MULTIPLICITY OF SECONDARIES IN HADRON-NUCLEUS COLLISIONS AND CONSTITUENT QUARK RESCATTERING

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A ratio of the relativistic secondary multiplicities for hadron-nucleus and hadron-nucleon interactions, $R_s(\mathrm{hA})$, is considered in the central region and a part of the target nucleus fragmentation region. The multiplicities are obtained from the experimental average numbers of relativistic charged or negative particles by subtraction of the projectile fragment numbers estimated theoretically. Two hypotheses on the A dependence of the secondary multiplicity in a constituent quark interaction with a nucleus are discussed. An assumption that this multiplicity is independent of A leads to $R_s(\mathrm{hA}) = \bar{\nu}_{\mathrm{hA}}/\bar{\nu}_{\mathrm{qA}}$. An alternative assumption that the qA multiplicity increases with A due to quark rescattering from several nucleons gives $R_s(\mathrm{hA}) = \bar{\nu}_{\mathrm{hA}}$. Comparison with experiment in the former case requires a great number of positively charged hadrons, probably protons, emitted from the nucleus. This number must rise significantly with both A and incident energy. The latter hypothesis is consistent with all data on $\langle n_s\rangle$ as well as $\langle n_-\rangle$ in pA collisions but disagrees by $\sim 20\%$ with $\langle n_-\rangle$ in π^-A interactions.

1. Introduction

One of the most promising ways for description of the multiparticle processes on nuclei is to exploit an idea of composite structure of hadrons. An assumption that hadrons consist of two or three spatially discreted constituent (or "dressed") quarks interacting with nucleons of the target independently of each other, enables one to describe quantitatively well, without introducing free parameters, both the secondary yields in the projectile fragmentation region [1, 2] and the multiplicity ratio in the central region for pA and πA interactions [3]. These results are independent of the mechanism of the constituent quark interaction with the nucleus.

On the other hand such mechanism itself is of obvious interest. To reveal its nature we consider in this paper a ratio of the multiplicities of relativistic charged particles off a nucleus and a nucleon in the central region

$$R_s(\mathrm{hA}) = \frac{\langle n_s^{\mathrm{c.r.}}(\mathrm{hA})\rangle}{\langle n_s^{\mathrm{c.r.}}(\mathrm{hN})\rangle}. \tag{1}$$

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The secondaries contributing into $\langle n_s^{c.r.}\rangle$ are formed from the partons which arise due to "disintegration" of a constituent quark as a consequence of its interaction with the target. As shown in Appendix A, an average number of constituent quarks interacting inelastically with the nucleus depends only on the magnitude of quark-nucleon inelastic cross section $\sigma_{qN}^{inel} \simeq \frac{1}{3}\sigma_{pN}^{inel} \simeq \frac{1}{2}\sigma_{\pi N}^{inel} \simeq 10$ mb and is of the form

$$N_q(\mathrm{hA}) = \bar{\nu}_{hA}/\bar{\nu}_{qA}, \tag{2}$$

where $\bar{\nu}_{hA}$ ($\bar{\nu}_{qA}$) is a number of inelastic interactions of the hadron h (quark q) passing through the nucleus A:

$$\bar{\nu}_{hA} = \frac{A \cdot \sigma_{hN}^{inel}}{\sigma_{hA}^{prod}}, \tag{3}$$

$$\bar{\nu}_{qA} = \frac{A \cdot \sigma_{qN}^{inel}}{\sigma_{qA}^{prod}}. \tag{4}$$

Here, σ^{prod} is a cross section of an inelastic interaction with production of at least one secondary hadron.

The $R_s(\mathrm{hA})$ ratio differs from $N_q(\mathrm{hA})$ by a factor that is the secondary multiplicity ratio in the quark-nucleus and quark-nucleon collisions:

$$R_s(\mathrm{hA}) = N_q(\mathrm{hA}) \frac{\langle n_s(\mathrm{qA})\rangle}{\langle n_s(\mathrm{qN})\rangle}. \tag{5}$$

Different models are distinguished by just the structure of the last factor in Eq. (5).

In Refs. [4–6], the $\langle n_s(\mathrm{qA})\rangle$ and $\langle n_s(\mathrm{qN})\rangle$ were assumed to coincide so that

$$R_s(\mathrm{hA}) = N_q(\mathrm{hA}) = \bar{\nu}_{hA}/\bar{\nu}_{qA}. \tag{6}$$

However, as seen from the data discussed in Ref. [3], this equality holds only for particles produced in the high-energy part of the central region. With reducing the momenta of secondaries, the $R_s(\mathrm{hA})$ ratio increases and becomes noticeably larger than $N_q(\mathrm{hA})$. Owing to the contribution of this part of the central region, the average total multiplicity ratio is also larger than $N_q(\mathrm{hA})$, especially for heavy nuclei [7]. Therefore the scheme of Refs. [4–6] must include an additional assumption on the significant cascade reproduction of relativistic secondaries in the low-energy part of the central region.

In the models considering a collision with a nucleus as a consequence of interactions with individual nucleons (or quarks) there are generally two basic mechanisms giving rise to increasing multiplicity: intranuclear cascade and rescattering of the incident constituents [8]. The first one is a series of repeated interactions of the secondaries in the final state, that occur after the primary interaction is over. Experimental data are inconsistent with cascade reproduction of fast secondaries. This is likely a consequence of their formation from the point-like partons at large longitudinal distances, just outside the nucleus [9]. On the other hand, slow secondaries with momenta up to a few GeV/c have (probably) enough time to be formed inside the nucleus and then to interact once more giving rise to the cascade.

Another mechanism possible is rescattering of incident quarks or, equivalently, presence of events with production of secondaries from several nucleons of a nucleus simultaneously[1]. In the picture of the constituent quark being represented by a cloud of the point-like quark-partons and gluons [11], it means that two or more wee partons of the cloud may interact with different nucleons. In such a case coherence of an initial state is probably destroyed to a large extent giving rise to production of a larger number of secondaries. Alternatively, one may say that in the collisions of hadrons with nuclei production of several multiperipheral ladders of secondary hadrons is probable while in the hadron-nucleon interaction, as a rule, only one ladder is being produced.

According to the Abramovsky–Gribov–Kancheli rules [12] the multiplicity ratio for a nucleus and a nucleon is then an average number of inelastic interactions of the quark inside the nucleus, i.e.

$$\frac{\langle n_s(\mathrm{qA})\rangle}{\langle n_s(\mathrm{qN})\rangle} = \bar{\nu}_{\mathrm{qA}}. \tag{7}$$

For the present energies which are not very high one has to account for conservation of energy in interactions with several nucleons. It gives a correction to Eq. (7) which is computed in Appendix B.

Neglecting for a moment this correction, there is a simple relation for the multiplicity ratio of the secondaries in the central region which follows from Eqs. (2), (5) and (7):

$$R_s(\mathrm{hA}) = \bar{\nu}_{\mathrm{hA}}. \tag{8}$$

It agrees qualitatively with the result of Refs. [13–15] where the ratio of the total multiplicities of relativistic charged particles off a nucleus and a nucleon has been found to be a function of the variable $\bar{\nu}_{\mathrm{hA}}$ only.

