pm 0.010, \qquad \theta_3 = 0.28^{+0.21}_{-0.28}. \tag{65}$$

Essentially the θ_3 value is an upper limit, $\theta_3 < 0.5$.

Barger et al. [31] and Shrock et al. [32] push further, using the second-order diagrams of Fig. 1 (including intermediate t) to calculate the K_L–K_S mass difference and a CP-violation parameter. In principle these give two more constraints on the mixing parameters, giving loose solutions in parameter-space. For example, Shrock et al. find two broad regions centred on the following values:

Solution I: $\delta \simeq 0$, $s_2 \simeq 0.2$ with

$$\left.\begin{aligned} d' &= 0.97d + 0.22s + 0.07b \\ s' &= -0.22d + 0.85s + 0.48b \\ b' &= -0.05d + 0.48s - 0.88b \end{aligned}\right\}, \tag{66}$$

Solution II: $\delta \simeq \pi$, $s_2 \simeq 0.5$ with

$$\left.\begin{aligned} d' &= 0.97d + 0.22s + 0.07b \\ s' &= -0.20d + 0.95s - 0.22b \\ b' &= -0.11d + 0.20s + 0.97b \end{aligned}\right\}, \tag{67}$$

where the very small *CP*-violating terms are omitted.

Note that the 2×2 Cabibbo submatrix of d′, s′, d, s is now distorted and non-unitary. In particular the strengths $|c\to d|$, $|c\to s|$ are no longer constrained to equal $|u\to s|$, $|u\to d|$. This affects charm decays and also charm production by neutrinos. The changes are not dramatic but may be measurable.

In both solutions $b\to c$ coupling is favoured over $b\to u$, so that cascade decays $b\to c\to s\to u$ are quite probable. Similarly in t-decay the most favoured first step is $t\to b$ leading to cascades again.

These parameterizations imply mean lifetimes [31, 32]

$$\tau_c(m=1.87)=(4-6)\times 10^{-13}\ \text{sec}$$

$$\tau_b(m=5)=(1-12)\times 10^{-14}\ \text{sec}$$

$$\tau_t(m=14)=(1.1-1.6)\times 10^{-17}\ \text{sec} \tag{68}$$

from approximate free-quark decays, including hard gluon corrections in the c case. The b-lifetime is enhanced (because it couples preferentially to t) and we may hope to see tracks eventually. Mean track length $\simeq \tau c\gamma \simeq$ 30–400 microns for typical $\gamma = 10$; we can detect decay lengths down to a few tens of microns in nuclear emulsions, a few hundred microns in high resolution bubble chambers.

t-flavour has not yet been found. Up to June 1979 there is no sign of any $e^+e^-\to t\bar{t}$ threshold up to $E_{c.m.}=27.4$ GeV [33], so the assumed mass $m_t = 14$ above is near the present lower limit. As mass increases $\tau(t)$ scales approximately with m_t^{-5}.

It may be unsafe to neglect further quarks; one conclusion from Eqs. (67) and (68) is that truncating the theory at four quarks can be misleading, and the same may be true for truncating at six. Also Wolfenstein [34] has criticised the theoretical assumptions made in Refs. [31–32], arguing that the mixing angles are little constrained beyond Eqs. (64)–(65). Nevertheless the solutions above are interesting if only for the points they illustrate and the questions they raise.

(c) Multilepton b and t signatures

Cascade b and t decays can give spectacular multilepton signals [35] since at each stage the virtual W-quantum can emit a charged lepton plus neutrino with probability 5–10% (except the $s\to u$ stage where hadronic enhancement pushes lepton modes down to 10^{-3}). Thus $b\to c l\bar{\nu}$, $c\to s\bar{l}\nu$ gives two charged leptons. As an extreme example $t\to b\bar{b}c$ can give 5 charged leptons, but with miniscule branching fraction $\leqslant 10^{-5}$.

