ARE LEPTON AND QUARK FAMILIES QUANTIZED DYNAMICAL SYSTEMS?

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Lepton and quark families $\{\nu_N\}$, $\{e_N\}$ and $\{u_N\}$, $\{d_N\}$ are conjectured to be quantum-dynamical systems in the space of generations $N = 0, 1, 2, \ldots$. Such a conjecture, called the "zeroth quantization", implies the form of lepton and quark mass spectra, if lepton and quark families are supposed to be almost in thermal equilibrium with the rest of the Universe. Toponium is predicted at about 38 GeV, while the next charged lepton at 28.5 GeV (in accord with the author's previous predictions made on a more phenomenological ground).

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In order to introduce the idea which is proposed in this paper, let us begin with the following pedagogical example. Consider a bound particle in an external potential (e.g. an electron in a hydrogen atom, where the proton is treated as infinitely heavy). Assume further that this system is in thermal equilibrium with a heat reservoir (e.g. our hydrogen atom may be put into a cavity with black-body radiation). Then the particle is in a mixed state given by the density matrix [1]

$$\varrho = \sum_n |n\rangle \varrho_n \langle n|, \quad \sum_n \varrho_n = 1, \tag{1}$$

where $H|n\rangle = E_n|n\rangle$ and

$$\varrho_n = \frac{1}{Z} \exp(-\beta E_n), \quad Z = \sum_n \exp(-\beta E_n), \quad \beta = 1/kT. \tag{2}$$

In the mixed state ϱ, the mass m of the particle is statistically distributed over the component states n with the weights ϱ_n

$$\mu_n = m\varrho_n, \quad \sum_n \mu_n = m. \tag{3}$$

Of course, if the state ϱ were a pure state n_0, i.e. if $\varrho = |n_0\rangle\langle n_0|$, then the total amount of mass m would be concentrated in the particular component state n_0, $\mu_n = m\delta_{nn_0}$.

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Obviously, *the numbers μ_n are not masses of the particle in the component states n, but masses of the fractions of the particle in these component states.* Note that the same as above concerning the mass m can be also said about the electric charge Q of the particle.

The above example concerns the familiar level of quantum theory which is the first quantization. It may suggest, however, some extensions in a new direction. Namely, one may ask the exciting question, whether there is possibly a more primordial level of quantum theory, where *particles are fractions of some new quantum-dynamical systems which might be called the "particle families"*. If it is so, the latter are to be identified with the experimentally observed (directly or indirectly) lepton or quark families $f = \nu, e, u, d$, each consisting of several generations $N = 0, 1, 2, \ldots$, viz. [2]

$$f = \{f_N\}, \quad f_N = \begin{cases} \nu_N = \nu_e, & \nu_\mu, \quad \nu_\tau, \ldots \\ e_N = e, & \mu, \quad \tau, \ldots \\ u_N = u, & c, \quad (t), \ldots \\ d_N = d, & s, \quad b, \ldots \end{cases} \tag{4}$$

where electric charges of f_N particles are independent of N, $Q_{f_N} = Q_f$ with $Q_f = 0, -1, \frac{2}{3}, -\frac{1}{3}$ for $f = \nu, e, u, d$, respectively. Such a level of quantum theory, where particle families are quantum-dynamical systems, may be called the "*zeroth quantization*" [3] because it is analogical to but logically precedes the first quantization, where particles are quantum-dynamical systems. On the other hand, in the second quantization logically following the first, particle fields are quantum-dynamical systems.

If the conjecture of the zeroth quantization is true, each lepton or quark family f can be described by the density matrix

$$\varrho_f = \sum_N |N\rangle \varrho_{f_N} \langle N|, \quad \sum_N \varrho_{f_N} = 1, \tag{5}$$

where $H_f|N\rangle = E_{f_N}|N\rangle$, the operator H_f being an unknown Hamiltonian of f family in the zeroth quantization. The density matrix (5) gives the statistical distribution of some mass M_f of f family over the generations $N = 0, 1, 2, \ldots$ with the weights ϱ_{f_N}

$$m_{f_N} = M_f \varrho_{f_N}, \quad \sum_N m_{f_N} = M_f < +\infty. \tag{6}$$

Here, *the numbers m_{f_N} are masses of f_N particles as the latter are the fractions of f family in the component states N*[1]. We can see from Eq. (6) that *the masses m_{f_N} of f_N particles depend on the state ϱ_f of f family*. The observed masses m_{f_N} correspond to the actual state ϱ_f which ought to be theoretically determined. Note that, in contrast to the mass $M_f = \sum_N m_{f_N}$, no electric charge $\sum_N Q_{f_N}$ can be ascribed to f family, because all f_N particles possess equal electric charges $Q_{f_N} = Q_f$, so that $Q_{fN} \neq (\sum_N Q_{f_N})\varrho_{f_N}$. Thus, electric charge as well as all