Thus, there are two basic hypotheses on the interaction mechanism of a constituent quark with a nucleus, namely:

(A) The hypothesis of Refs. [4–6] on the secondary multiplicity in the central region being independent of the kind of the target. Here, $\langle n_s(\mathrm{qA})\rangle = \langle n_s(\mathrm{qN})\rangle$ so that Eq. (6) must be valid, and its violation observed in the low-energy part of the central region should be attributed to the contribution of an intranuclear cascade (see for example Ref. [16]).

(B) The hypothesis on rescattering of the constituent quarks inside the nucleus, giving rise to Eq. (8) for the central region.

The physical difference between these two hypotheses consists in the treatment of interaction in the final state of a multiparticle process. In the (A) case one believes this interaction to be rather strong. Due to a competition of the "decay" and "gathering" processes there may be a limiting density of secondaries, which is independent of the target and the number of interacting nuclear nucleons. At the same time strong interaction in the final state is also the reason for importance of intranuclear cascading.

The hypothesis (B), on the contrary, implies the interaction in the final state to be of no importance. In a first approximation it can be neglected as well as the cascades, so that Eqs. (7) and (8) emerge.

[1] In Ref. [10] it was shown that even in a model with multiple scattering of an incident hadron there is a possibility to describe satisfactorily both the A dependence of the average multiplicity of relativistic charged particles and multiplicity distributions, totally neglecting the contributions of intranuclear cascades.

The predictions following from the two hypotheses differ, sometimes drastically. Therefore the problem of their comparison with experiment is in order. One may think the sharpest distinction between the (A) and (B) cases to be in the inclusive spectrum and correlation function structure. For instance, in both cases one expects a plateau in the $R_s(y) = [dn_s(\mathrm{hA})/dy]/[dn_s(\mathrm{hp})/dy]$ ratio for super-high energies and large y, however, there must be $R_s(y) = \bar{\nu}_{\mathrm{hA}}/\bar{\nu}_{\mathrm{qA}}$ in case (A) and $R_s(y) = \bar{\nu}_{\mathrm{hA}}$ in case (B). On the other hand, the energies reached so far are not high enough, and predictions for the actual spectra and correlations are ambiguous. For instance the Fermi motion of nucleons inside a nucleus practically does not influence the number of secondaries but distorts their distributions in y or η. In particular, it increases sharply the number of hadrons with the smallest y or η where there are very few particles for a hadron-nucleon collision. The intranuclear cascade contribution depends also on a number of additional assumptions such as a kind and proportion of the resonances produced, etc.

Much more unambiguous are the predictions for average multiplicity of relativistic charged (or negative) secondaries in the entire central region to which one may add also a relativistic part of the target fragmentation region. Though there are no explicit experimental data for the multiplicity of this kind, it can be obtained by subtraction of the number of hadrons in the projectile fragmentation region, estimated theoretically, from the experimental total multiplicity of relativistic charged secondaries. The purpose of this paper is just to analyse such data and to compare them with predictions of the hypotheses (A) and (B).

The paper is organized as follows. In Section 2 we estimate the number of hadrons in the projectile fragmentation region. Difference between the proton and neutron targets and an experimental restriction for detection of only the relativistic secondaries are also taken into account. As a result we obtain the relations connecting total multiplicities $\langle n_s(\mathrm{hA})\rangle$ and $\langle n_{\mathrm{ch}}(\mathrm{hp})\rangle$ or $\langle n_-(\mathrm{hA})\rangle$ and $\langle n_-(\mathrm{hp})\rangle$ in both cases (A) and (B).

Comparison of these relations with the data is carried out in Section 3. We find that both hypotheses (A) and (B) meet with some difficulties. In order to make (A) compatible with the data on $\langle n_s\rangle$ and $\langle n_-\rangle$ it is necessary to allow a number of relativistic positive particles, probably fast protons, to be emitted (knocked out) from the nucleus. This number has to increase with both A and incident energy. The hypothesis (B) is consistent with the main bulk of the data, the only exception being the measurements of $\langle n_-\rangle$ in π^-A collisions where there is a discrepancy of $\sim 20\%$.

In Section 4 we estimate from the available data the number of fast protons which is related to the excess of the positive charge among the relativistic secondaries off a nucleus. According to the data this number is found to be noticeably larger in π^-A than in pA collisions with no reasonable explanation of such phenomenon.

Section 5 is devoted to dependence of the total charged multiplicity in hA interactions on incident energy. The hypothesis (A) is shown to predict a weaker energy dependence than that found experimentally. The hypothesis (B) is consistent with the observed energy dependence.

The results obtained and the accuracy of our estimates are discussed in Section 6.

2. *Relations for multiplicities of secondaries*

Since only the sum of the secondaries in the central region and the fragmentation region of the projectile can be measured experimentally and we are interested in the former one, it is necessary to estimate somehow the latter.

Let us consider a collision of a pion with a nucleus shown in Fig. 1. In the Fig. 1a case when only one constituent quark of the pion interacts inelastically, secondary hadrons are produced both in the central region and in the fragmentation region in exactly the

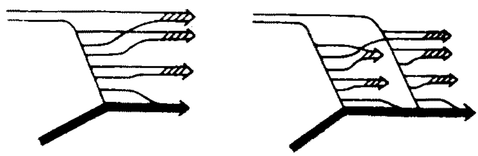

Fig. 1. Interaction of a pion with a nucleus. Either one (a) or two (b) constituent quarks interact and desintegrate into partons

same way as in the pion-nucleon interaction. In Fig. 1b, on the contrary, both quarks interact inelastically and are "absorbed". Therefore all the secondaries are in the central region or in the target fragmentation region here, and their number is twice as large as in the Fig. 1a case. Besides, the hadrons produced because of the valence quark-parton binding from different constituent clouds are also in the small x region in Fig. 1b. We adopt the simplest assumption that the number of such hadrons is equal to the number of the projectile fragments formed in the upper part of Fig. 1a. Then to get a ratio of 2 of the secondary multiplicities in Figs. 1b and 1a one has to subtract from them the average number of charged particles in the fragmentation region in Fig. 1a which coincides with that in the hadron-nucleon interaction, $\langle n^f_{ch}(\mathrm{hN})\rangle$. It means that the R_s ratio of Eq. (1) can be written as

$$R_s(\mathrm{hA}) = \frac{\langle n_s(\mathrm{hA})\rangle - \langle n^f_{ch}(\mathrm{hN})\rangle}{\langle n_s(\mathrm{hN_A})\rangle - \langle n^f_{ch}(\mathrm{hN})\rangle}, \tag{9}$$

where $\mathrm{N_A}$ is an "average" nucleon in the nucleus A.

There is no way to measure the magnitude of $\langle n^f_{ch}(\mathrm{hN})\rangle$ so we estimate it theoretically. According to Ref. [17], a quark-model calculation accounting for decay of both vector mesons and higher resonances gives $\langle n^f_{ch}(\pi^\pm \mathrm{N})\rangle \simeq 1.4$ and $\langle n^f_{ch}(\mathrm{K}^\pm \mathrm{N})\rangle \simeq 1.25$. For the incident proton beam a consideration of the measured difference of the total charged multiplicities in pp and πp collisions [18, 19] suggests $\langle n^f_{ch}(\mathrm{pN})\rangle \simeq 1.5$.