For example, in $e^+e^-\to b\bar{b}$ production we can study multi-electron signals [36, 37]

$$\begin{aligned} e^+e^- \to\ & e^\pm X \\ & e^+e^-X \\ & (e^+e^+ + e^-e^-)X \\ & (e^+e^+e^- + e^-e^-e^+)X \\ & e^+e^+e^-e^-X. \end{aligned} \tag{69}$$

The first two channels have contributions from $c\bar{c}$ and $\tau\bar{\tau}$ production that may be hard to subtract accurately. The other three channels have no such backgrounds (D^0–$\bar{D}^0$ mixing is observed to be very small), and their rates alone are enough to determine three basic parameters of b-decay (*i*) the branching fraction for $b \to cX$ (*ii*) the branching fraction for $b \to eX$ (*iii*) the degree of B^0–$\bar{B}^0$ mixing before decay. *CP*-violation can be detected and measured, e.g. through an asymmetry between e^+e^+X and e^-e^-X modes. Inclusive b-decays may be calculated as free-quark transitions to a first approximation [35].

In "unnatural" models (e.g. no t-quark, b a singlet) there are flavour changing neutral currents giving $b \to (s, d)e^+e^-$ decays. In principle such terms can be detected by comparing $e^+e^- \to e^+e^+e^-X$ that contains such contributions with $e^+e^- \to e^+e^+\mu^-X$ that does not [35].

A difficulty with multilepton modes is the small rate. Taking an optimistic luminosity of 5×10^{31} cm^2 sec^{-1} with $\Delta R(b\bar{b}) = \frac{1}{3}$ and a four-electron branching fraction 10^{-4} gives an idealized $e^+e^- \to e^+e^+e^-e^-X$ rate of 30 events per year at $E_{cm} = 12$ GeV. Realistic acceptance factors and accelerator running periods reduce this to more like one event per calendar year, making slow statistics. Fortunately the e^+e^+X and e^-e^-X same-sign dilepton signals are one hundred times stronger: when eventually seen, they should be a useful measure of b-production.

Another useful source of information is the shape of the single-lepton spectrum [38, 39] in $e^+e^- \to b\bar{b} \to l^\pm X$. $b \to u$ transitions give a rather hard $l^\pm$ spectrum. $b \to c$ transitions have a 13% lower endpoint and bring a large soft component from second-

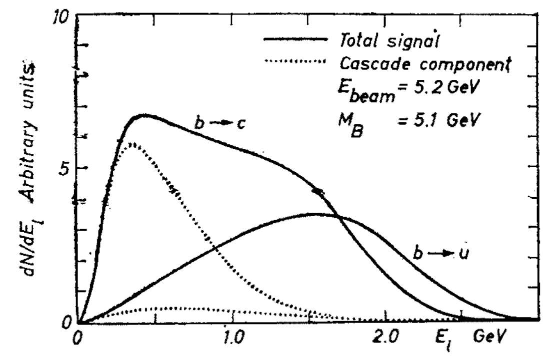

Fig. 3. Single lepton spectrum for $e^+e^- \to \bar{b}b \to l^\pm X$ from Ref. [39]

-stage cascades. (There is a cascade component in the first case too, from $b \to uc\bar{s}$ etc., but it is much smaller). By interpolating linearly between these extremes, we can determine the coupling ratio $|b \to u|/|b \to c|$ directly from the spectrum shape. The measurement is cleanest near threshold, preferably on a narow $\bar{b}b$ resonance. Figure 3 shows calculated spectrum shapes, from Ref. [39].

(d) Neutrino production of b, t

In principle production by neutrinos and antineutrinos would directly measure the weak couplings of b and t, but the expected cross-sections at present accelerators are pitifully small. Couplings to the more abundant valence quarks $u \to b$ and $d \to t$ are suppressed by the Cabibbo angle θ_1 (see Eq. (63)), and the high masses of b and t bring heavy threshold suppression factors [40, 41].

