INFRA-RED DIVERGENCES AND REGGE BEHAVIOUR IN QCD*

By T. Jaroszewicz

Institute of Nuclear Physics, Cracow**

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We analyze high energy behaviour of multi-gluon exchange amplitudes in the leading-ln s approximation in perturbation theory. Working in the Coulomb gauge and employing Ward identities we derive an integral equation for the n-gluon system in the exchange channel. We find that the Regge behaviour is associated with exponentiation of leading infrared divergences, and the position of the j-plane singularities is determined by the colour quantum numbers of the exchanged system.

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In high energy small momentum transfer interactions of hadrons the dominant feature is certainly the Regge behaviour, in particular the Pomeron responsible for diffraction. A question arises whether these properties can be explained in the framework of Quantum Chromodynamics (QCD), considered to be the leading candidate for the theory of strong interactions. On the qualitative level, QCD can account for the main features of the elastic scattering amplitudes: for instance, their approximate energy independence can be associated with the exchange of vector mesons (gluons), and the smallness of the real parts results from the fact that the one gluon exchange between hadrons is forbidden by colour conservation. On the quantitative level the problem is far from solved, although much work has been done in this field recently [1–8, 11, 12, 15]. These works concentrate on the perturbation theory in the weak coupling limit $g^2 \to 0$, $g^2 \ln s = \text{const}$, i.e. on the leading-logarithmic approximation (LLA), where all powers of $g^2 \ln s$ are summed. First, the vector-meson exchange channels have been studied; they are of course unphysical, but can be considered building blocks in constructing the Pomeron. It was found that these channels are governed by a Regge trajectory on which the vector meson lies, which means

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** Address: Instytut Fizyki Jądrowej, Radzikowskiego 152, 31-342 Kraków, Poland.

that the gauge vector meson reggeizes. In the vacuum channel another important fact was established, namely, that in the LLA the Pomeron is a fixed branch point in the complex angular momentum plane, with the intercept above unity.

Most of these results have been obtained by starting with a spontaneously broken gauge theory in the unitary gauge, and then by either directly evaluating individual Feynman diagrams [1, 2], or using unitarity and dispersion relations to calculate amplitudes in successive perturbative orders [3, 4, 7]. The latter methods are not specific to gauge theories and are applicable equally well to, say, the ϕ^3 theory. On the other hand, it was noted [8] that choice of a suitable gauge may considerably simplify the structure of the relevant Feynman diagrams (as in deep inelastic scattering in the axial gauge); then the leading energy dependence can be extracted by using the Ward identities.

In the present note we simplify and generalize this approach and further exploit the relationship between the high energy behaviour and the infra-red (IR) structure of gauge theories. We start directly with the Coulomb gauge in the massless SU(N) gauge theory (dimensionally regularized, when necessary). We briefly discuss the properties of multi--gluon exchange amplitudes in this gauge. Then we sketch the derivation of integral equations governing the corresponding two-particle n-gluon vertices; our equations are generalizations of those obtained in Refs [2, 4, 8, 11, 12]. Our derivation is based on a method analogous to that used by Grammer and Yennie [9] in analyzing the IR divergences in QED. Finally, we discuss the physical meaning and some properties of those equations, and their approximate solutions.

We consider elastic scattering of two objects, A and B, moving along the positive and negative z-axis, with momenta p_A and p_B respectively. We choose such a reference frame that p_A^+, p_B^- and $s \simeq 2p_A^+p_B^-$ are large, and $p_A^2 \simeq p_B^2 \simeq 0$ $\left(p^\pm = p_\pm = \frac{1}{\sqrt{2}}(p^0 \pm p^3)\right)$. We shall use the Coulomb gauge massless gluon propagators

$$D^{\mu\nu}(q) = \frac{1}{q^2+i\varepsilon}\left[-g^{\mu\nu}+\frac{(N\cdot q)(N^\mu q^\nu+q^\mu N^\nu)-N^2q^\mu q^\nu}{(N\cdot q)^2-N^2q^2}\right] \tag{1}$$

with $N_\mu = (1, 0, 0, 0)$ in our reference frame.