In a similar way, one can consider also the multiplicity ratio for negative secondaries. Here

$$R_-(\mathrm{hA}) = \frac{\langle n_-(\mathrm{hA})\rangle - \langle n^f_-(\mathrm{hN})\rangle}{\langle n_-(\mathrm{hN_A})\rangle - \langle n^f_-(\mathrm{hN})\rangle}, \tag{10}$$

where

$$\langle n^f_-(\mathrm{h^\pm N})\rangle = \tfrac{1}{2}\left[\langle n^f_{ch}(\mathrm{h^\pm N})\rangle \mp 1\right]; \tag{11}$$

TABLE I

Multiplicity of secondaries in the projectile fragmentation region [17]

Multiplicity \ h	p	π^+	K^+	π^-
$\langle n^f_{ch}(hN)\rangle$	1.5	1.4	1.25	1.4
$\langle n^f_-(hN)\rangle$	0.25	0.2	0.12	1.2
$\langle n^f_{\pi^0}(hN)\rangle$	0.25	0.7	0.5	0.7

the upper sign is for positively charged π^+, K^+ and p, and the lower one is for π^-, K^- and $\bar{p}$. The $\langle n^f_{ch}(hN)\rangle$ and $\langle n^f_-(hN)\rangle$ values used in what follows are listed in Table I.

Our next problem is related to presence in the denominators of Eqs. (9) and (10) of the $\langle n_s(hN_A)\rangle$ and $\langle n_-(hN_A)\rangle$ quantities for an "average" nucleon which we accept to be $N_A = \frac{1}{2}\,p+\frac{1}{2}\,n$ while experimentally both n_{ch} and n_- are measured on a hydrogen (proton) target. In the case of $p \to n\pi^+$ inelastic transition the charged number does not change but in the similar $n \to p\pi^-$ transition it increases by two units. Therefore assuming the probability of such transition to be $\simeq 1/3$ (see for example Ref. [20]) one finds

$$\langle n_{ch}(hN_A)\rangle = \langle n_{ch}(hp)\rangle - 0.17,$$

$$\langle n_-(hN_A)\rangle = \langle n_-(hp)\rangle + 0.17. \tag{12}$$

Furthermore, in nuclear experiments the multiplicity of shower particles, $\langle n_s\rangle$, is measured, that does not include the contribution of slow (non-relativistic) secondaries. Therefore their contribution has to be subtracted from the $\langle n_{ch}(hN_A)\rangle$ multiplicity on a nucleon. According to Ref. [21], when using the criteria adopted in the photoemulsion experiments there are $\simeq 0.48$ slow protons and $\simeq 0.14$ slow π^+ per one interaction with a proton. For a collision with a neutron there must be respectively $\simeq 0.14$ slow π^- and an unknown but small number of slow protons. As a result we obtain

$$\langle n_s(hN_A)\rangle = \langle n_{ch}(hN)\rangle - 0.45,$$

$$\langle n_-(hN_A)\rangle = \langle n_-(hN)\rangle - 0.07. \tag{13}$$

Combining Eqs. (12) and (13) with (9) and (10) gives finally

$$R_s(hA) = \frac{\langle n_s(hA)\rangle - \langle n^f_{ch}(hN)\rangle}{\langle n_{ch}(hp)\rangle - 0.6 - \langle n^f_{ch}(hN)\rangle},$$

$$R_-(hA) = \frac{\langle n_-(hA)\rangle - \langle n^f_-(hN)\rangle}{\langle n_-(hp)\rangle + 0.1 - \langle n^f_-(hN)\rangle}. \tag{14}$$

On the other hand, it is necessary to emphasize that the number of slow (not "shower") secondaries depends strongly on the experimental technique used. For instance, in a propane bubble chamber practically all the negative secondaries are detected, and only $\simeq 0.15$ protons are "slow", i.e. not included in the $\langle n_s(hA)\rangle$ number [22]. Therefore Eqs. (14) are reasonable to apply only to the electronic and photoemulsion experiments; for the bubble chamber ones the 0.6 and 0.1 figures in the denominators of Eqs. (14) have to be replaced by $\simeq 0.3$ and $\simeq 0.17$, respectively.

Let us now turn to our hypotheses (A) and (B). If the (A) is true then up to the contributions of intranuclear cascades there must be

$$\xi_s^{(A)} \equiv R_s(\text{hA}) \cdot \bar{\nu}_{qA}/\bar{\nu}_{hA} = 1,$$
$$\xi_-^{(A)} \equiv R_-(\text{hA}) \cdot \bar{\nu}_{qA}/\bar{\nu}_{hA} = 1. \tag{15}$$

In the (B) case the ν_{qA} in Eq. (15) is to be replaced by unity. Besides in the denominators of R_s and R_- it is necessary to add the terms $-D$ and $-D/2$, respectively, accounting for the energy losses in multiple rescattering processes. It gives

$$\xi_s^{(B)} \equiv R_s^{(B)}(\text{hA})/\bar{\nu}_{hA} = 1,$$
$$\xi_-^{(B)} \equiv R_-^{(B)}(\text{hA})/\bar{\nu}_{hA} = 1, \tag{16}$$

where

$$R_s^{(B)} = \frac{\langle n_s(\text{hA})\rangle - \langle n_{ch}^f(\text{hN})\rangle}{\langle n_{ch}(\text{hp})\rangle - 0.6 - \langle n_{ch}^f(\text{hN})\rangle - D},$$

$$R_-^{(B)} = \frac{\langle n_-(\text{hA})\rangle - \langle n_-^f(\text{hN})\rangle}{\langle n_-(\text{hp})\rangle + 0.1 - \langle n_-^f(\text{hN})\rangle - D/2}. \tag{17}$$

TABLE II

Corrections for the energy losses in multiple interactions of a quark with several nucleons (for $p_{lab} = 100$ GeV/c)

Nucleus	A	$\bar{\nu}_q$	D_1	D_2
C	12	1.25	0.037	0.075
CNO	14	1.26	0.038	0.077
Ne	20	1.34	0.047	0.097
Al	27	1.38	0.054	0.107
Cr	51	1.49	0.067	0.131
Em	60	1.51	0.069	0.135
Cu	63.5	1.52	0,071	0.137
AgBr	94	1.60	0.079	0.153
Ag	108	1.62	0.082	0.157
W	184	1.80	0.102	0.190
Pb	207	1.83	0.105	0.195
U	238	1.85	0.107	0.198

As shown in Appendix B, the D function can be written as

$$D = D(\bar{\nu}_{qA}, E) = D_1(\bar{\nu}_{qA}) \cdot \ln E + D_2(\bar{\nu}_{qA}). \tag{18}$$

An explicit form of D_1 and D_2 in terms of parameters determining the energy dependence of the charged multiplicity in hp collisions is given in Appendix B. The numerical values of D_1 and D_2 for different nuclei are presented in Table II. Just as in the case of Eq. (14), for the bubble chamber experiments the numbers 0.6 and 0.1 in the denominators of Eqs. (17) have to be replaced by 0.3 and 0.17, respectively.