Figure 4 illustrates upper limits for b-production on nuclear targets as a fraction of the total CC cross section, corresponding to the phenomenological upper bounds [31, 32]: $|u \to b|^2 \leqslant 0.013$, $|c \to b|^2 \leqslant 0.5$, $|t \to b|^2 \leqslant 1$. The upper limits on t-production are

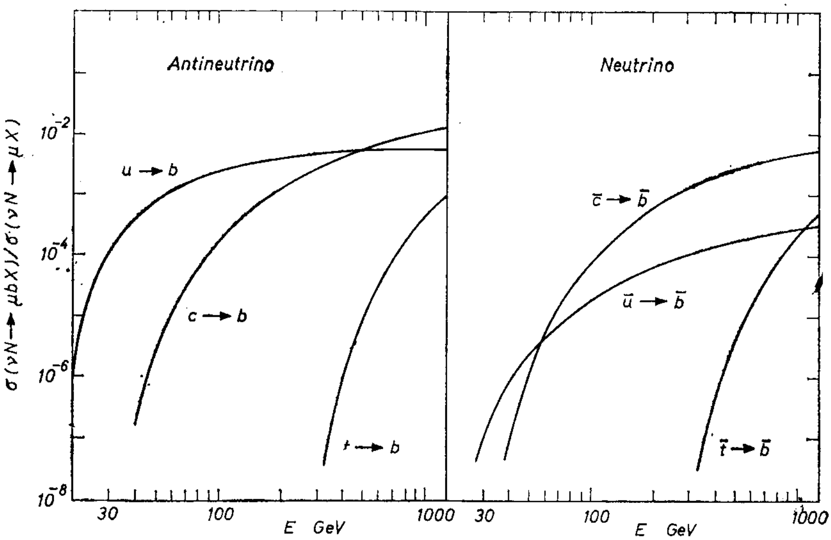

Fig. 4. Upper limits on b-production by ν and $\bar{\nu}$ from Ref. [40]

even more discouraging. There are also problems in detecting b and t flavours at such low production levels [41, 42].

Many theoretical models of quark masses and mixing angles (i.e. models of the Higgs sector) give much smaller mixings than the empirical limits used above: see Weyers' lectures [10].

8. *Grand unification*

It is attractive to suppose that our flavour gauge groups SU(2)×U(1) are commuting subgroups of a more far-reaching gauge group $\mathscr{G}$. If $\mathscr{G}$ also contains the colour group SU(3) of QCD,

$$\mathscr{G} \supset SU(3) \times SU(2) \times U(1) \tag{70}$$

we have "Grand Unification", uniting weak, electromagnetic and strong interactions in a single framework.

There are several advantages, compared to S–W plus QCD separately.

(*i*) One single gauge coupling (assuming $\mathscr{G}$ is simple) compared to three: α_s, α, $\sin^2\theta$.

(*ii*) Charge quantization. Previously this has been put in by hand; e.g. the hypercharge assignments $Y = \frac{1}{3}$ for $(u, d)_L$, $Y = \frac{4}{3}$ for u_R, $Y = -\frac{2}{3}$ for d_R have been imposed arbitrarily to get the conventional quark charges. There is nothing in SU(2) × U(1) to prohibit electric charges of $\pi/2$ or whatever.

(*iii*) Leptons and quarks appear in the same multiplet. At least this economizes multiplets.

(*iv*) Exotic gauge bosons appear, mediating lepton ↔ quark and even quark ↔ antiquark transitions. This may not be exactly an advantage, but it is interesting and makes protons unstable.

Pati and Salam [42] made the first suggestion $\mathscr{G}$ = SU(4) × SU(4), with the two gauge couplings reduced to one by a discrete symmetry. However their development employs integrally charged quarks and unconfined colour, outside the scope of the present lectures. Subsequent popular examples include $\mathscr{G}$ = SU(5) proposed by Georgi and Glashow [43], $\mathscr{G}$ = SO(10) proposed by Fritzsch and Minkowski [44] and E6 suggested by Gursey et al. [45].