[1] It is possible that $M_f = \langle H_f \rangle = \sum_N E_{fN}\varrho_{fN}$. Than $m_{fN} = E_{fN_0}\delta_{NN_0}$ if ϱ_f were a pure state N_0 i.e. if $\varrho_f = |N_0\rangle\langle N_0|$.

space-time quantities (e.g. position, momentum and spin) seem to be particle attributes. The first quantization is needed to localize in space-time the fractions of particle families which become particles thereby. Then the second quantization allows them to interact via local field interactions connected with the existence of particle charges.

To proceed further with the idea of the zeroth quantization and then pass to its applications we must be more specific about the Hamiltonian H_f and the distribution ϱ_{f_N}.

First of all let us note that *the particle family f appears as a one-dimensional quantum-dynamical system* since its states can be apparently numerated by one quantum number $N = 0, 1, 2, \ldots$. So, as a first guess, it is very natural to try for f family a one-dimensional oscillator with the canonical variables $q = (a+a^+)/\sqrt{2}$ and $p = (a-a^+)/i\sqrt{2}$, where $[a, a^+] = 1$. Then the Hamiltonian of f family is

$$H_f = \omega_f a^+ a + \theta(N_f - a^+ a)V_f, \tag{7}$$

where $\theta(x) = 0$ or 1 for $x \geqslant 0$ or $x < 0$ and $V_f \to \infty$ in order to provide the possible cut $N \leqslant N_f$ for the number of generations of f family. The eigenvalues of H_f representing the energy levels of f family are

$$E_{f_N} = \omega_f N \quad \text{for} \quad N \leqslant N_f. \tag{8}$$

Assumption (7) may be considered as the zero-order approximation in a weak coupling regime for the zeroth quantization. In this case, introducing the phase ϕ of $a^+ = \sqrt{a^+a}\exp(\pm i\phi)$ which is canonically conjugate to $\pm a^+a = -id/d\phi$, we get, respectively, for the dynamical systems considered on the level of the zeroth, first and second quantization the following co-ordinates:

$$\phi, \quad \vec{r}(\phi), \quad \psi(\vec{r}), \tag{9}$$

where $\vec{r}(\phi)$, or relativistically $x^\mu(\phi)$, represents a *string* reminding us of the dual model! However, in contrast to this model where the usual first quantization follows by averaging [4]

$$\vec{r} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi \vec{r}(\phi), \tag{10}$$

in our case the usual first quantization can be obtained for f_N particles ($N = 0, 1, 2, \ldots$) by the Fourier transformation

$$\vec{r}_N = \frac{1}{\sqrt{\pi}}\int_{-\pi}^{\pi} d\phi \vec{r}(\phi)\cos(N\phi). \tag{11}$$

So the effect of dual excitations around the usual first-quantization motion is absent in our case.

The Hamiltonian (7) of f family is perturbed by the interaction with the rest of the Universe which is responsible for setting up the distribution ϱ_{f_N} of f family. Let us suppose that, if the electric charges Q_f of f_N particles can be neglected, f family is in thermal equilibrium with the rest of the Universe considered as a heat and N-number reservoir. Then we get the grand canonical distribution

$$\varrho_{f_N}|_{Q_f=0} = \frac{1}{Z_f}\exp\,[\beta(\mu_f N - E_{f_N})], \tag{12}$$

where

$$Z_f = \sum_N \exp\,[\beta(\mu_f N - E_{f_N})], \quad \beta = 1/kT, \tag{13}$$

the constant μ_f being a chemical potential. The restriction to the approximation of $Q_f = 0$ is in Eq. (12) necessary, since electric charge contributes to particle mass and so it is a kind of "source" of the "field" ϱ_{f_N} in the space of generations $N = 0, 1, 2, \ldots$ (other virtual "sources" of ϱ_{f_N} are neglected here).