TABLE III

Check of relations (15) and (16) for charged multiplicities

p[GeV/c]	hA	$\langle n_s(\mathrm{hA})\rangle$	$\langle n_{ch}(\mathrm{hp})\rangle$	$\bar{\nu}_{hA}/\bar{\nu}_{qA}$	$\xi_s^{(A)}$	$\langle n_p\rangle_{add}$	$\bar{\nu}_{hA}$	$\xi_s^{(B)}$
21 [28]	pCNO	4.58±.18	4.11±.06	1.36±.07	1.13±.08	.3±.2	1.68±.07	1.01±.08
[29]	pCNO	5.10±.3			1.32±.13	.9±.4		1.18±.12
[28]	pEm	5.90±.18		1.68±.09	1.30±.09	1.0±.3	2.50±.08	1.06±.07
[29]	pEm	5.8±.2			1.27±.10	.9±.3		1.04±.07
[28]	pAgBr	6.37±.18		1.81±.09	1.34±.09	1.2±.3	2.86±.11	1.05±.08
[29]	pAgBr	6.2±.2			1.29±.09	1.1±.3		1.01±.08
24 [30]	pCNO	5.2±.2	4.25±.03	1.36±.07	1.26±.10	.8±.3	1.68±.07	1.13±.07
[30]	pEm	6.6±.1		1.68±.09	1.41±.08	1.5±.3	2.50±.08	1.13±.05
[30]	pAgBr	7.1±.1		1.81±.09	1.44±.08	1.7±.3	2.86±.11	1.12±.05
28 [31]	pNe	5.7±.2	4.52±.07	1.43±.07	1.22±.10	.7±.3	1.83±.07	1.06±.08
50 [32]	pC	6.40±.23	5.35±.11	1.33±.09	1.13±.07	.6±.4	1.65±.06	0.98±.06
[30]	pCNO	6.7±.2		1.36±.07	1.18±.09	.8±.4	1.70±.07	1.01±.07
[30]	pEm	8.7±.1		1.68±.09	1.32±.09	1.7±.4	2.51±.08	1.01±.06
[32]	pEm	8.91±.27			1.36±.10	1.9±.4		1.04±.07
[32]	pCu	9.27±.43		1.70±.09	1.41±.12	2.2±.6	2.54±.08	1.09±.09
[30]	pAgBr	9.5±.2		1.81±.09	1.36±.09	2.1±.4	2.88±.11	1.00±.07
[32]	pPb	11.67±.57		2.04±.10	1.53±.13	3.5±.7	3.65±.12	1.06±.09
60 [29]	pCNO	6.4±.5	5.60±.09	1.36±.07	1.11±.13	.5±.6	1.70±.07	0.88±.10
[29]	pAgBr	8.9±.5		1.81±.09	1.26±.12	1.5±.7	2.88±.11	0.85±.08
67 [29]	pCNO	6.3±.7	5.84±.08	1.36±.07	0.94±.15	−.3±.8	1.71±.07	0.80±.13
[30]	pCNO	7.0±.2			1.08±.08	.4±.4		0.92±.06
[29]	pEm	8.8±.3		1.68±.09	1.16±.08	1.0±.5	2.52±.08	0.88±.06
[30]	pEm	9.3±.1			1.24±.07	1.5±.4		0.94±.05
[33]	pEm	9.73±.23			1.31±.08	1.9±.4		0.99±.06
[29]	pAgBr	10.1±.4		1.81±.09	1.27±.09	1.8±.6	2.89±.11	0.92±.07
[30]	pAgBr	10.1±.2			1.27±.08	1.8±.5		0.92±.06
69 [34]	pCNO	7.53±.27	5.89±.07	1.36±.07	1.17±.08	.9±.4	1.71±.07	0.99±.07
[34]	pAgBr	10.53±.48		1.81±.09	1.32±.10	2.2±.7	2.89±.11	0.95±.08
100 [15]	pC	7.72±.16	6.37±.06	1.33±.07	1.09±.07	.5±.4	1.67±.06	0.93±.05

[15]	pCu	11.0 ± .32		1.70 ± .09	1.31 ± .09	2.2 ± .6	2.56 ± .08	0.97 ± .06
[15]	pPb	14.75 ± .38		2.04 ± .10	1.52 ± .09	4.5 ± .7	3.69 ± .12	1.00 ± .06
[15]	pU	15.94 ± .50		2.08 ± .11	1.63 ± .10	5.6 ± .7	3.85 ± .14	1.05 ± .07
200 [32]	pC	9.34 ± .18	7.68 ± .07	1.33 ± .07	1.06 ± .06	.4 ± .5	1.70 ± .06	0.87 ± 05
[30]	pCNO	10.5 ± .2		1.36 ± .07	1.19 ± .07	1.4 ± .5	1.74 ± .07	0.98 ± .05
[35]	pCNO	10.7 ± .5			1.21 ± .09	1.6 ± .7		1.00 ± .07
[36]	pAl	11.2 ± .35		1.49 ± .08	1.17 ± .08	1.4 ± .6	2.06 ± .06	0.91 ± .05
[30]	pEm	13.8 ± .2		1.68 ± .09	1.31 ± .08	2.9 ± .6	2.58 ± .08	0.94 ± .05
[32]	pEm	13.36 ± .25			1.27 ± .08	2.5 ± .6		0.90 ± .05
[33]	pEm	13.31 ± .28			1.26 ± .08	2.4 ± .6		0.90 ± .05
[35]	pEm	13.4 ± .2			1.27 ± .08	2.5 ± .6		0.91 ± .05
[32]	pCu	14.01 ± .34		1.70 ± .09	1.32 ± .08	3.0 ± .6	2.61 ± .08	0.95 ± .05
[30]	pAgBr	15.0 ± .2		1.81 ± .09	1.34 ± .08	3.4 ± .6	2.94 ± .11	0.92 ± .05
[37]	pAgBr	14.7 ± .5			1.31 ± .09	3.1 ± .8		0.90 ± .06
[32]	pAg	14.95 ± .34		1.85 ± .09	1.30 ± .08	3.1 ± .7	3.00 ± .10	0.90 ± .05
[32]	pPb	18.47 ± .40		2.04 ± .10	1.49 ± .09	5.6 ± .8	3.73 ± .12	0.94 ± .05
300 [35]	pCNO	12.1 ± .5	8.50 ± .12	1.36 ± .07	1.22 ± .09	1.9 ± .7	1.75 ± .07	0.99 ± .07
[37]	pNe	11.70 ± .18		1.43 ± .07	1.06 ± .07	.6 ± .6	1.90 ± .07	0.85 ± .05
[38]	pCr	13.8 ± 1.2		1.63 ± .09	1.18 ± .13	1.9 ± 1.4	2.45 ± .09	0.85 ± .10
[35]	pEm	15.3 ± .2		1.68 ± .09	1.28 ± .08	3.0 ± .7	2.59 ± .08	0.91 ± .04
[35]	pAgBr	16.7 ± .5		1,81 ± .09	1.31 ± .08	3.6 ± .8	2.96 ± .11	0.89 ± .06
[38]	pW	18.6 ± 1.5		2.00 ± .10	1.34 ± .15	4.3 ± 1.7	3.60 ± .15	0.84 ± .10
400 [39]	pCNO	12.0 ± .3	8.99 ± .14	1.36 ± .07	1.12 ± .07	1.1 ± .6	1.76 ± .07	0.91 ± .06
[40]	pEm	17.0 ± .21		1.68 ± .09	1.34 ± .08	3.9 ± .7	2.60 ± .08	0.94 ± .05
[41]	pEm	16.42 ± .17			1.29 ± .08	3.4 ± .7		0.91 ± .04
[42]	pEm	16.8 ± .4			1.32 ± .08	3.7 ± .8		0.93 ± .05
[39]	pAgBr	18.3 ± .3		1.81 ± .09	1.35 ± .08	4.3 ± .8	2.97 ± .11	0.90 ± .05
10.5 [43]	π^+Ne	4.23 ± .05	4.05 ± .10	1.21 ± .06	1.00 ± .09	.2 ± .2	1.55 ± .04	0.85 ± .06
50 [32]	π^+C	6.54 ± .18	5.89 ± .06	1.17 ± .06	1.14 ± .08	.6 ± .3	1.41 ± .04	0.99 ± .05
[36]	π^+Al	7.4 ± .2		1.24 ± .06	1.24 ± .08	1.1 ± .4	1.64 ± .05	1.02 ± .06
[32]	π^+Em	9.05 ± .23		1.34 ± .07	1.47 ± .09	2.4 ± .4	1.98 ± .08	1.11 ± .07
[32]	π^+Cu	9.76 ± .30		1.35 ± .07	1.59 ± .11	3.1 ± .5	2.00 ± .07	1.20 ± .07
[36]	π^+Cu	8.6 ± .25		1.59 ± .11	1.37 ± .08	1.9 ± .4		1.03 ± .06
[36]	π^+Ag	9.6 ± .3		1.42 ± .07	1.48 ± .09	2.7 ± .5	2.30 ± .09	1.05 ± .06
[32]	π^+Pb	11.26 ± .36		1.51 ± .08	1.68 ± .11	4.0 ± .5	2.76 ± .09	1.09 ± .07

