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GLUON JET PRODUCTION IN DEEP INELASTIC LEPTON HADRON COLLISIONS. MONTE CARLO STUDY OF JET FRAGMENTATION EFFECTS

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We study the cross sections for gluon jet production in deep inelastic leptoproduction as predicted by QCD perturbation theory. Using a nonperturbative chain decay model for parton jet fragmentation, we study the cross section for two-jet (quark and spectator) and three-jet (quark, gluon and spectator) production. The cross section is studied in terms of the thrust variable, transverse momentum and transverse momentum sum of hadrons relative to the momentum transfer direction, and in terms of the $z_{||}$ variable in the Breit frame. We find it possible to detect gluon jet effects for lepton energies starting with about 200 GeV and at as large as possible values of Q^2 and W^2. The transverse momentum sum is found to be the most useful variable for identifying gluon jets.

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1. Introduction

Quantum Chromodynamics predicts the emission of gluon jets in hard scattering processes. Besides e^+e^- annihilation into hadrons, deep inelastic lepton hadron scattering seems to be the process where the identification of events with gluon jets is most easy. The cross sections for gluon jet production in deep inelastic lepton hadron collisions were given in several papers [1–4]. Methods proposed to identify measurable effects due to gluon jet emission include: azimuthal asymmetries [1, 5, 6] thrust and spherocity distributions [7–9], transverse momentum distributions [10], average transverse momenta [11, 12] and transverse momentum sums [3, 13], energy flow distributions [4, 14, 15] and the distributions of $z_{||}$ in the Breit frame [16]. In most of these calculations, the hard gluon tail of the cross section, neglecting the fragmentation of the quark und gluon jets, is compared to the fragmented two-jet cross sections. Here we take the nonperturbative fragmentation of gluon and quark jets into account in two-jet (quark and spectator) as well as in three-jet (quark, gluon and spectator) events. We start from the deep inelastic cross sections as

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given by Peccei and Rückl [4] and select Monte Carlo three-jet events from the cross sections after defining a finite smoothed three-jet cross section (Section 3). The nonperturbative jet fragmentation is treated using a Monte Carlo chain decay model for jet fragmentation [17, 18, 19].

Our method only considers single gluon emission and neglects events with more than one gluon jet on one side; on the other side, it uses the nonperturbative fragmentation of parton jets with a phenomenological chain decay model, this treatment is more conventional than the approach of Binétruy [20] who includes the perturbative quark jet fragmentation according to leading log QCD. In his approach the quark jet fragmentation and the emission of any number of gluons is treated in the same scheme. But one might worry whether it is justified to treat the observed hadrons just like gluons and quarks of the QCD jet neglecting the hadronization process all together.

2. Deep inelastic lepton-parton cross sections

The cross sections for deep inelastic lepton-parton interactions with photon or weak boson exchange are calculated according to the diagrams given in Fig. 1. We introduce the usual scaling variables

$$x = -\frac{q^2}{2Pq}, \quad x_p = -\frac{q^2}{2p_a q},$$

$$y = \frac{Pq}{Pl} = \frac{p_a q}{p_a l},$$

$$z = \frac{Pp''}{Pq}, \quad z_p = \frac{p_a \cdot p'}{p_a \cdot q},$$

$$Q^2 = -q^2, \quad \nu = q \cdot P, \quad W^2 = (P+q)^2,$$

$$s = (P+l)^2 = 2m_p E_0. \tag{1}$$

E_0 is the laboratory energy of the initial lepton, W is the invariant total energy of the produced hadronic system. The cross sections for current-parton scattering can be written as

$$\frac{d\sigma}{dx_p dy dz_p d\phi} = \sigma^{(0)} + \sigma^{(1)} \cos\phi + \sigma^{(2)} \cos 2\phi, \tag{2}$$

where ϕ is the angle between the leptonic plane $(ll'q)$ and the hadronic plane $(p_a p' q)$. In the following we will only be interested in the cross sections averaged over the angle ϕ.

$$\frac{d\sigma}{dx_p dy dz_p} = 2\pi\sigma^{(0)} = 2\pi\left[\tfrac{1}{2}(1+(1-y)^2)\sigma^{\mathrm{T}} + (1-y)\sigma^{\mathrm{S}} \pm y\left(1-\frac{y}{2}\right)\sigma^{\mathrm{I}}\right]. \tag{3}$$