If $\mu_f \neq 0$, the grand canonical ensemble (12) implies the conservation of N number for the Universe as a whole. This conservation, appearing here on the level of the zeroth quantization, corresponds to the (sometimes approximate) generation conservations on the level of the second quantization (where N number for an antiparticle comes with minus sign into the balance). For example, the N-number conservation forbids such unwanted processes as $\mu^- \to e^-e^+e^-$, $\mu^- \to e^-\gamma$ and $\tau^- \to e^-e^+e^-$ or $\mu^-e^+e^-$ or $\mu^-\mu^+\mu^-$, $\tau^- \to e^-\gamma$ or $\mu^-\gamma$, but it would allow for $\tau^- \to \mu^-\bar{\nu}_e\nu_\mu$ and/or $\tau^- \to \mu^-e^+\mu^-$ if there were charged and/or neutral intermediate bosons of the generation $N = 1$ (which, however, would get presumably much larger masses than $W^\pm$ and Z belonging to the generation $N = 0$). The proton decay, as e.g. $p \equiv uud \to e^+\bar{\nu}_e\bar{\nu}_e$, is also allowed from the viewpoint of N-number conservation, depending on the existence of appropriate intermediate bosons (of the generation $N = 0$, but necessarily with a big mass). Of course, the generation conservations in weak processes involving quarks are only approximate in the cases where Cabibbo-like mixing appears.

Now, let us switch on the electric charges Q_f of f_N particles and suppose that the thermal equilibrium (12) is then perturbed linearly by a term proportional to $Q^2_{f_N} = Q^2_f$

$$\varrho_{f_N} = A_f \varrho_{f_N}|_{Q_f=0} + B_f Q_f^2, \tag{14}$$

where

$$A_f + B_f Q_f^2 \bar{N}_f = 1, \quad \bar{N}_f = N_f + 1 \tag{15}$$

due to the normalization condition $\sum_N \varrho_{f_N} = 1$. Here, because of the constant term in the distribution (14), it must be assumed that the number of generations of f family is finite, $N \leqslant N_f < +\infty$.

In consequence of Eqs (8) and (12)–(15) one can eventually write

$$\varrho_{f_N} = (1 - B_f Q_f^2 \bar{N}_f)\frac{1}{Z_f}\lambda_f^{2N} + B_f Q_f^2, \tag{16}$$

where

$$Z_f = \frac{\lambda_f^{2\bar{N}_f} - 1}{\lambda_f^2 - 1}, \quad \lambda_f^2 = \exp\,[\beta(\mu_f - \omega_f)]. \tag{17}$$

We can see that in the case of Eqs (8), (12) and (14) each f_N particle of N generation (being the fraction of an f family in N state) contains N identical Bose objects characterized thermodynamically — on the level of the zeroth quantization — by the energy ω_f and chemical potential μ_f. These objects, playing the role of excitation quanta, may be called the "*generatons*" as they are responsible for the higher generations $N > 0$ of leptons and quarks.

In order to compare the distribution (16) with experimental data, we multiply it by M_f, obtaining the mass spectrum of f family. Introducing into it the "initial" mass values m_{f_0} and m_{f_1} we obtain the following spectral formula:

$$m_{f_N} = (m_{f_0}+\kappa_f Q_f^2)\lambda_f^{2N}-\kappa_f Q_f^2, \tag{18}$$

where

$$\kappa_f Q_f^2 = -M_f B_f Q_f^2 = \frac{m_{f_1}-\lambda_f^2 m_{f_0}}{\lambda_f^2-1}. \tag{19}$$

As will shortly be seen, experimentally $\lambda_f > 1$ and $\kappa_f > 0$. The mass spectrum (18) satisfies the simple recurrence formula

$$m_{f_{N+1}} = \lambda_f^2 m_{f_N}+(\lambda_f^2-1)\kappa_f Q_f^2 \tag{20}$$

with an inhomogeneous term proportional to Q_f^2 which shows explicitly that electric charge is here a "source" of particle mass. The spectrum (18) and the recurrence formula (20) were previously proposed on a more phenomenological ground [5].

In the case of leptons, Eq. (20), if fitted to the masses m_e, m_μ and $m_\tau = 1782^{+2}_{-7}$ MeV [6], gives

$$\lambda_e = 3.993^{+0.004}_{-0.009}, \quad (\lambda_e^2-1)\kappa_e = 97.51^{+0.01}_{-0.04}\ \text{MeV} \tag{21}$$

and then predicts [5]

$$m_{e_3} = 28.5^{+0.2}_{-0.5}\ \text{GeV}, \quad m_{e_4} = 455^{+3}_{-10}\ \text{GeV} \tag{22}$$

and also

$$m_{\nu_N} = 0 \quad (N > 0) \tag{23}$$

if $m_{\nu_e} = 0$ (otherwise $m_{\nu_N} \simeq 16^N m_{\nu_e}$ when $\lambda_\nu \simeq \lambda_e$).