The properties of the Feynman diagrams in this gauge are best illustrated on the example of gluon–gluon scattering in the order g^4 (Fig. 1). A straightforward calculation shows that the energy dependence of these diagrams in the covariant (Feynman) gauge is: (a) $\sim g^4s$, (b) $\sim g^4s$, (c) $\sim g^4s(-\ln s+i\pi)$, (d) $\sim g^4s\ln s$, whereas in our Coulomb gauge: (a) $\sim g^4s\ln p_A^+$, (b) $\sim g^4s\ln p_B^-$, (c) $\sim i\pi g^4s$, (d) ~ 0. In the covariant gauge the $\ln s$ in the box diagram comes from one of the horizontal lines being far off-mass-shell. This contribution is suppressed in the Coulomb gauge; instead, the leading contribution comes from the horizontal lines being both on the mass-shell (note that the box and the crossed box diagrams are not simply related by the $s \leftrightarrow u$ crossing, because of additional dependence on N^μ). On the other hand, diagrams (a) and (b) now acquire the energy dependence from the N^μ-dependence of vertex corrections ($N\cdot p_A \sim p_A^+$, $N\cdot p_B \sim p_B^-$). The sum (a)+(b) in N^μ-independent because $\ln p_A^+ + \ln p_B^- \sim \ln s$.

This analysis can be extended to an arbitrary n-gluon exchange diagram. The corresponding amplitude is asymptotically equal (Fig. 2).

$$T_n = -2is \frac{(-i)^n}{n!} (2\pi)^{-2(n-1)} \int \prod_{i=1}^{n} \frac{d^2 q_i}{\vec{q}_i^{\,2}} \delta^2 \left(\sum_{i=1}^{n} \vec{q}_i - \vec{\Delta} \right)$$
$$\times f^A(p_A^+; \vec{q}_1, \ldots, \vec{q}_n) f^B(p_B^-; \vec{q}_1, \ldots, \vec{q}_n), \tag{2}$$

where $\vec{q}_i$'s are the transverse momenta of the exchanged gluons, and the colour indices are suppressed. The quantities f^A and f^B are the integrals of multiple discontinuities (in the mass variables) of the two-particle n-gluon vertex functions Γ, e.g.

$$f^A(p_A^+; \vec{q}_1, \ldots, \vec{q}_n) = -i(2\pi i)^{-(n-1)} (2p_A^+)^{-n} \int_0^{mp_A^+} d\sigma_1 \ldots d\sigma_{n-1}$$
$$\times \operatorname{disc}_{\sigma_1} \ldots \operatorname{disc}_{\sigma_{n-1}} \Gamma^A_{+\ldots+}(p_A; q_1, \ldots, q_n), \tag{3}$$

where $\sigma_i \equiv (p_A - q_1 - \ldots - q_i)^2 \simeq -2p_A^+(q_1^- + \ldots + q_i^-)$, the indices "+" in Γ^A are the Lorentz indices, and m is a mass scale (irrelevant in the LLA). The vertices f^A and f^B are analogous to the "impact factors" introduced by Cheng and Wu [10], except that they depend on the momenta p_A^+ or p_B^-.

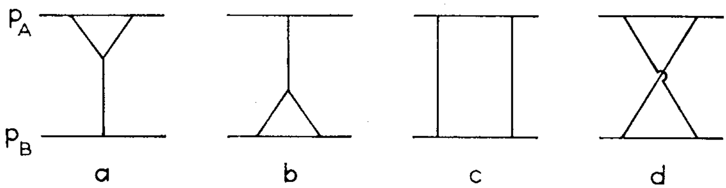

Fig. 1. Gluon-gluon scattering in the order g^4

The crucial feature of Eq. (2) is that, in agreement with the discussion of Fig. 1, the exchanged gluons do not give any logarithms of energy. Therefore, in the LLA, the contributing diagrams are only those with the smallest number of the exchanged (wee) gluons, allowed by the quantum numbers in the t-channel. The logarithms $\ln p_A^+$ and $\ln p_B^-$ come exclusively from the integration over the longitudinal momenta of fast "right-movers" and "left-movers" in the blobs f^A and f^B.

Note also that there are no crossed gluon lines in Fig. 2. Physically, this is so because in the LLA the relevant component of the gluon propagator (1) can be approximated by

$$D^{-+}(q) \simeq \frac{N^2}{(N \cdot q)^2 - N^2 q^2},$$

i.e. by an instantaneous Coulomb interaction. Fig. 2 thus represents a sequence of instantaneous Coulombic gluon exchanges.