TABLE III (continued)

p[GeV/c]	hA	$\langle n_s(hA)\rangle$	$\langle n_{ch}(hp)\rangle$	$\bar{\nu}_{hA}/\bar{\nu}_{qA}$	$\xi_s^{(A)}$	$\langle n_p\rangle_{add}$	$\bar{\nu}_{hA}$	$\xi_s^{(B)}$
100 [15]	π^+C	7.86±.15	6.62±.07	1.16±.06	1.21±.07	1.1±.4	1.42±.04	1.04±.05
[15]	π^+Cu	10.29±.26		1.35±.07	1.43±.09	2.6±.5	2.03±.07	1.05±.05
[15]	π^+Pb	13.21±.30		1.51±.08	1.69±.10	4.8±.5	2.81±.09	1.07±.06
[15]	π^+U	14.57±.39		1.53±.08	1.86±.12	6.1±.6	3.00±.10	1.12±.06
200 [32]	π^+C	9.48±.27	8.0±.2	1.16±.06	1.16±.08	1.1±.5	1.43±.05	0.99±.07
[32]	π^+Cu	13.61±.77		1.35±.07	1.51±.14	4.1±.9	2.06±.07	1.08±.09
[32]	π Ag	14.31±.70		1.42±.07	1.54±.13	4.6±.9	2.37±.09	1.02±.08
[32]	π^+Pb	16.17±.61		1.51±.08	1.63±.13	5.7±.8	2.84±.10	0.99±.07
50 [32]	K^+C	6.31±.52	5.7±.2	1.13±.06	1.16±.14	.7±.6	1.31±.05	1.06±.14
[32]	K^+Cu	9.26±1.45		1.29±.07	1.61±.32	3.0±1.5	1.80±.07	1.29±.26
[32]	K^+Pb	11.35±1.29		1.44±.07	1.82±.27	4.6±1.4	2.40±.09	1.30±.21
100 [15]	K^+C	6.91±.33	6.65±.31	1.13±.06	1.04±.11	.3±.6	1.36±.06	0.92±.11
[15]	K^+Cu	8.89±1.10		1.29±.07	1.24±.21	1.4±1.2	1.86±.07	0.95±.17
[15]	K^+Pb	12.92±.79		1.44±.07	1.69±.18	4.8±1.0	2.49±.10	1.14±.13
[15]	K^+U	12.93±1.33		1.45±.07	1.68±.24	4.7±1.5	2.55±.15	1.11±.17
10.5 [43]	π^-Ne	3.91±0.4	3.68±.04	1.21±.06	1.21±.06	.1±.2	1.60±.04	0.89±.04
17 [29]	π^-CNO	4.1±.4	4.27±.05	1.17±.06	1.02±.16	0±.5	1.48±.05	0.87±.14
[29]	π^-Em	5.3±.3		1.34±.07	1.28±.12	.9±.4	2.05±.09	0.98±.10
[29]	π^-AgBr	5.7±.3		1.40±.07	1.35±.12	1.1±.4	2.30±.09	0.99±.09
20 [44]	π^-Al	5.25±.15	4.60±.05	1.24±.06	1.19±.08	.6±.3	1.69±.06	0.98±.06
[44]	π^-Cu	5.85±.15		1.35±.07	1.27±.09	.9±.3	2.06±.08	0.96±.06
[44]	π^-Ag	6.30±.15		1.42±.07	1.33±.09	1.2±.3	2.36±.10	0.94±.06
[44]	π^-Pb	6.70±.15		1.51±.08	1.35±.09	1.4±.3	2.84±.10	0.89±.05
25 [45]	π^-Ne	5.67±.14	4.86±.04	1.21±.06	1.12±.07	.4±.3	1.61±.04	0.91±.05
37.5 [44]	π^-C	5.90±.15	5.36±.10	1.16±.06	1.15±.08	.6±.3	1.46±.04	0.98±.06
[44]	π^-Al	6.75±.15		1.24±.06	1.28±.09	1.2±.3	1.70±.06	1.03±.06
[44]	π^-Cu	7.65±.15		1.35±.07	1.38±.09	1.7±.3	2.07±.08	1.02±.06
[44]	π^-Ag	8.45±.15		1.42±.07	1.48±.10	2.4±.4	2.37±.10	1.02±.06
[44]	π^-Pb	8.85±.15		1.51±.08	1.47±.10	2.4±.4	2.86±.10	0.94±.06
40 [46]	π^-C	6.32±.06	5.52±.03	1.16±.06	1.11±.07	.5±.3	1.46±.04	0.93±.04
50 [28]	π^-CNO	6.21±.17	5.78±.04	1.17±.06	1.08±.07	.4±.3	1.49±.05	0.91±.05