In the case of quarks, if taking the small current masses $m_u \simeq 0 \simeq m_d$ (of a few MeV) and assuming that

$$m_u \ll \tfrac{4}{9}\kappa_u, \quad m_d \ll \tfrac{1}{9}\kappa_d, \tag{24}$$

one gets from Eq. (18)

$$m_{u_N} \simeq \tfrac{4}{9}\kappa_u(\lambda_u^{2N}-1), \quad m_{d_N} \simeq \tfrac{1}{9}\kappa_d(\lambda_d^{2N}-1) \quad (N > 0). \tag{25}$$

Thus, if

$$\lambda_u \simeq \lambda_d, \quad \kappa_u \simeq \kappa_d, \tag{26}$$

one concludes that $m_{u_N} \simeq 4m_{d_N}$ and in particular

$$m_c \simeq 4m_s \simeq \tfrac{4}{9}(\lambda_u^2-1)\kappa_u, \quad m_t \simeq 4m_b \simeq \tfrac{4}{9}(\lambda_u^4-1)\kappa_u \tag{27}$$

(cf. the mass relations discussed on a different ground in Ref. [7]. Hence, if $m_c \simeq 1.5$ GeV and $m_b \simeq 5$ GeV, one obtains

$$m_s \simeq 0.38 \text{ GeV}, \quad m_t \simeq 20 \text{ GeV}, \tag{28}$$

predicting toponium $t\bar{t}$ at about 38 GeV [5] as $m_{t\bar{t}} \simeq 4m_{b\bar{b}} = 38$ GeV for $m_{b\bar{b}} = 9.5$ GeV. One also obtains

$$\lambda_u \simeq 3.5, \quad (\lambda_u^2-1)\kappa_u \simeq 3.4 \text{ GeV}. \tag{29}$$

Then from Eqs (25) and (26) one predicts

$$m_{u_3} \simeq 250 \text{ GeV}, \quad m_{d_3} \simeq 62 \text{ GeV}. \tag{30}$$

One can verify a posteriori that the assumption (24) is satisfied for m_u very well, while for m_d with the factor 5 or so (for m_u and m_d equal to a few MeV).

Note that the mass spectrum (18) and its normalization condition $\sum m_{f_N} = m_f < +\infty$ confirm the necessity of a finite number of lepton and quark generations, $N \leqslant N_f < +\infty$, since $\lambda_f > 1$ (see Eqs (21), (26) and (29) and in the spectrum (18) there is a constant term. Explicitly, the normalization condition gives

$$(m_{f_0}+\kappa_f Q_f^2)Z_f - \kappa_f Q_f^2 \bar{N}_f = M_f. \tag{31}$$

It would be very natural to speculate [5] that the upper bound for N should be caused by the gravitational cut for the magnitude of particle masses

$$m_{f_N} \lesssim \sqrt{\frac{\hbar c}{2G}} = \frac{m_{\mathrm{PL}}}{\sqrt{2}}, \tag{32}$$

where $m_{\mathrm{PL}} = 1.2211027 \times 10^{19}$ GeV$/c^2$ is the Planck mass. In this case (excitingly enough!) our spectral formula (18) and the values (21) and (29) obtained for λ_f and κ_f by their fitting to experimental data give for leptons and quarks very close upper bounds

$$\bar{N}_e = 18, \quad \bar{N}_u = 19 \tag{33}$$

which become even identical,

$$\bar{N}_e = \bar{N}_u = 18, \tag{34}$$

if $\lambda_u \simeq 3.6$ instead of $\lambda_u \simeq 3.5$ (then $m_b \simeq 5.2$ GeV and $m_t \simeq 21$ GeV instead of $m_b \simeq 5$ GeV and $m_t \simeq 20$ GeV). However, it is well known that there exist various arguments for a much lower number of lepton and quark generations, among them the important perturbative argument [8] based on one-loop corrections to the relation $m_{\mathrm{W}}^2/m_{\mathrm{Z}}^2 \cos^2\theta_{\mathrm{W}} = 1$ in the standard Glashow-Salam-Weinberg theory.

Let us finally remark that because of the rise of experimental mass spectrum m_{f_N} with N (for $f \neq \nu$), the actual distribution $\varrho_{f_N} = m_{f_N}/M_f$ also increases with N, showing that the probability of finding the component state N in the actual state ϱ_f of f family rises with N. It does not imply, however, that heavy fermions should be more abundant in our present Universe than light ones, as the former decay into the latter via the electroweak interactions (including Cabibbo-like mixing). These decays cannot be described on the level of the zeroth quantization (similarly as they cannot be on the level of the first quantization) and require the second quantization to be introduced (with particle masses consistent with the zeroth quantization considered as a zero-order approximation or an effective approach).

In conclusion, we would like to stress that the zeroth quantization provides a new mechanism of particle excitations which may be responsible for the generation of mass differences in the experimentally known lepton and quark families. This quantum--statistical mechanism has nothing to do with the *radial* or *orbital* excitations connected with a hypothetic composite structure of leptons and quarks [9].

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