EXPERIMENTAL STUDY OF THE $K^+ \to \pi^+\pi^0\pi^0$ DECAY MATRIX ELEMENT BETWEEN 1 and 45 MeV OF THE π^+ KINETIC ENERGY

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(Received April 11, 1980; final version received July 1, 1980)

The π^+ kinetic energy spectrum in the $K^+ \to \pi^+\pi^0\pi^0$ decay is analysed between 1 and 45 MeV. Three forms of the decay matrix element are fitted. Existence of possible $\pi^0\pi^0$ interaction is discussed.

PACS numbers: 14.40 Dt, 14.40.Fu

1. Introduction

The most recent study of the π^+ kinetic energy spectrum in the τ' decay of the K^+ meson, $K^+ \to \pi^+\pi^0\pi^0$ has been done in two separate experiments performed with the use of the nuclear emulsion and bubble chamber techniques [9–10]. The aim of that work was to investigate the possible influence of the σ dipion state on the decay matrix element. The hypothetical $\pi\pi$ resonance σ with $J^P = 0^+$ and $I = 0$ was suggested by L. Brown and P. Singer [12]. Its mass was estimated to be around 400 MeV with a width of about 80 MeV. Such a particle was convenient in the description of the nucleon–nucleon medium range forces (one boson exchange potential). For a good fit to the data the mass of the scalar meson was required to be about 400 ± 500 MeV [13]. However, its existance as a resonance has never been confirmed and after $\varepsilon(1300)$ was discovered, interest in σ disappeared.

In the $K^+ \to \pi^+\pi^0\pi^0$ decay one may expect to observe the final state strong interactions between the two neutral pions because of the following:

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(1) The volume in which the final state particles appear is of the order of R^3, where R is the range of the forces;

(2) They have low relative momenta (the energy released in this decay is $Q = 84.2$ MeV);

(3) The interaction of the two neutral pions is attractive (positive phase shift in the s-wave and $I = 0$ state [14]). The maximum value of the $\pi^0\pi^0$ invariant mass is 354 MeV in the $K^+ \to \pi^+\pi^0\pi^0$ decay, corresponding to the zero kinetic energy of the positive pion. This region of the experimental spectrum may be influenced by the σ dipion state if it plays any role in this decay.

The spectrum, which spreads from 0 to 53.3 MeV in kinetic energy has been covered separately by the above mentioned experiments [9–10] as follows: nuclear emulsion 1–29 MeV and bubble chamber 11–45 MeV, yielding 4639 and 2519 events respectively.

TABLE I

Matrix element	Constraint τ'/τ	Emulsion expt.	B. ch. expt.
$1+g\frac{s_1-s_0}{m_+^2}$	no	$g = 0.405 \pm 0.027$	$g = 0.324 \pm 0.018$
	yes	$g = 0.411 \pm 0.017$	—
$1+s\frac{s_1-s_0}{m_+^2}+h\frac{s_1-s_0^2}{m_2^2}$	no	$g = 0.403 \pm 0.110$ $g = 0.001 \pm 0.094$	$g = 0.335 \pm 0.027$ $h = 0.02 \pm 0.04$
	yes	$g = 0.413 \pm 0.027$ $h = -0.009 \pm 0.037$	—

In the present paper a combined analysis of the spectra from these experiments is carried out over the total available range of the π^+ kinetic energy, i.e. from 1 to 45 MeV. The motivation for doing so is as follows:

(1) Results of the two independent analyses seem to be in agreement (see Table I, Fig. 5 and the text for definition of the parameters);

(2) The two parameter fit would be more efficient if done over the whole energy range.

The $K^+ \to \pi^+\pi^0\pi^0$ decay matrix element can be expanded in terms of the Mandelstam variables:

$$M \sim 1+g\frac{s_1-s_0}{m_+^2}+h\left[\frac{s_1-s_0}{m_+^2}\right]^2+j\left[\frac{s_2-s_3}{m_+^2}\right]^2, \qquad (1)$$

where $s_i = (p_k-p_i)^2$, $s_0 = \frac{1}{3}\sum_{i=1}^{3} s_i$, p_k and p_i are the K^+ and the π meson four-momenta (index "1" refers to the positive pion, indices "2" and "3" refer to the neutral pions, m_+ is the π_+ mass). Parameters g, h and j are unknown and must be determined by fitting.