We are now ready to derive the leading-ln p_A^+ equation for the vertex f^A, or rather for its Mellin transform

$$f^A(E; \vec{q}_1, \ldots, \vec{q}_n) = -\int_m^\infty dp_A^+ (p_A^+)^{E-1} f^A(p_A^+; \vec{q}_1, \ldots, \vec{q}_n),$$

where E is related to the angular momentum j by $j = 1-E$. (Analogous equation can of

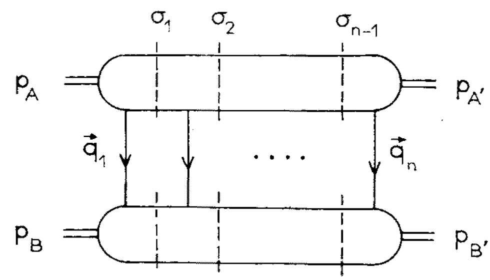

Fig. 2. General structure of the n-gluon exchange amplitude

course be derived for f^B). Our starting point is the almost obvious recursion of Fig. 3, where (a) is the lowest order inhomogeneous term, and the other terms represent all possible ways of adding a gluon loop which can yield $g^2 \ln p_A^+$. The dots after (c) indicate all remaining loop insertions with uncut gluon lines, and after (g) — all remaining insertions with a cut gluon line joining two different "cells". The logarithms $\ln p_A^+$ result from the k^+-integrations in the region $|\vec{k}| \ll k^+ \ll p_A^+$. It can be verified now that for the gluons in the blob f^A the Coulomb gauge is equivalent (in the LLA) to the light-like axial gauge with the gauge-defining vector n^μ taken along p_B^μ (so that $n \cdot k = k^+$),

$$D^{\mu\nu}(k) = \frac{1}{k^2 + i\varepsilon}\left[-g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k}\right]. \tag{4}$$

This simplification is justified, provided we limit the integration regions to $k^- \ll |\vec{k}| \ll k^+$.

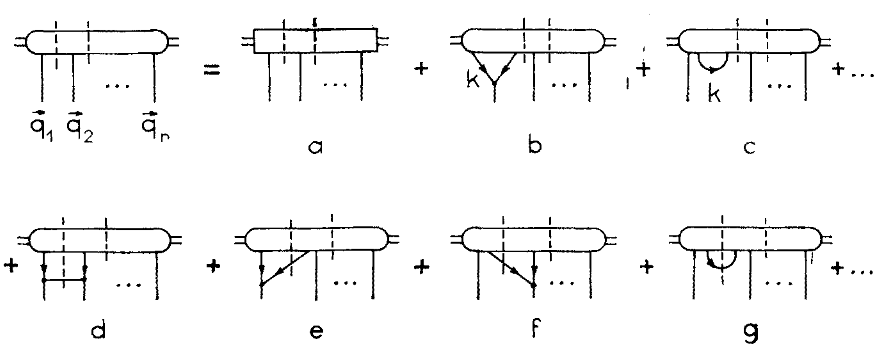

Fig. 3. Integral equation for the two-particle n-gluon vertex

Using (4) the relevant product of propagators in the diagram (b) is found to be

$$D^{\mu\varrho}(k) g_{\varrho\sigma} D^{\sigma\nu}(q_1 - k) = \frac{1}{k^2 + i\varepsilon} \frac{1}{(q_1 - k)^2 + i\varepsilon} \frac{k \cdot (q_1 - k)}{n \cdot k \, n \cdot (q_1 - k)} + \text{nonleading terms.} \tag{5}$$

Now we are taking the crucial step, which is the replacement of n^μ by k^μ / k^- [1]. The justification is as follows: The k^- loop integral can be thought of as an integral of the discontinuity of $n^\mu n^\nu \Gamma^A_{\mu\nu\ldots}$ in the mass variable $(p_A - k)^2 \simeq -2p_A^+ k^-$. Because of the Ward identities the product $k^\mu n^\nu \Gamma^A_{\mu\nu\ldots} \equiv G$ reduces to the vertex function with the number of gluon legs less by one, and obviously has no singularities in the variable $(p_A - k)^2$. Instead, there is a (principal-value) pole at $k^- = 0$, whose residue is proportional to $k^- n^\nu \Gamma^A_{-\nu\ldots} + k^i n^\nu \Gamma^A_{i\nu\ldots} \equiv H$ at $k^- = 0$. This quantity can be expressed by the dispersion relation in k^- in terms of its discontinuity. Since $(1/k^-) \operatorname{disc} H = -\operatorname{disc} n^\mu n^\nu \Gamma^A_{\mu\nu\ldots}$ (which follows from $\operatorname{disc} G = 0$), the dispersion integral reproduces the original one (in the LLA).