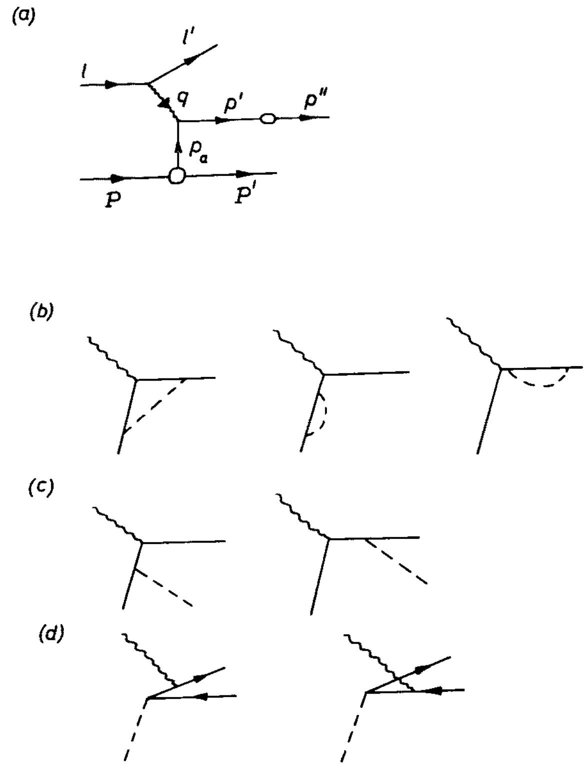

Fig. 1. (a) Deep inelastic lepton-nucleon scattering; (b) Virtual gluon corrections; (c) Gluon bremsstrahlung corrections; (d) Quark antiquark pair creation

The $\pm$ sign in front of the interference term σ^{I} refers to scattering of ν and $\bar{\nu}$, respectively. For electron or muon scattering, the interference term vanishes. The cross section according to the graphs in Fig. 1a, b and c is obtained as follows [4].

$$\sigma^{\mathrm{T}} = \delta(1-x_{\mathrm{p}})\delta(1-z_{\mathrm{p}})\left[1-\frac{2\alpha_{\mathrm{s}}}{3\pi}\left(\frac{1}{2}+\frac{2\pi^2}{3}\right)\right]+\frac{2\alpha_{\mathrm{s}}}{3\pi}\,\delta(1-x_{\mathrm{p}})$$

$$-\frac{2\alpha_{\mathrm{s}}}{3\pi}\,\delta(1-z_{\mathrm{p}})\left[1+\frac{2(1+x_{\mathrm{p}}^2)}{1-x_{\mathrm{p}}}\ln x_{\mathrm{p}}\right]+\frac{2\alpha_{\mathrm{s}}}{3\pi}\left[2(x_{\mathrm{p}}z_{\mathrm{p}}+1)+\frac{z_{\mathrm{p}}^2+x_{\mathrm{p}}^2}{(1-x_{\mathrm{p}})_+\,(1-z_{\mathrm{p}})_+}\right],$$

$$\sigma^{\mathrm{S}} = \frac{2\alpha_{\mathrm{s}}}{3\pi}\,[4x_{\mathrm{p}}z_{\mathrm{p}}],$$

$$\sigma^{\mathrm{I}} = \sigma^{\mathrm{T}}+\frac{2\alpha_{\mathrm{s}}}{3\pi}\,[2(x_{\mathrm{p}}+z_{\mathrm{p}})-2(x_{\mathrm{p}}z_{\mathrm{p}}+1)]. \tag{4}$$

In (4) the terms in $\ln(q^2/-p'^2)$ and $\ln(q^2/p_a^2)$ corresponding to the mass singularities of the incoming and outgoing partons have been droped already. The terms with $\delta(1-x_{\mathrm{p}})$

$\delta(1-z_p)$ refer to the Born term and virtual corrections to it. The terms with $\delta(1-x_p)$ and $\delta(1-z_p)$ refer to situations with the gluon parallel to the outgoing quark or parallel to the incoming quark. The last term without δ function is the finite contribution to the cross section with the gluon parallel neither to the incoming nor to the outgoing quark. The term with

$$\sigma_3 = \frac{2\alpha_s}{3\pi} \frac{z_p^2+x_p^2}{(1-x_p)_+ (1-z_p)_+} \tag{5}$$

in the cross section (4) gives a finite contribution to the cross section after integration over x_p and z_p.