TABLE II

Author	Year	Technique	Statistics
Kalmus et al.	1964	HLBC	1792
Bisi et al.	1965	HLBC + HBC	2027
Davison et al.	1969	HLBC + E	4048
Pandoulas et al.	1970	E	198
Aubert et al.	1972	HLBC	1365
Lucas et al.	1973	HBC	574
Sheaff	1975	HLBC	5635
Smith et al.	1975	W	27000
Braun et al.	1976	HLBC	3263
Bertrand et al.	1976	E	4639

Abbreviations: HLBC — Heavy Liquid Bubble Chamber; HBC — Hydrogen Bubble Chamber; E — Nuclear Emulsion; W — Multiwire proportional chambers.

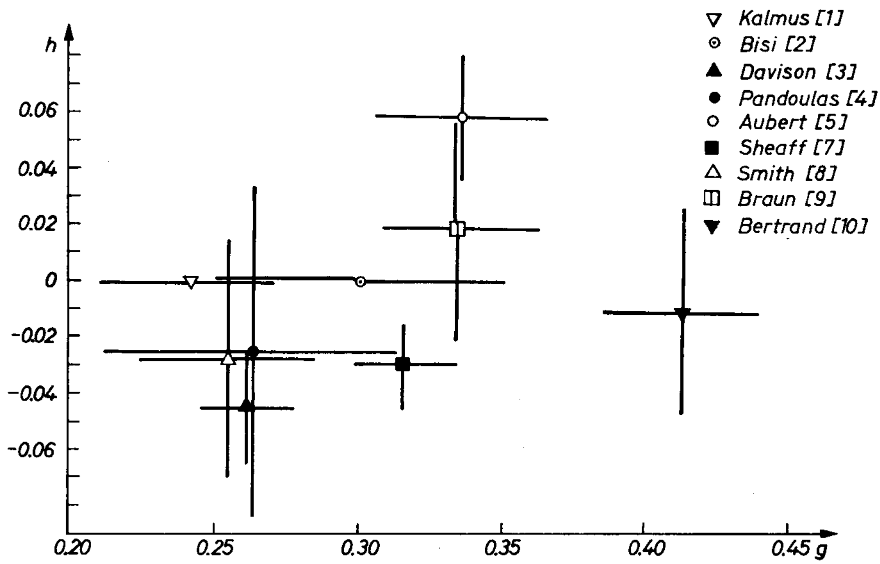

Fig. 1. Compilation of the existing data on the parameters g and h of the $K^+ \to \pi^+\pi^0\pi^0$ decay matrix element (Eq. (1))

The compilation of the existing world results is presented in Table II and Fig. 1. It is expected from the theory of weak interactions and the PCAC hypothesis [15–18] that the matrix element of the $K^+ \to \pi^+\pi^0\pi^0$ decay should be linearly dependent on the positive pion energy. This, in terms of the unknown parameters means that $h, j \ll g$.

If the decay proceeds via a σ intermediate state, the matrix element may be written in the form:

$$M \sim [(s_1 - M_\sigma^2 + \tfrac{1}{4}\Gamma_\sigma^2)^2 + M_\sigma^2 \Gamma_\sigma^2]^{-1/2}, \qquad (2)$$

where M_σ and Γ_σ are the mass and the width of the hypothetical intermediate scalar meson σ. These parameters also have to be determined by fitting.

The total number of events used for the combined spectrum analysis was 7158, the largest statistics ever reached in experiments using visual techniques (see Table II). It represents almost a twofold increase as compared with the statistics of Davison et al. [3]. Although Smith et al. [8] yielded about 27000 events in a counter experiment, their data display large errors in the low energy part of the spectrum which is of crucial importance for investigating the σ resonance influence on this decay. Experimental results in the σ resonance analysis are given only by Bisi et al. [2] and Davison et al. [3]:

M_σ [MeV]	Γ_σ [MeV]	Ref.
350	80	[2]
342.3±3.6	76.5±4.7	[3]

The experimental spectra of the positive pion kinetic energy obtained by these authors show a substantial deviation from linearity at low energy and thus allow for a fit of a non-linear function such as given by (2). No other results exhibit this feature leaving the σ hypothesis rather in doubt.