(a) $\mathscr{G}$ = SU(5)

In this theory [43] the first generation of leptons and quarks are assigned to a 5-plet and a 10-plet representation,

$$[5] = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \bar{\nu} \end{bmatrix}_R \quad [10] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \bar{u}_3 & -\bar{u}_2 & -u_1 & -d_1 \\ -\bar{u}_3 & 0 & \bar{u}_1 & -u_2 & -d_2 \\ \bar{u}_2 & -\bar{u}_1 & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{bmatrix}_L . \tag{71}$$

showing the quark colour indices 1, 2, 3. Note that quarks, antiquarks and antileptons appear on the same footing. The gauge bosons correspond to the generators of SU(5), in the adjoint 24-plet representation. Beside 12 conventional bosons (γ, $W^\pm$, Z, 8 gluons) there are 12 exotic gauge bosons; through them any fermion can transmute into any other fermion in the same multiplet. After spontaneous symmetry breaking the exotic bosons must be superheavy to suppress unobserved exotic processes.

The second and third generations of fermions (ν_μ, μ, c, s), (ν_τ, τ, t, b) appear in two more 5-plets plus two more 10-plets, and mixing occurs in general.

This scheme is very economical: no new fermions are needed to fill the multiplets. No right-handed neutrinos appear, so neutrinos are automatically massless — a bonus. There are no anomalies, the 5-plet and 10-plet contributions cancel. The Weinberg angle is predicted, $\sin^2\theta = \frac{3}{8}$.

(b) How to deduce $\sin^2\theta$

From Eq. (16) the internal symmetry operators of the electromagnetic and weak currents are (within a common normalization),

$$(J^A) = g \sin\theta T^3 + \tfrac{1}{2} g' \cos\theta Y = T^3 + \tfrac{1}{2} Y,$$

$$(J^Z) = g \cos\theta T^3 - \tfrac{1}{2} g' \sin\theta Y = T^3 \cot\theta - \tfrac{1}{2} Y \tan\theta. \tag{72}$$

Under grand unification the A and Z fields belong to a common multiplet, and their associated currents too transform like the adjoint 24-plet representation. Hence, summing over any complete multiplet of fermions, the current operators above must be traceless and have the same normalization:

$$\mathrm{trace}\,(J^{\mathrm{A}}) = \mathrm{trace}\,(J^{\mathrm{Z}}) = 0, \qquad \mathrm{trace}\,(J^{\mathrm{A}})^2 = \mathrm{trace}\,(J^{\mathrm{Z}})^2. \tag{73}$$

With Eq. (72) this gives

$$\tan^2\theta = \sum (T^3)^2 / \sum \tfrac{1}{4} Y^2, \qquad \sin^2\theta = \sum (T^3)^2 / \sum Q^2, \tag{74}$$

summing over the particles in any multiplet. For example the 5-plet in Eq. (71) has $\sum (T^3)^2 = \frac{1}{2}$ and $\sum Q^2 = \frac{4}{3}$ giving $\sin^2\theta = \frac{3}{8}$ as advertized.

Eq. (74) is a handy formula. Incidentally it shows that any two grand unified theories that have exactly the same particles — in no matter how many multiplets — must have the same $\sin^2\theta$ in the symmetry limit.

(c) Renormalization

Coupling constants are generally defined with respect to a mass scale μ, characterizing the momenta at a three-particle vertex. Coupling constants $g_i(\mu)$ change as μ changes, according to the renormalization group equations (for recent reviews see Ref. [46]).