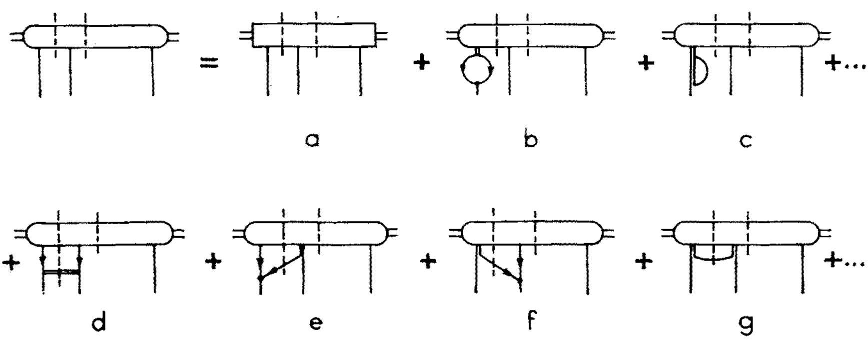

Fig. 4. Integral equation of Fig. 3 after applying Ward identities

Hence, the replacement $n^\mu \to k^\mu / k^-$, and then the Ward identities, give, in the standard graphical notation, the term (b) in Fig. 4, which involves the loop integral

$$\int dk^+ \int d^2 k \frac{\vec{k} \cdot (\vec{q}_1 - \vec{k})}{k^+} \int \frac{dk^-}{k^-} \frac{1}{k^2 + i\varepsilon} \frac{1}{(q_1 - k)^2 + i\varepsilon} = i\pi \int \frac{dk^+}{k^+} \int d^2 k \frac{\vec{k} \cdot (\vec{q}_1 - \vec{k})}{\vec{k}^2 (\vec{q}_1 - \vec{k})^2}.$$

Then, Fig. 3(c) is reduced to Fig. 4(c) in two steps: first, using the propagator (4), we apply Ward identities to the terms proportional to k^μ and k^ν; secondly, we replace n^ν and n^μ by k^ν / k^- and k^μ / k^- and proceed as before. Diagrams (e), (f) and (g) are treated along the same lines, except that some of the gluons are put on mass-shell, as indicated in the figure. In the diagram (d) we approximate the corresponding product of propagators as in Eq. (5).

Finally, the rules for Fig. 4 turn out to be:

1. for every internal single line: propagator $1/\vec{k}^2$;
2. for double lines: propagator $= 1$;

[1] This is analogous to the method of Ref. [9], where one replaces the full photon propagator by an expression involving factors k^μ, k^ν, such that it gives the same IR divergences as the original propagator.

3. for a cut single line: additional factor -1;
4. for each vertex: factor g and a group factor $-iC_{abc}$ with indices taken counter-clockwise;
5. for two internal single lines $\vec{k}_1$, $\vec{k}_2$ joining at one vertex or connected by a double line: factor $\vec{k}_1 \cdot \vec{k}_2$;
6. for a diagram with any number of cut lines: overall factor 2;
7. overall integration $-(2\pi)^{-3} \int dk^+/k^+ \int d^2k$ or, in the Mellin transform, $E^{-1}(2\pi)^{-3} \int d^2k$.

Thus the equation of Fig. 4 reads (suppressing the group factors)

$$Ef^A(E; \vec{q}_1, \ldots, \vec{q}_n) = \Phi^A(\vec{q}_1, \ldots, \vec{q}_n)$$

$$+g^2(2\pi)^{-3} \int d^2k_1 \left[\frac{\vec{k}_1 \cdot (\vec{q}_1 - \vec{k}_1)}{\vec{k}_1^2(\vec{q}_1 - \vec{k}_1)^2} + \frac{1}{\vec{k}_1^2}\right] f^A(E; \vec{q}_1, \ldots, \vec{q}_n) + \ldots$$