In order to sample Monte Carlo events we have to introduce a cut-off or to define a smoothed cross section. We choose the latter and define the smoothed cross section according to the following two requirements:

(*i*) The value of the integral

$$I_3 = \int_x^1 dx_p \int_0^1 dz_p \frac{F\left(\frac{x}{x_p}\right)}{x_p} \frac{(x_p^2+z_p^2)}{(1-x_p)_+ (1-z_p)_+} \tag{6}$$

should be unchanged.

(*ii*) For $x_p < 1-\Delta x$ and $z_p < 1-\Delta z$ and especially in the limit $z_p \to 0$ and $x_p \to 0$ or $x_p \to x$ the parton model cross sections describing the emission of hard gluons should have the form

$$\sigma_3 = \frac{2\alpha_s}{3\pi} \frac{z_p^2+x_p^2}{(1-x_p)(1-z_p)}\,. \tag{7}$$

We define the smoothed cross section

$$\bar{\sigma}_3 = \frac{2\alpha_s}{3\pi} \frac{z_p^2+x_p^2}{(1-x_p)(1-z_p)} G(x_p, z_p, a(x)). \tag{8}$$

According to the requirements (*i*) and (*ii*), the function $G(x_p, z_p, a(x))$ should cancel the poles at $x_p = 1$ and $z_p = 1$ and should have the limiting behaviour

$$G(x_p, z_p, a(x)) \xrightarrow[\substack{x_p\to x \\ z_p\to 0}]{} 1. \tag{9}$$

The ansatz

$$G(x_p, z_p, a(x)) = \cos\left(\frac{\pi}{2} z_p\right) \exp\left(a(x) z_p^2\right)$$
$$\times \cos\left(\frac{\pi}{2} \frac{(x_p-x)}{1-x}\right) \exp\left(a(x)\left(\frac{x_p-x}{1-x}\right)^2\right) \tag{10}$$

fulfills both requirements. The unknown function $a(x)$ can be determined from the condition

$$I_3 = \int_x^1 dx_p \int_0^1 dz_p \frac{F\left(\frac{x}{x_p}\right)}{x_p} \frac{(x_p^2+z_p^2)}{(1-x_p)(1-z_p)} G(x_p, z_p, a(x)), \tag{11}$$

where $F\left(\frac{x}{x_p}\right)$ is one of the quark model parton distributions. In Fig. 2. we plot the functions $a(x)$ determined in this way which we will use subsequently.

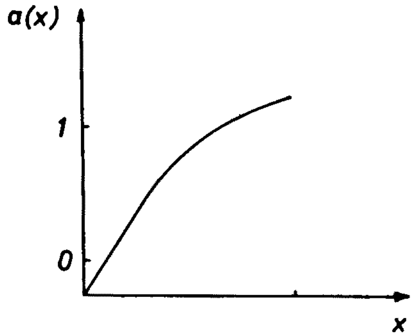

Fig. 2. The function $a(x)$, see Eq. (11)

3. *The Monte Carlo calculation*

Using the QCD-improved parton model, we write the jet production cross section in deep inelastic lepton hadron collisions.

$$\frac{d\sigma}{dxdydz_pdx_p} = \sum_a \int d\xi \delta(x-\xi x_p) f_a(\xi, Q^2) \frac{d\sigma}{dx_pdydz_p}. \tag{12}$$

We use Q^2 dependent quark distributions $f_a(\xi, Q^2)$ according to the parametrization [21]. It is practical to introduce new variables. The energy fractions of the 3 (or 2) jets are given by

$$x_1 = 2E_1/W, \quad x_2 = 2E_2/W, \quad x_3 = 2E_3/W, \tag{13}$$

$$x_1+x_2+x_3 = 2.$$

Jet 1 is the quark jet, jet 2 is the gluon jet and jet 3 is the spectator jet. The x_i variables are related to x, x_p and z_p