2. *Relative normalization of the emulsion and bubble chamber data*

A. Model independent procedure (method A)

The range of the π^+ kinetic energy spectrum covered by two techniques overlap between 11 and 29 MeV. The emulsion experiment yielded roughly twice as many events per 1 MeV energy bin in the overlap region compared with that of the bubble chamber. In order to find a scale factor a, normalizing the bubble chamber data to the emulsion ones, the chi-squared given by the following formula was minimized:

$$\chi^2(a) = \sum_i \frac{(N_{E_i} - aN_{B_i})^2}{\sigma_{E_i}^2 + a^2\sigma_{B_i}^2}, \tag{3}$$

where N_{E_i}, σ_{E_i}; N_{B_i}, σ_{B_i} are the numbers of events and their errors in the i-th energy bin of emulsion and bubble chamber data respectively. The best fit value of the scale factor was found to be $a = 2.117 \pm 0.064$. For each bin in the overlap region a weighted mean $\tilde{N}_i$ of the two numbers of events was calculated according to the following formula:

$$\tilde{N}_i = \frac{N_{E_i}\sigma_{E_i}^{-2} + aN_{B_i}(a\sigma_{B_i})^{-2}}{\sigma_{E_i}^{-2} + (a\sigma_{B_i})^{-2}}. \tag{4}$$

The error $\Delta\tilde{N}_i$ on this number is given by the expression:

$$\Delta\tilde{N}_i = \frac{a\sigma_{E_i}\sigma_{B_i}}{\sqrt{\sigma_{E_i}^2 + a\sigma_{B_i}^2}}. \tag{5}$$

The 3% error on the scale factor a has been neglected in the above formula because it is small compared to errors on the number of events (10%) and its influence on the results of the final fit is negligible. The experimental evaluation of the matrix element obtained in this way is presented in Fig. 2 with its errors.

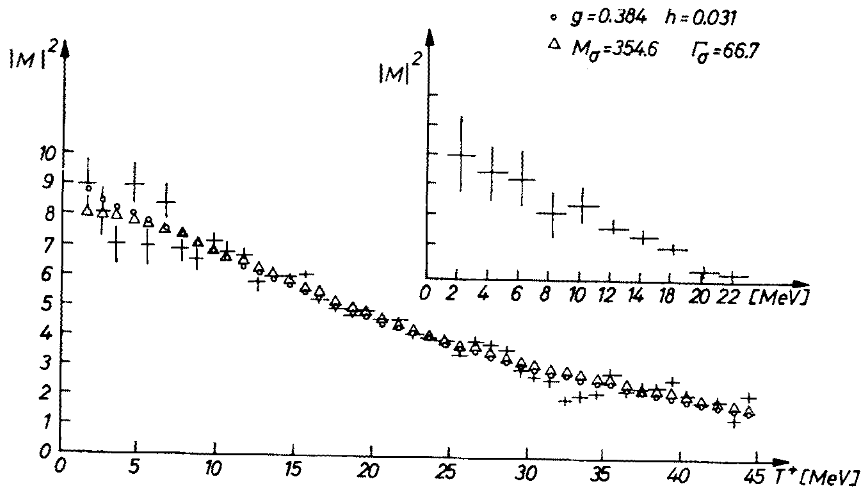

Fig. 2. Experimental matrix element squared with errors

B. Model dependent procedures

B1 Unconstrained probability normalization (method B1)