$$+g^2(2\pi)^{-3} \int d^2k_1 d^2k_2 \delta^2(\vec{q}_1 + \vec{q}_2 - \vec{k}_1 - \vec{k}_2) \left[\frac{2\vec{k}_1 \cdot \vec{k}_2}{\vec{k}_1^2 \vec{k}_2^2} - \frac{2\vec{k}_1 \cdot (\vec{q}_1 - \vec{k}_1)}{\vec{k}_1^2(\vec{q}_1 - \vec{k}_1)^2}\right.$$

$$\left. - \frac{2\vec{k}_2 \cdot (\vec{q}_2 - \vec{k}_2)}{\vec{k}_2^2(\vec{q}_2 - \vec{k}_2)^2} - \frac{2}{(\vec{q}_1 - \vec{k}_1)^2}\right] f^A(E; \vec{k}_1, \vec{k}_2, \vec{q}_3, \ldots, \vec{q}_n) \text{ (* group weight).}$$

The transverse momentum integrals, whenever divergent, have to be understood here as continued to $d > 2$ dimensions. Rearranging the terms sligtly and reintroducing the group weights we obtain

$$Ef^A_{a_1 \ldots a_n}(E; \vec{q}_1, \ldots, \vec{q}_n) = \Phi^A(\vec{q}_1, \ldots, \vec{q}_n) + \sum_i^n N\varepsilon(\vec{q}_i) f^A_{a_1 \ldots a_n}(E; \vec{q}_1, \ldots, \vec{q}_n)$$

$$+ \sum_{i<j}^n C_{a_icb_i} C_{a_jcb_j} g^2(2\pi)^{-3} \int d^2k_i d^2k_j \delta^2(\vec{q}_i + \vec{q}_j - \vec{k}_i - \vec{k}_j)$$

$$\times \left[\frac{(\vec{q}_i + \vec{q}_j)^2}{\vec{k}_i^2 \vec{k}_j^2} - \frac{\vec{q}_i^2}{\vec{k}_i^2(\vec{q}_i - \vec{k}_i)^2} - \frac{\vec{q}_j}{\vec{k}_j^2(\vec{q}_j - \vec{k}_j)^2}\right]$$

$$\times f^A_{a_1 \ldots b_i \ldots b_j \ldots a_n}(E; \vec{q}_1, \ldots, \vec{k}_i, \ldots, \vec{k}_j, \ldots, \vec{q}_n) \qquad (6)$$

with

$$\varepsilon(\vec{q}) = \tfrac{1}{2} g^2(2\pi)^{-3} \int d^2k \frac{\vec{q}^2}{\vec{k}^2(\vec{q} - \vec{k})^2}.$$

In the case of $n = 2$ our equation reduces to the one of Refs [2, 3, 8] or Ref. [11], depending on the quantum numbers in the t-channel. For $n = 1$ it coincides with the equation of Ref. [8], and for $n = 3$ in the symmetric colour singlet channel in SU(3) with the equation recently derived in Ref. [12].

In general, Eq. (6) can be interpreted as a Reggeon calculus equation for a system of n reggeized gluons (cf. [13]). The factors $N\varepsilon(\vec{q}_i)$ are then the Reggeon energies and the terms in the sum over i, j represent $2 \to 2$ Reggeon vertices. Since the function f^A is not the full amplitude but its multiple discontinuity, Eq. (6) refers to the *unsignatured* two--particle n-Reggeon amplitude.

We shall discuss now some properties of the equation (6) and its solutions.

(*i*) *IR finiteness in colour singlet exchange channels.* It was noted in Ref. [6] that for $n = 2$ equation (6) has no IR divergences in the vacuum quantum number exchange channel, when additionally the external particles are colour singlets. Here we make a more general observation (cf. also Ref. [15]). In Eq. (6) the divergences in the $\vec{k}_i$, $\vec{k}_j$ integrals come from two regions:

(α) $\vec{k}_i \simeq \vec{q}_i$. These divergences cancel with the divergences in the Reggeon energies $N\varepsilon(\vec{q}_i)$.This can be seen by using the identity

$$\sum_j C_{a_j c b_j} f_{a_1 \ldots b_j \ldots a_n} = 0$$

(valid when $f_{a_1 \ldots a_n}$ transforms as a colour-singlet tensor), and writing

$$\sum_i^n N\varepsilon(\vec{q}_i) f^A_{a_1 \ldots a_n}(E; \vec{q}_1, \ldots, \vec{q}_n) \equiv \sum_{i<j}^n C_{a_i c b_i} C_{a_j c b_j} g^2 (2\pi)^{-3} \int d^2k_i d^2k_j \delta^2(\vec{q}_i + \vec{q}_j - \vec{k}_i - \vec{k}_j)$$
$$\times \left[\frac{1}{(\vec{q}_i - \vec{k}_i)^2} \frac{\vec{q}_i^{\,2}}{\vec{k}_i^2 + (\vec{q}_i - \vec{k}_i)^2} + (i \to j) \right] f^A_{a_1 \ldots b_i \ldots b_j \ldots a_n}(E; \vec{q}_1, \ldots, \vec{q}_n).$$

Now the sum of the integrand here and in Eq. (6) is regular at $\vec{k}_i \simeq \vec{q}_i$. We can also see that this is in fact due to the pairwise cancellation of virtual IR divergences in Fig. 3(b), (c), ... with real divergences in Fig. 3(e), (f), (g), ...; they cancel independently of whether or not the external particles are colour singlets.

(β) $\vec{k}_i \simeq 0$. To show the absence of these divergences we note that in the LLA we have to consider only the minimum number of exchanged gluons, i.e. $n = 2$ in the charge conjugation $C = +1$, and $n = 3$ in the $C = -1$ colour singlet channels [12]. Then, if the external particles A and A' are colour singlets, one can see that each gluon emission from the blob f^A necessarily changes the group representation to which the fast-moving system belongs. Therefore the amplitude f^A vanishes if any of the gluon momenta $\vec{k}_i$ is zero. This is in fact a property of the lowest order term Φ^A, which propagates through the integral equation (6).

(*ii*) *Deep inelastic limit in the 2-gluon channel.* This amounts to taking only the leading $\ln \vec{q}_1^{\,2}$ (at zero total momentum transfer, $\vec{\Delta} = 0$). In this limit only the ladder diagrams, generated by Fig. 3(d), survive. The other terms generate diagrams with more than two gluons in the t-channel (i.e. two-particle-irreducible), which have no mass singularities [14].

(*iii*) *IR-finite cross-sections with colour exchange.* The simplest example is the one gluon exchange with momentum transfer $\vec{q}$, and with the initial system A being a colour singlet. It is convenient here to transform to the reference frame where A has transverse momentum $\vec{q}$, so that the final momentum is directed along the z-axis. The IR-finite cross--section can then be defined by summing over final states containing an arbitrary number of particles with transverse momenta less than a certain cut-off λ. This "jet" cross-section, or rather its Mellin transform, is proportional to the quantity $F_\lambda(E, \vec{q})$ satisfying

$$EF_\lambda(E; \vec{q}) = \Phi(\vec{q}) + 2N\varepsilon(\vec{q}) F_\lambda(E; \vec{q})$$
$$+ Ng^2(2\pi)^{-3} \int d^2k\, \theta(\lambda - |\vec{k} - \vec{q}|) \left[-2 \frac{\vec{q}^{\,2}}{\vec{k}^2 (\vec{q} - \vec{k})^2} \right] F_\lambda(E; \vec{k}).$$

This equation is essentially Eq. (6) specialized to $n = 2$ in the vacuum channel, with the integration over intermediate momenta restricted by the definition of our "jet". Here the divergences in $\varepsilon(\vec{q})$ are cancelled by those in the $\vec{k}$-integral, as discussed in ($i\alpha$) above. The effective IR-finite Regge trajectory is then approximately $j = 1 - N\varepsilon_\lambda(\vec{q})$, with

$$\varepsilon_\lambda(\vec{q}) \simeq \frac{g^2}{8\pi^2}\,\theta(\vec{q}^{\,2} - \lambda^2)\ln \vec{q}^{\,2}/\lambda^2.$$

IR-finite cross-sections can be similarly defined for exchange of gluon systems in higher representations.

(*iv*) *n-gluon exchange amplitudes.* In the LLA we can only consider such t-channel quantum numbers, for which n is a *minimum* number of the exchanged gluons. For the SU(2) gauge group ("isospin") such is the case of isospin $I = n$ exchange. For SU(3) the corresponding representations are $\{8\}$, $\{27\}$, $\{64\}$, ... for $n = 1, 2, 3, \ldots$. For SU(N) they are denoted by $D(n, 0, \ldots, 0, n)$; note that any subsystem of r gluons must then belong to the representation $D(r, 0, \ldots, 0, r)$.