$$x_1 = \frac{x}{1-x} \frac{1-x_p}{x_p}(1-z_p)+z_p,$$

$$x_2 = \frac{x}{1-x} \frac{1-x_p}{x_p} z_p+(1-z_p),$$

$$x_3 = 1-\frac{x}{1-x} \frac{1-x_p}{x_p}. \tag{14}$$

Furthermore, we introduce Q^2 and W^2 as variables

$$x = \frac{Q^2}{W^2+Q^2}; \qquad y = \frac{W^2+Q^2}{2ME_0} \tag{15}$$

and transform the cross section (12) into the form

$$\frac{d\sigma}{dQ^2dW^2dx_1dx_3} = \frac{1}{2ME(W^2+Q^2)} \frac{Q^2W^2}{x_3(Q^2+W^2(1-x_3))^2} \frac{d\sigma}{dxdydz_pdx_p}. \tag{16}$$

The Monte Carlo sampling of events proceeds in two steps

(*i*) Sampling of two- or three-jet events according to the cross section (16).

(*ii*) Fragmentation of the jets into final hadrons.

In step (*i*) we select weighted two- or three-jet events in the region

$$Q_1^2 \leqslant Q^2 \leqslant Q_2^2, \qquad W_1^2 \leqslant W^2 \leqslant W_2^2 \tag{17}$$

and in the case of three-jet events x_1 and x_3 in the triangle

$$0 \leqslant x_1 \leqslant 1 \qquad 1-x_1 \leqslant x_3 \leqslant 1. \tag{18}$$

The weight of the corresponding event becomes

$$W_i = (Q_2^2-Q_1^2)(W_2^2-W_1^2)\frac{d\sigma}{dQ^2dW^2dx_1dx_3}. \tag{19}$$

In step (*ii*) we fragment the two or three jets using a Monte Carlo chain decay model for parton jet fragmentation. Such programmes were described by Field and Feynman [17], Sjöstrand and Söderberg [18] and by Ritter and Ranft [19]. The method used in the last programme allows to take quantum number and energy momentum conservation into account and to include also baryons; however so far it is implemented only for two-jet events. Since we are here not interested in any flavour dependent quantities, we use the programme [18] to sample the two- and three-jet events. We add one hadron to each event to restore momentum conservation.

4. *Results*

We present here some results for the following cross sections in deep inelastic electron or muon scattering

(*i*) As function of the thrust variable

$$\frac{d\sigma}{dT}; \qquad T = \max\frac{\sum |p_{\parallel i}|}{\sum |\vec{p}_i|}. \tag{20}$$

We calculate the thrust of the three-jet events according to the formula $T = \max(x_1, x_2, x_3)$. The thrust of the fragmented events is calculated using a steepest descent iterative procedure [19] which converges well starting the iteration from the direction determined in the unfragmented jet case.

(*ii*) As function of the transverse momentum of final hadrons relative to the direction of the momentum transfer vector $\vec{q}$

$$\frac{d\sigma}{dp_\perp}. \tag{21}$$

(*iii*) As function of the variable $\pi_\perp$ [13]

$$\frac{d\sigma}{d\pi_\perp}; \qquad \pi_\perp = \frac{(\sum p_{\perp i})^2}{W^2}. \tag{22}$$

This variable is rather similar to the spherocity variable S, but has the advantage, that no minimization procedure is necessary.

(*iv*) As function of the $z_\parallel$ variable in the Breit frame

$$\frac{d\sigma}{dz_\parallel}; \qquad z_\parallel = 2\,\frac{\sum_i p^{\mathrm{B}}_{\parallel i}}{Q}. \tag{23}$$