Suppose that grand unified symmetry $\mathcal{G}$ is exact above some large mass scale M, and that all the exotic gauge bosons (not associated with SU(3)×SU(2)×U(1)) have masses near M. No gauge bosons can have masses $> M$ or $\mathcal{G}$ would be broken there; we could let some of the exotics have masses much less than M, but the way to minimize their effects is to keep them all near M. Then for mass scales $\mu \ll M$ the symmetry breaks down to SU(3)×SU(2)×U(1) and the corresponding three coupling constants evolve according to [47, 48]

$$(g_i(\mu))^{-2} = (g(M))^{-2} + 2b_i \ln (M/\mu),$$

$$b_3 = -11/(16\pi^2) + b_1, \quad b_2 = -11/(24\pi^2) + b_1, \quad b_1 = f/(24\pi^2), \tag{75}$$

where f is the number of quark flavours. The couplings start from a common value near $\mu = M$ but diverge as μ decreases as sketched in Fig. 5. Here g_3 is the gluon coupling,

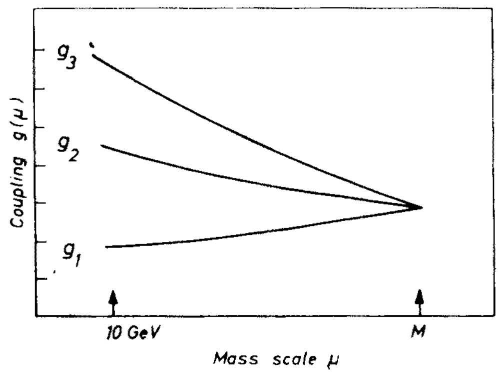

Fig. 5. Dependence of couplings on mass scale

g_2 is the SU(2) coupling g and g_1 is the U(1) coupling g' normalized appropriately for the $\mathscr{G}$-symmetry limit (see §8(b) above):

$$g_3 = (4\pi\alpha_s)^{1/2}, \quad g_2 = e/\sin\theta, \quad g_1 = (5/3)^{1/2}e/\cos\theta,$$
$$\tan\theta = (3/5)^{1/2}g_1/g_2. \tag{76}$$

Eq. (75) contains the leading terms that dominate behaviour for small g_i; in this approximation the g_i evolve independently. However, we have to stop using Eq. (75) around $\mu \sim 10$ GeV, since the gluon coupling is known to get big if we go much further. From the general characteristics of Eq. (75) indicated in Fig. 5, we see that θ decreases as μ decreases — which is just as well, since the $\mathscr{G}$-symmetry value for $\sin^2\theta$ at $\mu = M$ is much too big.

Eq. (75) is attractively simple to solve. There are three parameters M, $g(M)$ and f and one might believe it was possible to arrange α_s, α, θ independently. Not so, there is an important constraint

$$2\alpha = \alpha_s(3\sin^2\theta - 0.6\cos^2\theta) \tag{77}$$

holding at any mass scale μ. If we take $\alpha = 1/137$ we find the following sets of corresponding pairs:

$$\alpha_s = 0.20, \quad 0.10, \quad 0.05$$
$$\sin^2\theta = 0.19, \quad 0.21, \quad 0.25 \tag{78}$$

which look remarkably close to the physical facts. The underlying assumption is simply that the SU(3), SU(2) and U(1) couplings have independently migrated from a common grand-unified origin. It seems to check out well.

To find the grand-unified mass scale M, input α and α_s (or θ) at μ and integrate:

$$M = \mu\exp[3\pi(3\cos^2\theta - 5\sin^2\theta)/(55\alpha)] = 4.4\times10^{13}\text{ GeV}, \tag{79}$$

for $\mu = 10$, $\sin^2\theta = 0.22$, $\alpha_s = 0.08$. For academic interest, the grand-unified coupling is

$$g(M) = e[0.6\cos^2\theta + f(\sin^2\theta - 0.6\cos^2\theta)]^{-1/2} \tag{80}$$

independent of the mass scale of e and θ.

The approximations above are somewhat simplified. The most complete study to date is by Goldman and Ross [49] who include higher-order corrections to Eq. (75) plus effects of fermion and gauge boson thresholds, plus Higgs scalar corrections, plus the change of α between $\mu \simeq 0$ where it is measured ($\alpha = 1/137.0$) and $\mu = 2m_W$ where it is input ($\alpha = 1/130.4$). They find e.g.

$$M = 2.7\times10^{14}\text{ GeV} \tag{81}$$

for $\alpha_s(\mu = 10) = 0.22$ with six quark flavours.