	[45]	π^-Ne	6.88±.13		1.21±.06	1.11±.07	.5±.3	1.62±.04	0.89±.04
	[28]	π^-Em	8.01±.19		1.34±.07	1.31±.08	1.5±.4	2.07±.09	0.94±.06
	[28]	π^-AgBr	8.54±.19		1.40±.07	1.35±.08	1.8±.4	2.33±.09	0.92±.05
60	[29]	π^-CNO	7.4±.5	6.05±.10	1.17±.06	1.27±.13	1.3±.6	1.49±.05	1.05±.10
	[34]	π^-CNO	7.42±.24			1.27±.09	1.3±.4		1.06±.07
	[29]	π^-Em	8.6±.2		1.34±.07	1.33±.09	1.8±.4	2.07±.09	0.96±.06
	[47]	π^-Em	8.59±.18			1.33±.09	1.8±.4		0.96±.06
	[29]	π^-AgBr	9.3±.3		1.40±.07	1.39±.10	2.2±.5	2.33±.09	0.95±.07
	[34]	π^-AgBr	8.89±.3			1.32±.09	1.8±.5		0.90±.07
100	[32]	π^-C	7.9±.2	6.78±.08	1.16±.06	1.17±.08	1.0±.4	1.46±.06	0.98±.06
	[32]	π^-Cu	10.3±.3		1.35±.07	1.38±.09	2.4±.5	2.10±.08	0.98±.07
	[32]	π^-Pb	13.2±.3		1.51±.08	1.63±.10	4.6±.5	2.91±.11	0.99±.06
	[32]	π^-U	14.6±.4		1.53±.08	1.80±.11	5.9±.6	3.10±.12	1.04±.06
175	[32]	π^-C	9.0±.3	7.68±.15	1.16±.06	1.15±.08	1.0±.5	1.47±.05	0.96±.06
	[32]	π^-Cu	12.6±.5		1.35±.07	1.46±.10	3.5±.7	2.11±.07	1.02±.07
	[32]	π^-Pb	16.3±.6		1.51±.08	1.74±.12	6.3±.8	2.97±.11	1.02±.07
200	[32]	π^-C	8.99±.29	8.02±.12	1.16±.06	1.09±.08	.6±.5	1.47±.05	0.90±.06
	[48]	π^-CNO	9.4±.5		1.17±.06	1.14±.10	1.0±.7	1.50±.05	0.93±.07
	[36]	π^-Al	10.5±.3		1.24±.06	1.22±.09	1.6±.6	1.73±.07	0.93±.06
	[49]	π^-Cr	12.53±.64		1.32±.07	1.40±.11	3.2±.8	2.00±.09	1.01±.08
	[32]	π^-Em	12.46±.37		1.34±.07	1.37±.09	3.0±.6	2.09±.09	0.96±.07
	[47]	π^-Em	11.94±.23			1.31±.08	2.5±.6		0.91±.06
	[50]	π^-Em	12.01±.10			1.32±.08	2.6±.5		0.92±.05
	[32]	π^-Cu	12.60±.52		1.35±.07	1.38±.11	3.1±.7	2.12±.08	0.96±.07
	[48]	π^-AgBr	13.0±.4		1.40±.07	1.38±.10	3.2±.7	2.36±.09	0.90±.05
	[32]	π^-Ag	13.86±.48		1.42±.07	1.46±.11	3.9±.7	2.45±.08	0.94±.06
	[49]	π^-W	14.58±1.01		1.49±.08	1.47±.14	4.2±1.2	2.85±.09	0.87±.08
	[32]	π^-U	16.31±.49		1.51±.08	1.64±.11	5.8±.8	2.94±.09	0.96±.06
200	[32]	$\bar{p}$C	9.19±.85	8.0±.3	1.33±.07	0.98±.13	−0.2±1.0	1.76±.12	0.78±.12
	[32]	$\bar{p}$Cu	13.49±2.23		1.70±.09	1.20±.25	2.0±2.4	2.85±.11	0.78±.17
	[32]	$\bar{p}$Ag	17.96±2.12		1.85±.09	1.51±.23	5.5±2.3	3.25±.15	0.95±.15
	[32]	$\bar{p}$Pb	16.04±2.83		2.04±.10	1.21±.27	2.5±3.0	3.99±.15	0.71±.16

TABLE IV

Check of relations (15) and (16) for multiplicities of negative secondaries

p[GeV/c]	hA	$\langle n_-(\text{hA})\rangle$	$\langle n_-(\text{hp})\rangle$	$\bar{\nu}_{\text{hA}}/\bar{\nu}_{\text{qA}}$	$\xi_-^{(A)}$	$\langle n_-\rangle_{\text{add}}$	$\bar{\nu}_{\text{hA}}$	$\xi_-^{(B)}$
21 [28]	pCNO	1.68±.13	1.055±.03	1.36±.07	1.16±.13	.2±.2	1.68±.07	1.05±.12
[28]	pEm	2.19±.13		1.68±.09	1.27±.12	.4±.2	2.50±.08	1.06±.09
[28]	pAgBr	2.37±.13		1.81±.09	1.29±.11	.5±.2	2.86±.11	1.04±.09
28 [31]	pNe	2.01±.10	1.26±.035	1.43±.07	1.04±.09	.1±.2	1.83±.07	0.92±.08
300 [37]	pNe	4.95±.09	3.25±.06		1.04±.06	.2±.3	1.90±.07	0.83±.04
10.5 [43]	π^+Ne	1.37±.02	1.025±.05	1.21±.06	0.97±.08	0±.1	1.55±.04	0.85±.07
10.5 [43]	π^-Ne	2.08±.02	1.84±.02	1.21±.06	0.90±.06	−.1±.1	1.60±.04	0.78±.04
25 [45]	π^-Ne	2.91±.07	2.43±.02	1.21±.06	1.01±.07	0±.2	1.61±.04	0.83±.04
40 [46]	π^-C	3.24±.03	2.76±.03	1.16±.06	1.02±.06	0±.2	1.46±.04	0.86±.03
50 [28]	π^-CNO	3.33±.13	2.89±.02	1.17±.06	1.02±.08	0±.2	1.49±.09	0.85±.08
[45]	π^-Ne	3.51±.07		1.21±.06	1.03±.06	.1±.2	1.62±.04	0.83±.04
[28]	π^-Em	3.89±.13		1.34±.07	1.12±.08	.3±.2	2.07±.09	0.82±.06
[28]	π^-AgBr	4.06±.14		1.40±.07	1.14±.08	.4±.2	2.33±.09	0.79±.06

3. *Comparison with experiment*

In Table III and IV relations (15) and (16) are compared to the experimental data on multiplicity of relativistic charged and negative secondaries. The $\langle n_{ch}(\text{hp})\rangle$ and $\langle n_{-}(\text{hp})\rangle$ values have been taken from the compilation of Ref. [23] where the further references to the original experiments can be found. Exceptions are the data at 100 and 60 GeV/c where the more recent measurements of Refs. [24, 25] have been used, and at 40 GeV/c where preference has been given to an analysis of Ref. [20]. The $\bar{\nu}_{\text{hA}}$ values have been computed according to Eq. (3) by us ng the latest measurements of $\sigma_{\text{hA}}^{\text{prod}}$ in Ref. [26] and $\sigma_{\text{hp}}^{\text{inel}}$ from Ref. [27]. The average numbers of inelastically interacting quarks $N_q = \bar{\nu}_{\text{hA}}/\bar{\nu}_{\text{qA}}$ have