If one assumes a certain form of the decay matrix element M depending on the unknown parameters, the normalization constants C_N can be evaluated separately for both the emulsion and the bubble chamber data by the following formula:

$$C_N = N_{ev}(\int |M|^2 \phi(T^+) dT^+)^{-1}, \tag{6}$$

where N_{ev} is the total number of events in a given experiment. The integration of the modulus squared of the matrix element with the phase space function $\phi(T^+)$ is carried over the respective energy regions for both sets of data. The theoretical number of events in the i-th energy bin $N_{ev_i}^{theor}$ would be given by the formula:

$$N_{ev_i}^{theor} = C_N \int_{T_{i-1}}^{T_i} |M|^2 \phi(T^+) dT^+. \tag{7}$$

The chi-squared is calculated separately for the emulsion and the bubble chamber data as follows:

$$\chi^2 = \sum_i \frac{(N_{ev_i}^{theor} - N_{ev_i}^{expt})^2}{\sigma_i^{expt\,2}}, \tag{8}$$

where the sum is taken over the respective energy range and $N_{ev_i}^{expt}$ is the experimentally found number of events in the i-th energy bin, corrected for scanning losses. The emulsion and the bubble chamber chi-squared values are added. The resulting χ^2, which depends only on the unknown parameters entering the matrix element is then minimized.

B2. Normalization constrained by the branching ratio (method B2)

In this method, the emulsion normalization constant C_N^{em} is considered as an unknown parameter constrained by the equation

$$C_N^{em} = \frac{N_\tau R\Omega/\varepsilon}{\int_0^{T_{max}} |M|^2\phi(T^+)dT^+}, \tag{9}$$

where N_τ is the number of 30261 τ decays, i.e. $K^+ \to \pi^+\pi^+\pi^-$, found in the emulsion experiment; R [1] — is the τ'/τ branching ratio; ε — is the scanning efficiency; Ω — is the fraction of the solid angle into which the τ' decays were accepted.

This constraint will appear as an additional factor in the χ^2 expression [8]. Note that in using equations [6] and [9] the branching ratio R can be in turn considered as an unknown parameter constrained by

$$R = \frac{N_{ev}^{em} \int_0^{T_{max}} |M|^2\phi(T^+)dT^+}{N_\tau\Omega \int_{em} |M|^2\phi(T^+)dT^+}, \tag{10}$$

where N_{ev}^{em} is the total number of τ' events found in the emulsion experiment and the denominator integral is performed over the emulsion energy interval.

3. *Results of the combined analysis and conclusions*

The best fit values of the parameters of the decay matrix element are presented in Table III, whereas the fitted curves and the one standard deviation contours are in Fig. 2, 3 and 4. As can be seen in the inset of Fig. 2 (histogram with double binning), the kinetic

TABLE III

Matrix element	Method A	Method B1	Method B2
$1+g\dfrac{s_1-s_0}{m_+^2}$	$g = 0.394\pm0.014$ $\chi^2/n_D = 52.1/42$	$g = 0.387\pm0.016$ $\chi^2/n_D = 56.7/61$	$g = 0,376\pm0.016$ $\chi^2/n_D = 62.8/60$
$1+g\dfrac{s_1-s_0}{m_+^2}+h\left[\dfrac{s_1-s_0}{m_+^2}\right]^2$	$g = 0.388\pm0.015$ $h = 0.031\pm0.021$ $\chi^2/n_D = 50.0/41$	$g = 0.381\pm0.016$ $h = 0.030\pm0.019$ $\chi^2/n_D = 54.7/60$	$g = 0.370\pm0.016$ $h = 0.037\pm0.021$ $\chi^2/n_D = 59.5/59$
$((s_1-M_\sigma^2+\frac{1}{4}\Gamma_\sigma^2)^2+M_\sigma^2\Gamma_\sigma^2)^{-\frac{1}{2}}$	$M_\sigma = 354.6\pm3.2$ $\Gamma_\sigma = 66.7\pm2.6$ $\chi^2/n_D = 48.4/41$	$M_\sigma = 354.9\pm3.2$ $\Gamma_\sigma = 68.0\pm3.0$ $\chi^2/n_D = 52.8/60$	$M_\sigma = 356.0\pm3.4$ $\Gamma_\sigma = 69.6\pm3.0$ $\chi^2/n_D = 57.0/59$

[1] The world value [11] : $R = 0.309\pm0.009$.