These huge values for the grand unification mass are approaching the Planck mass

$$m_{\text{Planck}} = \left[\frac{\hbar c^5}{G_{\text{grav}}}\right]^{1/2} = 1.2\times 10^{19}\ \text{GeV} \tag{82}$$

and suggest that gravity may come into the story somewhere.

(d) Proton instability

Exotic gauge bosons (and Higgses) allow transitions like $u\bar{u} \to e^+\bar{d}$, $ud \to e^+\bar{u}$, $ud \to \bar{\nu}\bar{d}$ and hence proton decays [47–50] by $p \to e^+\pi^0$, $\nu\pi^+$ etc., as sketched in Fig. 6. The decay amplitudes are of order M^{-2}, so on dimensional grounds we expect $\tau_p \sim M^4/m_p^5$. For the SU(5) case Buras et al. [48] estimate the numerical coefficient in this formula to get

$$\tau_p \simeq (10^3 - 10^4) M^4/m_p^5 \tag{83}$$

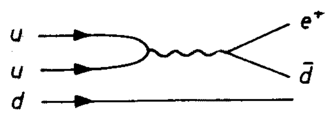

Fig. 6. Typical proton decay diagram

and Ref. [50] also gives a coefficient 10^3. For an input M like Eq. (81) this gives $\tau_p \sim (1.5$–$15)\times 10^{29}$ years (using $\hbar\ \text{GeV}^{-1} = 2.1\times 10^{-32}$ years). This lies right on the experimental bound of Reines and Crouch [51]:

$$\tau_p(\text{exp}) > 2\times 10^{30}\ \text{years}. \tag{84}$$

However this determination was based on muonic decay modes, arguably $\lesssim 10\%$ of all decays, in which case the true lower limit is a factor 10 smaller [50].

On the theoretical side, Goldman and Ross [49] estimate a conservative upper limit for SU(5) models to be

$$\tau_p(\text{theory}) < 10^{32}\ \text{years} \tag{85}$$

so small improvements on either side may bring a confrontation or a discovery. This has therefore become a hot topic. However, there is still an amusing theoretical escape route; the mixing matrices for exotic transitions (analogous to Eq. (63)) are unknown and independent of the normal weak sector, and could in principle suppress proton decays [52].

Note that neutrons, normally stable when bound in stable nuclei, would have analogous decays $n \to e^+\pi^-$, etc. too.

(e) $\mathscr{G}$ = SO(10)

This scheme [44] has a classification similar to SU(5) that is a subgroup. All fermions of a given generation appear in a single 16-plet representation, that breaks up into a 10-plet plus a 5-plet plus a singlet of SU(5).

$$[16] = [10] + [5] + [1] \tag{86}$$

Using a single representation is an advantage over SU(5). However, the extra singlet is just the right-handed neutrino — and we lose the previous zero mass prediction. Also, if just one representation occurs, why is it [16] rather than the fundamental [10]?

Since we have the same particles as in SU(5) (except for ν_R with $T = Y = 0$), the $\sin^2 \theta$ prediction is the same: see Eq. (74). The SU(3) × SU(2) × U(1) renormalization story is also the same.