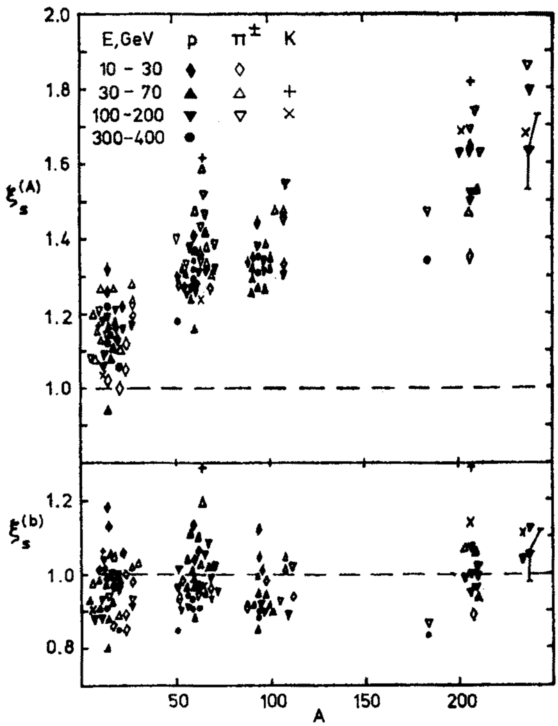

Fig. 2. The ξ_s values from the data of Table III on multiplicities of the relativistic charged hadrons

been calculated by means of formulae given in Appendix A; they are also included in Tables III and IV. The data on $\langle n_s(\text{hA})\rangle$ have been taken from Refs. [15, 28–50].

In Fig. 2 the $\xi_s^{(A)}$ and $\xi_s^{(B)}$ ratios are plotted versus A. They should be unity provided hypothesis (A) or (B) is valid, respectively. The $\xi_s^{(B)} = 1$ equality is seen to be actually the case to a good accuracy. An $\xi_s^{(B)}$ = const assumption gives $\xi_s^{(B)} = 0.97$ with χ^2/N.D. = 171/126 in spite of not all the experiments being made by the same technique.

To this respect one may note that the equality $\xi_s^{(B)} = 1$ explains in a natural way the success of the attempts to parametrize the total multiplicity ratio, $\langle n_s(\text{hA})\rangle/\langle n_{ch}(\text{hp})\rangle$

by a linear function of $\bar{\nu}_{hA}$. From Eqs. (16) and (17) it follows

$$\frac{\langle n_s(\mathrm{hA})\rangle}{\langle n_{ch}(\mathrm{hp})\rangle} = a+b\bar{\nu}_{hA},$$

$$a = \frac{\langle n^f_{ch}(\mathrm{hN})\rangle}{\langle n_{ch}(\mathrm{hp})\rangle}, \qquad b = 1-\frac{\langle n^f_{ch}(\mathrm{hN})\rangle+0.6+D}{\langle n_{ch}(\mathrm{hp})\rangle}. \tag{19}$$

Due to the increase of $\langle n_{ch}(\mathrm{hp})\rangle$ with energy the parameter a must decrease to zero and b rise to the limiting value of unity. At the present energies, $a+b < 1$. For 100 GeV/c using an average value $D = 0.3$ in Eq. (19) gives $a = 0.24$, $b = 0.63$ while the best fit to the data (with $\langle n_{ch}(\mathrm{hp})\rangle$ replaced by $[\langle n_{ch}(\mathrm{hp})\rangle-0.5]$ that increases both a and b by $\simeq 8\%$) results in $a = 0.47$, $b = 0.67$ [15]. For 400 GeV/c, $a = 0.17$ and $b = 0.73$ are predicted. Owing to the A dependence of D, a very weak deviation is expected from the linear dependence on ν_{hA}.

As to the alternative $\xi_s^{(A)} = 1$ equality, it is violated badly. For instance, $\xi_s^{(A)} \simeq 1.3$–1.4 for $A \simeq 100$, and $\xi_s^{(A)} > 1.5$ for $A > 200$. In the framework of the hypothesis (A), the deviation of $\xi_s^{(A)}$ from unity is possible to explain only by presence in $\langle n_s(\mathrm{hA})\rangle$ of some contribution of additionally produced hadrons. These hadrons must be mostly positively charged. In an opposite case they would contribute to the $\langle n_-(\mathrm{hA})\rangle$ as well, but the experimental number of additional negative hadrons, as shown in Table IV, is rather small. (Unfortunately, the data on $\langle n_-\rangle$ are there for not very high momenta only, up to 50 GeV/c, and not for the heaviest nuclei). Therefore one may believe the main part of additionally produced hadrons to consist of fast protons with momenta greater than 1 GeV/c (slower protons are not included in $\langle n_s(\mathrm{hA})\rangle$). These protons are either knocked out of the nucleus in an intranuclear cascade process or emitted due to Fermi motion. Denoting their number by $\langle n_p(\mathrm{hA})\rangle_{add}$ we have to replace $\langle n_s(\mathrm{hA})\rangle$ in the numerator of Eq. (14) for $R_s(\mathrm{hA})$ by the $\langle n_s(\mathrm{hA})\rangle-\langle n_p(\mathrm{hA})\rangle_{add}$ difference. Then, the condition $\xi_s^{(A)} = 1$ enables one to find the $\langle n_p(\mathrm{hA})\rangle_{add}$ values that are also given in Table III. One finds that this quantity increases with both A and energy, and for $p_{lab} > 30$ GeV/c can be approximated by an expression

$$\langle n_p(\mathrm{hA})\rangle_{add} = 0.05\cdot A^{2/3}\ln p_{lab}/5, \tag{20}$$

where p_{lab} is measured in GeV/c. The values $\langle n_p(\mathrm{hA})\rangle_{add}$ required by the condition $\xi_s^{(A)} = 1$ for the photoemulsion data are shown in Fig. 3, as well as the fit of Eq. (20). Both the rapid increase of $\langle n_p(\mathrm{hA})\rangle_{add}$ with p_{lab} and its large magnitude for heavy nuclei appear not very probable. For instance if one extrapolates the momentum dependence of Eq. (20) up to higher energies it will give even for an interaction with light nuclei of air at $p_{lab} = 10^5$ GeV/c as large value as $\langle n_p\rangle_{add} \simeq 3$.