energy rises monotonically towards the low energy end. Such behaviour suggests that the introduction of an intermediate dipion state σ is not necessary to describe the matrix element, although the fit was convergent. The obtained σ mass is practically equal to the maximal value of the invariant mass of the $\pi^0\pi^0$ system 354.1 MeV, corresponding to the

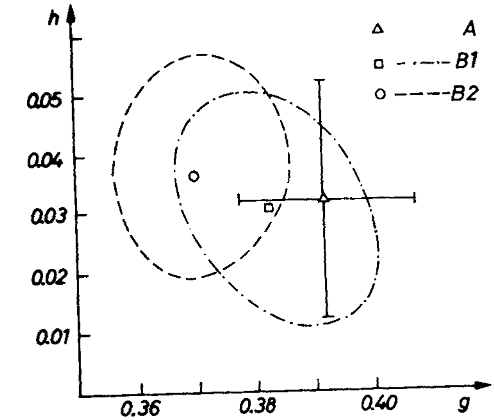

Fig. 3. One standard deviation contours for the parameters g and h

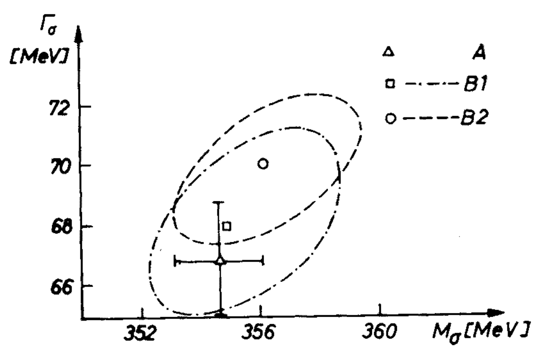

Fig. 4. One standard deviation contours for the paramaters M_σ and Γ_σ

zero kinetic energy of the positive pion. This happens so because the decay matrix element shows no maximum in the low energy region, as displayed by the results of [2] and [3].

Thus it is justified to accept (1) as the correct form of the $K^+ \to \pi^+\pi^0\pi^0$ decay matrix element.

The value of the parameter j in (1) is consistent with zero [9]. The values of the parameter h obtained in this work are one order of magnitude smaller than g.

Finally, it should be noted that the separate bubble chamber results [9] and the emulsion fits with the constraint using the world value $R = 0.309 \pm 0.009$ are in disagreement (see Fig. 5). The emulsion fit performed with the value R [2] $= 0.3309 \pm 0.0016$ agrees with

[2] Quoted by Chiang et al [19] as a result of the statistically most significant experiment done so far.

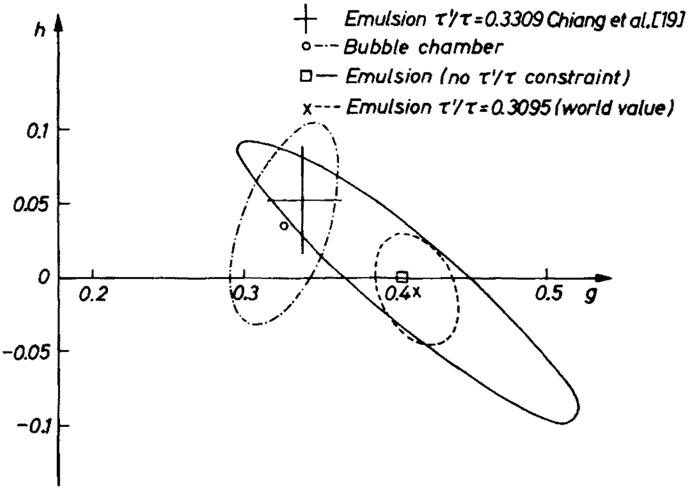

Fig. 5. Comparison of separate bubble chamber and emulsion data analyses

the bubble chamber within one standard deviation contour as shown on Fig. 5. This fact is in turn reflected by the value of R obtained from Eq. (10) for the emulsion only, $R = 0.3989 \pm 0.0059$, which is in good agreement with previously quoted value of Chiang et al.

The authors wish to thank Professor J. Zakrzewski for fruitful discussions and careful reading of the manuscript.

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