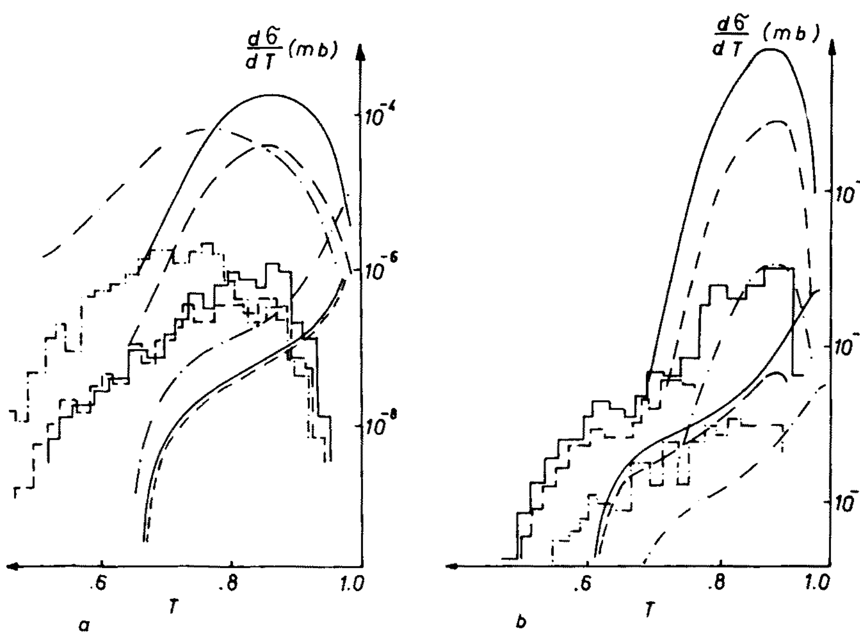

Fig. 3. The cross section $d\sigma/dT$(mb) vs T for different E_0, Q^2 and W^2. The upper curves describe the fragmented 2 jet events, the lower curves describe the unfragmented 3 jet events and the histograms describe the fragmented 3 jet events. (a) solid lines: $E_0 = 200$ GeV, $Q^2 = (3\text{–}4)$ GeV^2, $W^2 = (300\text{–}350)$ GeV^2; dashed lines: $E_0 = 200$ GeV, $Q^2 = (10\text{–}20)$ GeV^2, $W^2 = (300\text{–}350)$ GeV^2; dashed-dotted lines: $E_0 = 50$ GeV, $Q^2 = (3\text{–}4)$ GeV^2, $W^2 = (70\text{–}90)$ GeV^2; (b) $E_0 = 600$ GeV and $W^2 = (900\text{–}1000)$ GeV^2; solid lines $Q^2 = (3\text{–}5)$ GeV^2; dashed lines: $Q^2 = (10\text{–}15)$ GeV^2; dashed-dotted lines: $Q^2 = (60\text{–}70)$ GeV^2.

