SATURATION EFFECT AND DETERMINATION OF NUCLEAR MATTER DENSITY DISTRIBUTION FROM OPTICAL POTENTIAL

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A refined double folding procedure with density dependence of the effective nucleon-nucleon interaction included is used to calculate the real part of the alpha particle – ^{48,40}Ca potentials. We show that the experimentally determined difference between rms radii of the (real) potentials implies a larger size of the nuclear matter distribution of the ^{48}Ca nucleus as compared to the ^{40}Ca nucleus.

1. *Introduction*

The matter (neutron) density distributions in nuclei have been a topic of considerable interest in recent years [1]. Some information has been obtained by scattering of strongly interacting projectiles such as 1 GeV protons or 100 MeV alpha particles. But the poor knowledge of the strong interaction between a projectile and target nucleus implies some model dependence of the results. Investigations of the closed-shell isotopes ^{40}Ca and ^{48}Ca are of particular interest because the Ca-nuclei offer a large value of neutron excess.

Optical model fits to the elastic scattering cross sections of 104 MeV alpha particles from ^{48,40}Ca using a Fourier–Bessel description of the real potential show [2] that the rms radius of the α-^{48}Ca potential is greater than the α-^{40}Ca one by $\langle r^2\rangle^{1/2}_{\mathrm{Pot}(48)} - \langle r^2\rangle^{1/2}_{\mathrm{Pot}(40)} = 0.13 \pm 0.04$ fm. Moreover, the volume integrals per nucleon pair for both potentials agree within the experimental error of ± 3 MeV $\cdot$ fm^3. Assuming that the difference between mean-square (ms) radii of the potentials is equal to the difference beetween ms radii of the matter density distributions it was found that $\langle r^2\rangle^{1/2}_{\mathrm{m}}(48) - \langle r^2\rangle^{1/2}_{\mathrm{m}}(40) = 0.17 \pm 0.07$ fm.

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In the present paper we investigate whether the difference of potential rms radii necessarily implies a difference between the rms radii of the corresponding nuclear density distributions or whether it can mostly be ascribed to the density dependence of the effective nucleon-nucleon interaction.

2. *Double folding potentials*

Recently, the double folding procedure has been refined so that in addition to an exchange term [3] the density dependence of the effective nucleon–nucleon interaction [4] has been included. The density dependence originates from the presence of the surrounding nucleons changing the intermediate processes that might occur in the interaction of two nucleons.

The importance of this saturation effect has recently been demonstrated for 104 MeV alpha particle scattering [4]. Exchange effects have been shown to be of minor importance for that energy.

The basis of our discussion is the refined double folding model described in detail in Ref. [4] where

$$V_{\alpha-\mathrm{Ca}}(r) = \iint \varrho_\alpha(Z_\alpha)\varrho_{\mathrm{Ca}}(Z_{\mathrm{Ca}})t_\varrho(\vec{r}, \vec{Z}_\alpha, \vec{Z}_{\mathrm{Ca}})d\vec{Z}_\alpha d\vec{Z}_{\mathrm{Ca}}. \tag{1}$$

All quantities and the coordinates in Eq. (1) are defined in Ref. [4]. The effective nucleon-nucleon interaction depends actually on the local density $\varrho(\vec{r}, \vec{Z}_\alpha, \vec{Z}_{\mathrm{Ca}})$ of the overlapping system. Due to the Pauli distortion this local density is assumed to be intermediate between the arithmetic sum of ϱ_α and ϱ_{Ca} (sudden approximation: maximum compression) and the adiabatic case $\varrho = \varrho_{\mathrm{Ca}}$ (no compression). For simplicity we parametrized as follows $\varrho(r, \vec{Z}_\alpha, \vec{Z}_{\mathrm{Ca}}) = m\varrho_\alpha(\vec{Z}_\alpha)+\varrho_{\mathrm{Ca}}(\vec{Z}_{\mathrm{Ca}})$ with m $(0 \leqslant m \leqslant 1)$ accounting for the compression of nuclear matter in the overlap region. With the value of the parameter $m = 0.5$ the experimental $^{40}\mathrm{Ca}(\alpha, \alpha)^{40}\mathrm{Ca}$ data have been well reproduced also in the "rainbow" region whereby the values of the potential volume integral per nucleon pair and rms radius have been reasonable [4].