(f) $\mathscr{G}$ = E6

This exceptional group [45, 53] is specially interesting because it suggests a classification with no t-quark. E6 contains three independent SU(3) subgroups that may be identified with colour and with left and right-handed flavour groups

$$E6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R. \tag{87}$$

(To recover the low-energy $SU(2)_L \times U(1)_{L+R}$ flavour groups, each flavour SU(3) should break down to SU(2) × U(1) and $SU(2)_R$ should have very heavy gauge bosons). Anyway, corresponding to Eq. (87) the 27-plet fundamental representation of E6 decomposes into

$$[27] = [1, 3^*, 3]+[3^*, 3, 1]+[3, 1, 3^*]. \tag{88}$$

The first term on the right is colour-singlet and presumably represents leptons; the other two are colour-antitriplet and triplet, representing quarks and antiquarks. If we now display $SU(3)_L$ triplets vertically, $SU(3)_R$ triplets horizontally, and suppress colour indices, these multiplets can be identified as

$$\begin{pmatrix} \bar{N}_T^0 & T^- & e^- \\ T^+ & N_T^0 & \nu_e \\ e^+ & \bar{\nu}_e & N^0 \end{pmatrix}_L, \quad \begin{pmatrix} u \\ d \\ h \end{pmatrix}_L, \quad (\bar{u}, \bar{d}, \bar{h})_L. \tag{89}$$

Notice that $SU(3)_R$ couples to R-quarks and hence L-antiquarks. (Variations of these assignments are also suggested [45, 53].)

The usual $SU(2)_L$ doublets, related by normal CC transitions, are vertical pairs in the top and middle rows of Eq. (89) (inverted in the lepton case that is 3*). Here we recognize the usual $(u, d)_L$ and $(\nu_e, e^-)_L$ doublets, plus two new doublets $(N_T^0, T^-)_L$ and $(T^+, \bar{N}_T^0)_L$. If we identify $T = \tau$, $N_T^0 = \nu_\tau$, this implies that the $\tau \to \nu_\tau$ coupling has both L and R components (not excluded by data [54]). There is also a new neutrino N^0 and a new quark h with charge $Q = -\frac{1}{3}$ that may be identified with the b-quark; there must be some d–h mixing to let it decay. The u_R, d_R, h_R components are singlets of $SU(2)_L$ as usual. This is the e-family.

In a similar way the μ-family includes c and s quarks, plus further new leptons and another new b-type quark. Notice we need only two families to accommodate the known fermions, compared to three for SU(5) or SO(10). This is a *topless scheme* as advertised; the presence of b and τ does not imply any extra $Q = \frac{2}{3}$ quark. However, there is an *extra b-quark* plus several new leptons. The scheme is not "natural" in the sense of §4, and b → d neutral currents are expected.

Should a new t-quark be found, however, E6 is not necessarily dead. An alternative subgroup and representation decomposition is

$$E6 \supset SO(10) \times U(1), \qquad [27] = [1]+[10]+[16]. \tag{90}$$

A 27-plet of Higgs fields can achieve this breakdown, and at the same time give a high mass to the 10-plet fermions, leaving just the original 16-plet of SO(10) with a t-quark in the third generation. Then why bother with E6 at all? Because [27] is the fundamental representation for E6, and this approach explains why the non-fundamental [16] of SO(10) appears [53].

Incidentally E6 is anomaly-free and gives $\sin^2 \theta = 3/8$ in the symmetry limit.

9. *Economy of parameters*

These lectures have centred on the structure of the currents and their couplings. Here there is enormous economy. The universal V–A CC model for six leptons and six quarks has one coupling strength and four mixing parameters (plus four more if neutrinos have mass), whereas in general we can imagine 36 different $V \pm A$ charged currents with up to 1286 independent couplings (removing 10 arbitrary relative phases). The $SU(2) \times U(1)$ gauge model then determines all NC couplings through at most two parameters (θ and ϱ), whereas in general we can imagine 72 $V \pm A$ neutral currents with up to 2628 independent couplings.

Grand unified theories explain charge quantization, justifying 24 hypercharge assignments previously put in by hand, and fix $\sin^2 \theta$. They bring in a lot more heavy gauge bosons, but their masses and mixing angles scarcely enter ordinary physics except for nucleon instability.

Nevertheless there still remain all those fermion mixing parameters plus twelve fermion masses, that come ultimately from the Higgs couplings. More simplification can be expected from more specific theories of the Higgs sector. But that is another story, for which you should listen to Jacques Weyers [10].

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