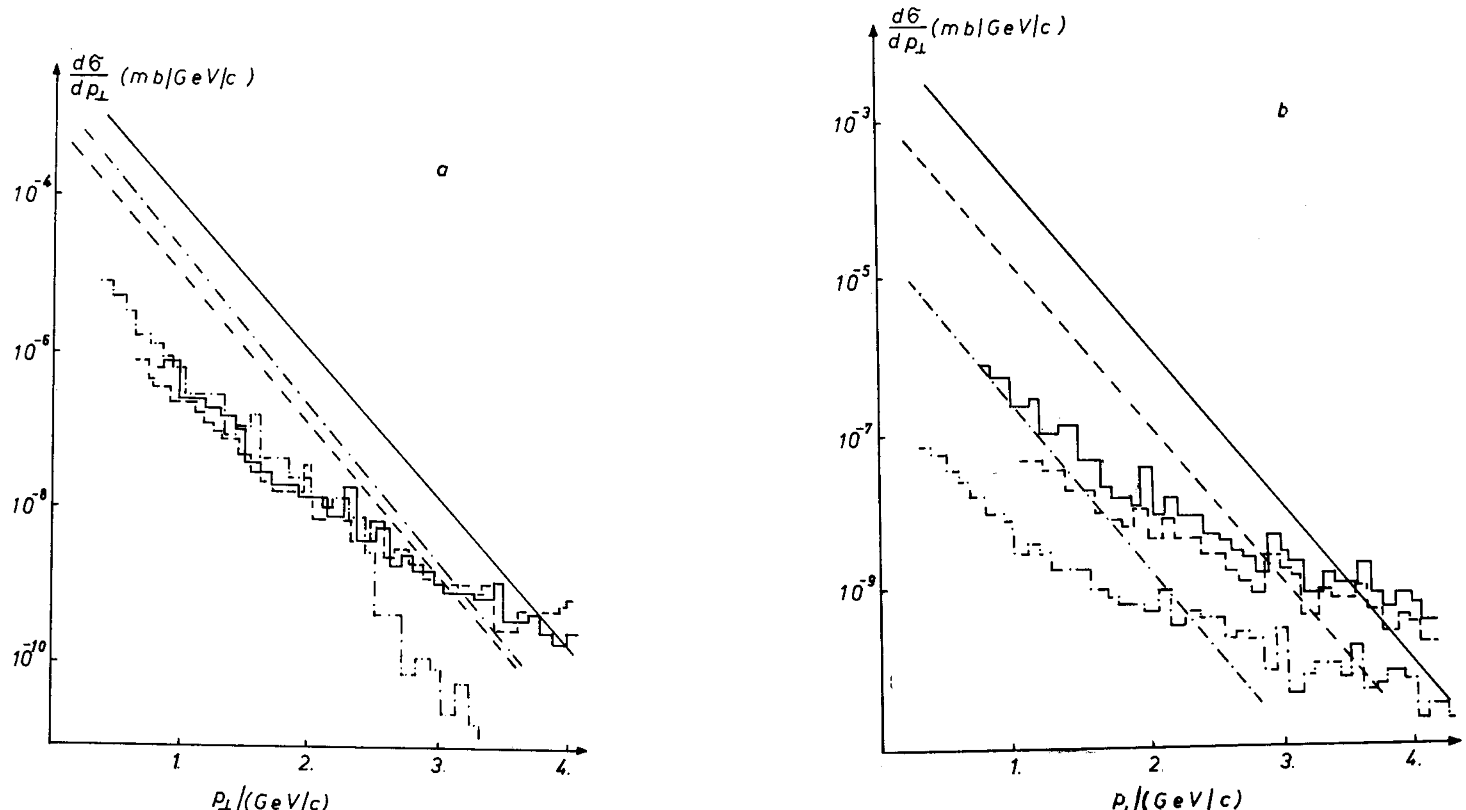

Fig. 4. The cross section $d\sigma/dp_\perp$ (mb/GeV/c) as a function of the transverse momentum $p_\perp$ (GeV/c) for different E_0, Q^2 and W^2. The curves describe the fragmented 2 jet events and the histograms describe the fragmented 3 jet events. (a) solid lines: E_0 = 200 GeV, Q^2 = (3–4) GeV^2, W^2 = (300–350) GeV^2; dashed lines: E_0 = 200 GeV, Q^2 = (10–20) GeV^2, W^2 = (300–350) GeV^2; dashed-dotted lines: E_0 = 50 GeV, Q^2 = (3–4) GeV^2, W^2 = (70–90) GeV^2; (b) E_0 = 600 GeV and W^2 = (900–1000) GeV^2; solid lines: Q^2 = (3–5) GeV^2; dashed lines: Q^2 = (10–20) GeV^2; dashed-dotted lines: Q^2 = (60–70) GeV^2