The alpha particle–^{48}Ca potential has been calculated using Eq. (1) with two alternative assumptions about the rms-radius of the matter density distribution ϱ_{m} for the ^{48}Ca nucleus[1].

(A) The rms-radius of $\varrho_{\mathrm{m}}(^{48}\mathrm{Ca})$ was assumed to be equal to that of $\varrho_{\mathrm{m}}(^{40}\mathrm{Ca})$ as it is the case for the *charge* distributions.

(B) The rms-radius of $\varrho_{\mathrm{m}}(^{48}\mathrm{Ca})$ was adopted from other analyses [2], [5] of 104 MeV alpha particle scattering yielding a larger rms radius of ^{48}Ca by an amount of 0.18 fm.

In both cases the influence of the density dependence of t_ϱ (Eq. (1)) was studied by introducing two extreme shapes for the ^{48}Ca matter density distribution.

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[1] In Eq. (1) for ^{40}Ca we used the point matter density distribution $\varrho_{\mathrm{m}} = \frac{40}{20}\varrho_{\mathrm{p}}$ whereby ϱ_{p} was deduced from the charge distribution ϱ_{ch} by unfolding the charge form factor of the proton.

(A, a) $\varrho_m(^{48}Ca) = \frac{48}{20}\,\varrho_p\,(^{48}Ca)$ where we use a three parameter Fermi distribution $\varrho_p(^{48}Ca)$ derived from Ref. [6]. This provides a considerable increase of the central density in the ^{48}Ca nucleus as compared to the ^{40}Ca one.

(A, b) The shape of $\varrho_m^{A,b}\,(^{48}Ca)$ has been adjusted so that the same value of the central density was fixed for ^{48}Ca as for the ^{40}Ca nucleus and the rms (^{48}Ca) radius was equal to that of the proton distribution used in case (A, a).

(B, a) $\varrho_m(^{48}Ca)$ was taken from Ref. [5].

(B, b) The shape of $\varrho_m^{B,b}\,(^{48}Ca)$ has been adjusted so that the same value of the central density was fixed for ^{48}Ca as for the ^{40}Ca nucleus and the rms (^{48}Ca) radius taken from Ref. [2] and [5].

In Fig. 1 we compare the matter density distribution of ^{40}Ca and that of ^{48}Ca obtained in procedure (A, b) and (B, b). In the cases (A, a) and (A, b) the resulting rms radii of the folded potentials are equal and the values of the volume integral per nucleon pair of the

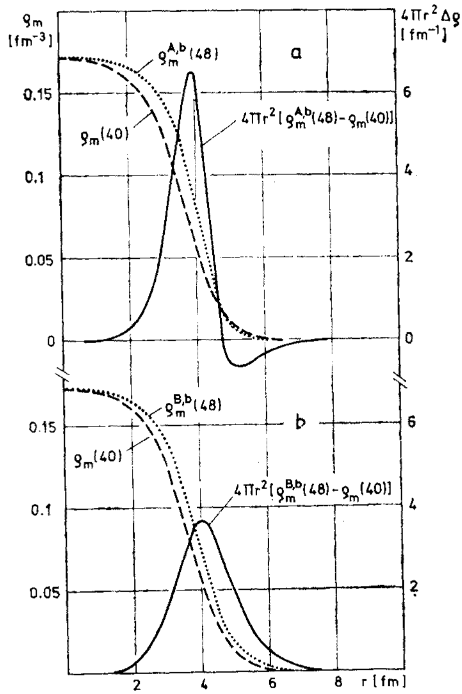

Fig. 1. Comparison of the matter density distribution $\varrho_m(40)$ of the ^{40}Ca nucleus (dashed line) with model matter density distributions $\varrho_m^{A,b}(48)$ and $\varrho_m^{B,b}(48)$ of the ^{48}Ca nucleus (dotted line) — see text. Radially weighted density difference is represented by solid line

$^{48}Ca\,(\alpha, \alpha)\,^{48}Ca$ potential are obviously too low, as one can see from Table I. This means that the refined double folding model is not able to reproduce the experimentally determined α–^{48}Ca potentials if the rms radius of the matter density distribution for ^{48}Ca nucleus is assumed to be equal to that of the proton density distribution. Moreover, applying