In such a channel the vertex $f^A(E, \vec{q}_1, \ldots, \vec{q}_n)$ obeys Eq. (6) with no summation over colour indices and with $C_{a_icb_i}C_{a_jcb_j}$ replaced by -1. We shall now evaluate the corresponding amplitude T_n for large momentum transfer in the leading logarithms $\ln \vec{\Delta}^2$ (cf. analogous calculation for $n = 2$ in Ref. [13]). To this end it is sufficient to solve the equation for f^A with the strong ordering of its arguments, say $\vec{\Delta}^2 \simeq \vec{q}_1^{\,2} \gg \vec{q}_2^{\,2} \gg \ldots \gg \vec{q}_n^{\,2}$, summing the leading logarithms of $\vec{q}_i^{\,2}$. The integral in Eq. (6) then becomes approximately (for $\vec{q}_i^{\,2} \gg \vec{q}_j^{\,2}$)

$$g^2(2\pi)^{-3}\int \frac{d^2k_j}{\vec{k}_j^2}\left[1 - \frac{\vec{q}_j^{\,2} + \vec{k}_j^2}{(\vec{q}_j - \vec{k}_j)^2}\right] f^A(E; \vec{q}_1, \ldots, \vec{k}_i, \ldots, \vec{k}_j, \ldots, \vec{q}_n)$$

$$\simeq -\frac{g^2}{4\pi^2}\ln \vec{q}_j^{\,2} f^A(E; \vec{q}_1, \ldots, \vec{q}_n). \tag{7}$$

The logarithmic divergence from $\vec{k}_j \simeq \vec{q}_j$ will eventually cancel in an appropriately defined cross-section, as discussed in (*iii*) above; it is in this sense that the $\ln \vec{q}_j^{\,2}$ in Eq. (7) and below should be understood. With Eq. (7) the solution for f^A is

$$f^A(E; \vec{q}_1, \ldots, \vec{q}_n) \simeq \Phi^A(\vec{q}_1, \ldots, \vec{q}_n)\left\{E - \sum_{i=1}^{n}(N-2+2i)\frac{g^2}{8\pi^2}\ln \vec{q}_i^{\,2}\right\}^{-1}.$$

The n-gluon exchange amplitude T_n becomes then proportional to

$$T_n \sim \frac{s}{\vec{\Delta}^2}\int dx_1\delta(x_1 - \ln \vec{\Delta}^2)\int_0^{x_1} dx_2 \ldots \int_0^{x_{n-1}} dx_n \exp\left\{-\sum_{i=1}^{n}(N-2+2i)\frac{g^2}{8\pi^2}x_i \ln s\right\}, \tag{8}$$

where $x_i = \ln \vec{q}_i^{\,2}$, and the lower cut-off in $\vec{q}_i^{\,2}$ is provided by vanishing of Φ^A and Φ^B (for the same reasons as discussed in ($i\beta$)). In an IR-finite cross-section we will therefore find a family of n logarithmic-type branch points in the j-plane at

$$j = j_r \equiv 1 - r(r+N-1)\varepsilon_\lambda(\vec{\Delta}), \qquad r = 1, 2, \ldots, n. \tag{9}$$

The singularity at j_r is generated by a system of r gluons, each carrying transverse momentum of order $\vec{\Delta}$, while the remaining ones have much smaller momentum. The factor in front of ε_λ in Eq. (9) can be recognized as the quadratic Casimir operator (C_2) eigenvalue for the group representation $D(r, 0, \ldots, 0, r)$. This means that the considered system acts coherently as a single object. In terms of the Reggeon calculus, the Reggeon–Reggeon interactions are important: for non-interacting Reggeons we would get branch points at $j = 1 - rN\varepsilon_\lambda(\vec{\Delta})$.

To summarize, we have described a simple and general method of deriving integral equations governing the behaviour of multigluon exchange amplitudes in the leading $-\ln s$ approximation. The derivation is based directly on the gauge invariance of the theory (convenient choice of gauge and the Ward identities). It also makes apparent the IR structure of contributing Feynman diagrams, in particular cancellations of virtual and real IR divergences. The solutions of the derived equations exhibit Regge behaviour due to exponentiation (see Eq.(8)) of the leading IR divergences associated with the exchange of colour quantum numbers.

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