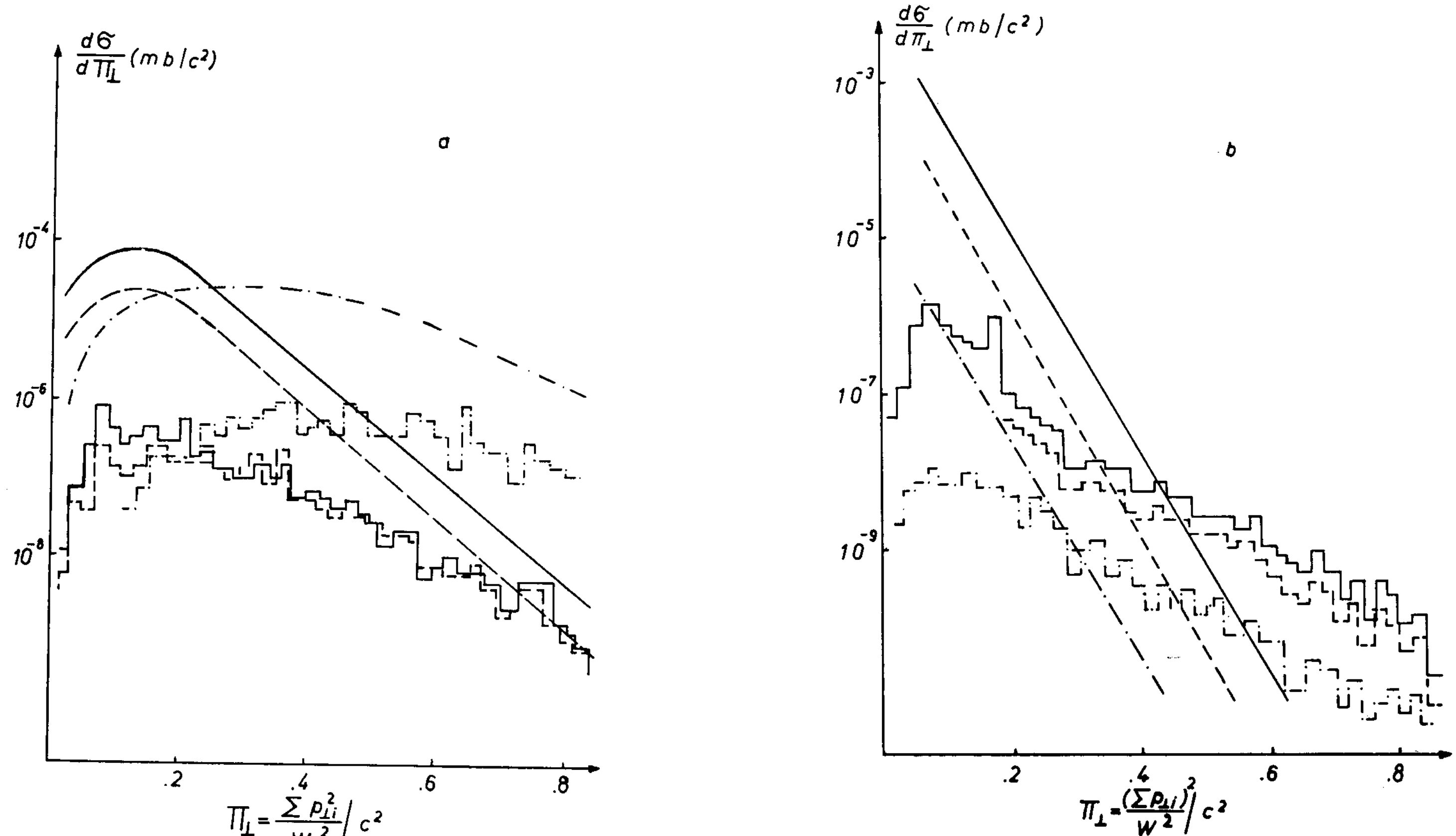

Fig. 5. The cross section $d\sigma/d\pi_\perp$ (mb/c^2) vs $\pi_\perp = (\Sigma p_\perp)^2/W^2(1/c^2)$ for different E_0, Q^2 and W^2. The upper curves describe the fragmented 2 jet events the histograms describe the fragmented 3 jet events.(a) solid lines: $E_0 = 200$ GeV, $Q^2 = (3–4)$ GeV^2, $W^2 = (300–350)$ GeV^2; dashed lines: $E_0 = 200$ GeV, $Q^2 = (10–20)$ GeV^2, $W^2 = (300–350)$ GeV^2; dashed-dotted lines: $E_0 = 50$ GeV, $Q^2 = (3–4)$ GeV^2, $W^2 = (70–90)$ GeV^2; (b) $E_0 = 600$ GeV and $W^2 = (900–1000)$ GeV^2; solid lines: $Q^2 = (3–5)$ GeV^2; dashed lines: $Q^2 = (10–15)$ GeV^2; dashed-dotted lines: $Q^2 = (60–70)$ GeV^2

The sum in (23) runs over all hadrons from the quark jet in two-jet events and over all hadrons from the quark and gluon jets in three-jet events. For unfragmented two-jet events one finds [16]

$$\frac{d\sigma}{dQ^2 dW^2 dz_{\parallel}} = \delta(1-z_{\parallel})\frac{d\sigma}{dW^2 dQ^2}. \tag{24}$$

In [16] it was found that a certain fraction of the unfragmented three-jet events has the property, that $z_{\parallel} < 0$; this was proposed to serve as a method to identify three-jet events containing one gluon jet.

We perform Monte Carlo calculations at there different energies of the primary electron or muon beam, $E_0 = 50$, 200 and 600 GeV.