TABLE I

The rms-radii and the volume integrals per nucleon pair of the calculated potentials. The procedures (A, a), (A, b) etc. are explained in the text

Double folding potential	Matter density distribution in target nucleus	$\langle r^2\rangle^{1/2}_{\mathrm{Pot}}$ (fm)	J_N (MeV · fm³)
^{40}Ca(α, α)^{40}Ca	$\varrho_{\mathrm{m}}(40) = \frac{A}{Z}\,\varrho_{\mathrm{p}}(40)$	4.22	298.0
^{48}Ca(α, α)^{48}Ca	(A, a) $\varrho_{\mathrm{m}}(48) = \frac{A}{Z}\,\varrho_{\mathrm{p}}(48)$	4.23	280.3
	(A, b) $\langle r^2\rangle^{1/2}_{\mathrm{m}}(48) = \langle r^2\rangle^{1/2}_{\mathrm{p}}(48)$ $\varrho_0(48) = \varrho_0(40)$	4.21	283.6
	(B, a) Taken from Ref. [5]	4.37	294.5
	(B, b) $\langle r^2_{\mathrm{m}}\rangle^{1/2}$ adopted from Ref. [5] $\varrho_0(48) = \varrho_0(40)$	4.36	294.6

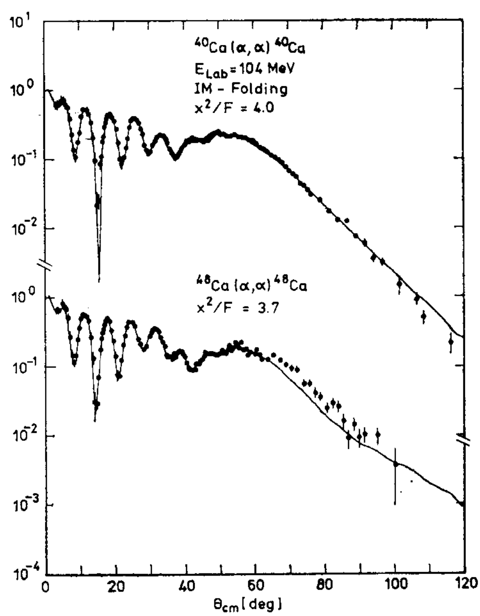

Fig. 2. Differential elastic scattering cross sections of ^{40,48}Ca(α, α) normalized to the Rutherford scattering and density dependent double folding model description

folded potentials (A, a) and (A, b) for description of the measured differential cross sections considerably worse reproductions have been obtained as compared to the $^{40}Ca(\alpha, \alpha)^{40}Ca$ case.

On the other hand we calculated the $^{48}Ca(\alpha, \alpha)^{48}Ca$ potential using the matter density distribution reported in Ref. [5] (case B). Thereby one obtains excellent agreement with the experimental data (Fig. 2) and with the empirical value [2] for the difference of the rms radii of the potentials *independent of the assumption about the central density* of ^{48}Ca (procedures (B, a), (B, b)). The resulting values of the volume integral per nucleon pair are equal for both target nuclei under consideration as one can see in Table I. The only "free" parameter m of the real potential which has been adjusted by means of the ^{40}Ca scattering cross sections cannot account for these findings since we varied it over the full reasonable range of $0 \leqslant m \leqslant 1$ and a change of the potential rms radius of only 0.06 fm has been observed.

3. *Conclusion*

The real part of the optical potential for alpha particle scattering from $^{48,40}Ca$ has been calculated using a refined double folding procedure. The model was calibrated on one isotope (^{40}Ca) and then used for the other (^{48}Ca) assuming that it does not change significantly over this restricted range of mass numbers.

We showed that the empirically determined difference between the rms radii of the alpha particle–^{48} ^{40}Ca potentials can only be reproduced in terms of a microscopic model if the additional assumption of a larger rms radius of the ^{48}Ca nucleus as compared to ^{40}Ca is included. Considering the fact that the proton distributions in ^{48}Ca and ^{40}Ca are very similar, the difference reveals a neutron halo in ^{48}Ca (solid line in Fig. 1b). This is in agreement with various Hartree–Fock calculations [7].

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