LETTERS TO THE EDITOR

ON LARGE DIMENSIONLESS NUMBERS IN COSMOLOGY

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Grabińska and Zabierowski's (1980) alleged new large dimensionless numbers are shown to be formally identical to Dirac's original large numbers, if one uses for the Hagedorn maximal universal temperature its expression derived in the framework of Hagedorn's statistical thermodynamics of strong interactions.

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The puzzling coincidences between large dimensionless numbers built from physical constants noticed by Weyl [1], Eddington [2] and Dirac [3] continue to attract serious attention from the physicists and cosmologists. They have been at the basis of the development of new cosmological theories characterized by a gravitational constant, G, changing with time, like Dirac's [4] recent theory and Canuto et al. [5] scale-covariant theory of gravitation.

Dirac's first coincidence relates the ratio of the electric and gravitational forces between an electron and a proton

$$N_1 = \frac{e^2}{Gm_e m_p} \simeq 10^{40}, \tag{1}$$

(where e is the electric charge, m_e, the electron mass and m_p, the proton mass) and the ratio of the radius of the observable universe to the size of an atom (or equivalently, the age of the universe expressed in terms of atomic units of time), i.e.

$$N_2 = \frac{m_e c^3}{He^2} \simeq 10^{40}, \tag{2}$$

where H is the Hubble constant.

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Dirac's second coincidence relates the mass of the universe, M_U, in units of the proton mass, i.e.

$$N_3 = \frac{M_U}{m_p} \simeq \frac{c^3}{HG} \simeq 10^{80}, \tag{3}$$

(the second equality of (3) being deduced from the Einstein field equations for a Friedmann–Robertson–Walker universe (Ref. [6])) and the square of N_1 (or N_2).

Dirac's explanation of these coincidences implies that they are always valid and that the values of the large dimensionless numbers vary with atomic time (Dirac's large number hypothesis) with the consequence that the gravitational constant G decreases with time as t^{-1}.

Alternative explanations for the relations $N_1 \simeq N_2$ and $N_3 \simeq N_1^2$ have been suggested, a more methaphysical one, first suggested by Dicke [7] and developed later by Carter [8], appealing to the anthropic principle (for a thorough review of the astrophysical and cosmological implications of the anthropic principle, see Carr and Rees [6]).

Grabińska and Zabierowski [9] have recently presented new large dimensionless numbers, N_4 and N_5, built from the same universal physical constants as N_1, N_2 and N_3 but containing also the Boltzmann constant, k, and Hagedorn's maximal universal temperature, T_H (for a review of Hagedorn's statistical thermodynamics of strong interactions, see [10] and references therein). They show that the coincidences existing between N_4 and N_5 and Dirac's large numbers N_1 and N_3 respectively are in excellent agreement with his large number hypothesis and they insist moreover on the fact that these alleged new coincidences imply some relation between gravitational and strong interactions and not only between gravitational and electromagnetic interactions as is the case for Dirac's original numbers N_1, N_2 and N_3.

The expressions for N_4 and N_5 given by Grabińska and Zabierowski are respectively

$$N_4 = h m_p^{-5/2} k^{1/2} T_H^{1/2} G^{-1} \simeq 10^{40}, \tag{4}$$

where h is Planck's constant, and

$$N_5 = c^5 H^{-1} G^{-1} (kT_H)^{-1} \simeq (10^{40})^2. \tag{5}$$

(Note that a "-1" exponent, relative to kT_H, is missing in Grabińska and Zabierowski's original expression (7)).

We wish to point out that Grabińska and Zabierowski's large dimensionless numbers N_4 and N_5 are not really new large numbers in the sense of Dirac, but are formally equivalent to N_1 and N_3, respectively.

In fact, N_4 can be written as follows:

$$N_4 = \alpha_G^{-1} \left(\frac{kT_H}{m_p c^2}\right)^{1/2}, \tag{6}$$

where $\alpha_G = \dfrac{Gm_p^2}{hc}$ is the gravitational fine structure constant.

It is easily seen (Ref. [6] and [11]) that, due to the value of the fine structure constant, $\frac{e^2}{\hbar c}$, α_G^{-1} and N_1 are both equal to $\simeq 10^{40}$ and are thus equal in the framework of Dirac's large number hypothesis, based on the order of magnitude arguments and which implies that large numbers approximately equal are considered as equal in Dirac's sense. On the other hand, Hagedorn's maximal universal temperature is known to be about one pion mass (in units where $k = c = 1$), i.e. in traditional units

$$kT_H = m_\pi c^2. \tag{7}$$

Taking account of (7), one sees that N_4 is formally equal (in Dirac's sense) to N_1, the ratio $\frac{m_p}{m_\pi}$ being quite negligible with respect to 10^{40}.

As regards Grabińska and Zabierowski's second large number, N_5, the introduction of equation (7) in the expression (5) for N_5, leads to:

$$N_5 = \frac{c^3 H^{-1} G^{-1}}{m_\pi}, \tag{8}$$

i.e. (cf. equation (3))

$$N_5 \sim \frac{M_U}{m_p} = N_2, \tag{9}$$

(the symbol $\sim$ meaning that the equality is approximate, the difference between m_p and m_π being neglected).

In conclusion, Grabińska and Zabierowski's alleged new dimensional coincidences are, in reality, not new coincidences. In fact, as we have shown, the large numbers N_4 and N_5 are intrinsically identical to Dirac's large numbers N_1 and N_3, as far as we adopt the order of magnitude framework characteristic of Dirac's large number hypothesis. This result has been obtained from Hagedorn's expression (7) for the universal maximal temperature, T_H, which receives a natural physical explanation in the framework of his statistical thermodynamics of strong interactions, independently of any cosmological consideration associated with the large dimensionless numbers.

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