In Fig. 3a (50 and 200 GeV) and 3b (600 GeV) we present thrust distributions $d\sigma/dT$ obtained for two-jet and three-jet events. It is obvious, there is no hope to isolate events with gluon jets at $E_0 = 50$ GeV, there is some marginal chance at $E_0 = 200$ GeV, but

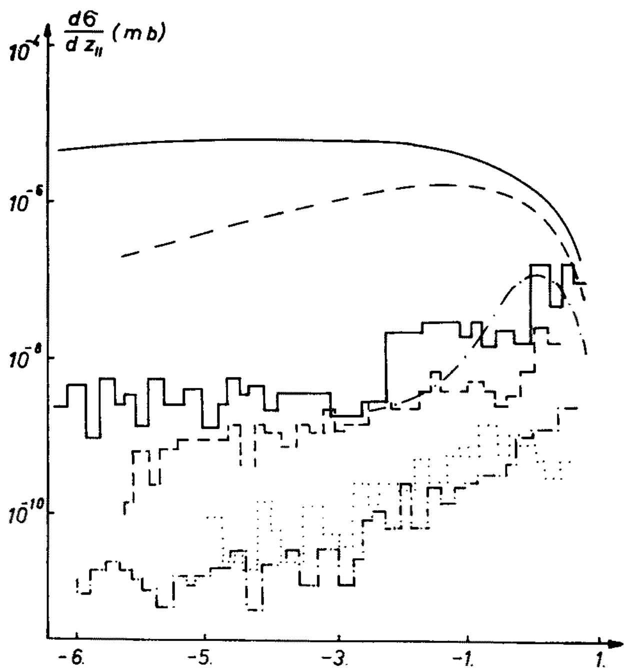

Fig. 6. The cross section $d\sigma/dz_{\parallel}$ (mb) vs $z_{\parallel}$ for $E_0 = 600$ GeV and $W^2 = (900\text{–}1000)$ GeV^2. The curves describe the fragmented 2 jet events, the histograms describe the unfragmented 3 jet events and the dotted histogram describes the fragmented 3 jet events for $Q^2 = (60\text{–}70)$ GeV^2; solid lines $Q^2 = (3\text{–}5)$ GeV^2; dashed lines: $Q^2 = (10\text{–}15)$ GeV^2; dashed-dotted lines: $Q^2 = (60\text{–}70)$ GeV^2

at $E_0 = 600$ GeV all events with hadronic energies $W \approx 30$ GeV and thrust values $T \lesssim 0.75$, especially at bigger Q^2, are likely to result from events with one gluon jet.

In Fig. 4a (50 and 200 GeV) and 4b (600 GeV) we present transverse momentum distributions $d\sigma/dp_{\perp}$ relative to the $\vec{q}$ axis. We find at $E_0 = 50$ GeV the two-jet cross section dominating at all $p_\perp$ values. At $E_0 = 200$ GeV and $p_\perp \gtrsim 3$ GeV/c at bigger Q^2 values, the three-jet events start to dominate. At $E_0 = 600$ GeV, depending on Q^2, the three-jet events dominate for $p_\perp > 2$–3 GeV/c.

In Fig. 5a (50 and 200 GeV) and 5b (600 GeV) we present the $\pi_\perp$ distribution $d\sigma/d\pi_\perp$. We find again only for $E_0 \geqslant 200$ GeV some chances to find events with gluon jets. At $E_0 = 600$ GeV the difference in shape of the 2 jet and 3 jet curves is significant.

In Fig. 6 we present $z_\parallel$ distributions in the Breit frame only at $E_0 = 600$ GeV. Including jet fragmentation, already the two-jet events spread out to $z_\parallel$ values $z_\parallel < 0$. This is partly also due to the non perfect energy momentum conservation in the jet fragmentation model used, but such effects will be present in experiments due to the unreliable separation of quark and spectator jets and due to missing neutrals. Our conclusion is, that $z_\parallel$ is not a useful variable to isolate events with gluons jets.

We conclude: At energies below $E_0 \approx 200$ GeV the chances to isolate gluon jets in deep inelastic lepton hadron collisions are rather small. At higher energies, higher W^2 and Q^2 values, this separation is possible. The best variable to identify events with gluon jets seems to be the transverse momentum sum $\pi_\perp$.

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