Let us now turn to the data on negative multiplicity in Table IV. The number of experiments here is unfortunately much smaller than in Table III, and they are confined to the comparatively small momenta ($\lesssim 50$ GeV/c) and A. In Fig. 4, the ratios $\xi_-^{(A)}$ and $\xi_-^{(B)}$ are shown, that must be unity provided the hypothesis (A) or (B) is correct. The situation is far from being clear. The experimental data are generally consistent with hypothesis (A)

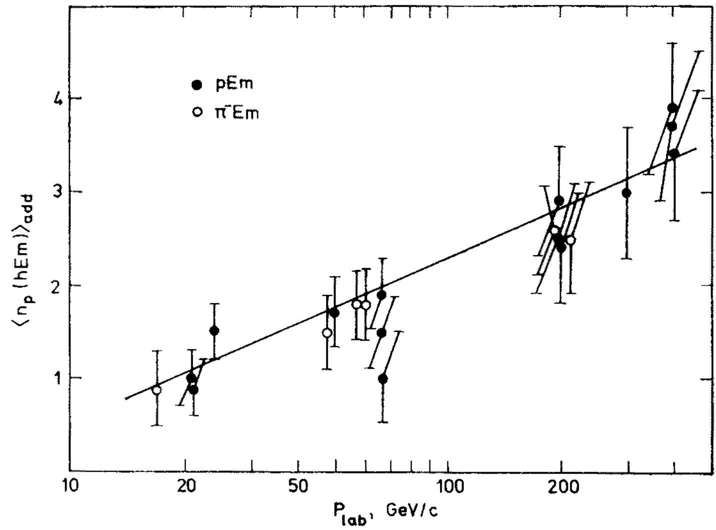

Fig. 3. Multiplicity of "additional" charged hadrons (protons) required by the $\xi_s^{(A)} = 1$ equality as a function of the incident energy for the photoemulsion data

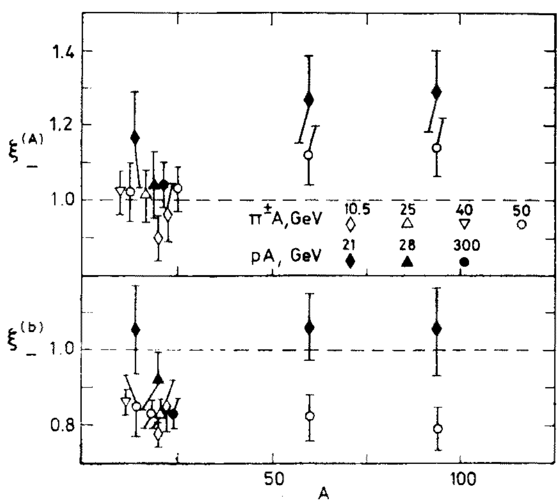

Fig. 4. The ξ_- values from the data of Table IV on multiplicities of the negative charged secondaries

although one may note some tendency for increasing $\xi_-^{(A)}$ with A, that follows from the data of Ref. [28]. On the other hand, the number of "additional" negative secondaries is not large everywhere.

In the case of hypothesis (B), the equality $\xi_-^{(B)} = 1$ is valid for the proton-nucleus interactions with the only exception of the preliminary data of Ref. [37] on pNe interactions at 300 GeV/c. However for the π^- beam all the data are consistent with $\xi_-^{(B)} \simeq 0.8$ instead

TABLE V

Check of relations (21) for the π^0 number

p[GeV c]	hA	$\langle n_{\pi^0}(\mathrm{hA})\rangle$	$\langle n_{\pi^0}(\mathrm{hN})\rangle$	$\bar{\nu}_{\mathrm{hA}}/\bar{\nu}_{\mathrm{qA}}$	$\xi_{\pi^0}^{(\mathrm{A})}$	$\langle n_{\pi^0}\rangle_{\mathrm{add}}$	ν_{hA}	$\xi_{\pi^0}^{(\mathrm{B})}$
10.5 [43]	$\pi^{\pm}$Ne	$1.77 \pm .05$	$1.62 \pm .08$	$1.21 \pm .06$	$0.96 \pm .11$	$0 \pm .2$	$1.57 \pm .04$	$.83 \pm .10$
40 [51]	π^{-}C	$2.91 \pm .04$	$2.47 \pm .06$	$1.16 \pm .06$	$1.08 \pm .08$	$.2 \pm .2$	$1.46 \pm .04$	$.91 \pm .06$

of unity. Generally the data of Refs. [28, 45, 46] on π^-–nucleus collisions seem to be more reliable, for they have been obtained at higher energy than in the p-nucleus experiments, where an uncertainty in the correction for the energy conservation must be less important.

Relations similar to Eqs. (14) and (17) can be written also for production of any definite hadron however the data are practically absent. The only exception is production of π^0 mesons whose multiplicity was measured in π^-C interactions at 40 GeV/c [51] and in $\pi^\pm$Ne collisions at 10.5 GeV/c [43]. The results of these experiments are given in Table V together with the ratios

$$\xi_{\pi^0}^{(A)} = \frac{\langle n_{\pi^0}(\mathrm{hA})\rangle - \langle n^{\mathrm{f}}_{\pi^0}(\mathrm{hN})\rangle}{\langle n_{\pi^0}(\mathrm{hN})\rangle - \langle n^{\mathrm{f}}_{\pi^0}(\mathrm{hN})\rangle} \cdot \frac{\bar{\nu}_{\mathrm{qA}}}{\bar{\nu}_{\mathrm{hA}}},$$

$$\xi_{\pi^0}^{(B)} = \frac{\langle n_{\pi^0}(\mathrm{hA})\rangle - \langle n^{\mathrm{f}}_{\pi^0}(\mathrm{hN})\rangle}{\langle n_{\pi^0}(\mathrm{hN})\rangle - \langle n^{\mathrm{f}}_{\pi^0}(\mathrm{hN})\rangle - \frac{1}{2}D} \cdot \frac{1}{\bar{\nu}_{\mathrm{hA}}}. \tag{21}$$

These ratios have been calculated by using the quark-model estimate $\langle n^{\mathrm{f}}_{\pi^0}(\pi^\pm\mathrm{N})\rangle = 0.7$ [17]. The $\langle n_{\pi^0}(\pi^\pm\mathrm{N})\rangle$ numbers have been taken from Refs. [43, 51]. In both cases, (A) and (B), ξ_{π^0} is of an intermediate value between ξ_s and ξ_-.

4 *Relativistic protons*

Thus we observe that $\xi_s^{(A)}$ is systematically higher than $\xi_-^{(A)}$ and, to a lesser extent, $\xi_s^{(B)}$ (π^-A) is higher than $\xi_-^{(B)}$(π^-A). As already mentioned, this can be an indication of an "additional" production of relativistic protons off nuclei. On the other hand we can estimate the number of relativistic protons in a direct way from the $\langle n_s(\mathrm{hA})\rangle$ and $\langle n_-(\mathrm{hA})\rangle$ values measured as

$$\langle n_{\mathrm{p}}(\mathrm{hA})\rangle = \langle n_s(\mathrm{hA})\rangle - 2\langle n_-(\mathrm{hA})\rangle \mp 1 + \eta(\mathrm{h} \to \mathrm{p}). \tag{22}$$

Here, the term ∓ 1 subtracts the charge of an incident hadron h so that the first three terms determine excess of the positive charge of the relativistic secondaries, and

$$\eta(\mathrm{h} \to \mathrm{p}) = \begin{cases} 0 & \mathrm{h} = \pi^\pm, \mathrm{K}^\pm \\ 2/3 & \mathrm{h} = \mathrm{p} \end{cases} \tag{23}$$

is an average number of protons due to fragmentation of hadron h. Thus computed magnitudes of $\langle n_{\mathrm{p}}(\mathrm{hA})\rangle$ are given in the third column of Table VI; in the fourth one the results of direct measurements are given whenever they are available. One can notice a discrepancy in the $\langle n_{\mathrm{p}}\rangle$ for the $\pi^\pm$Ne experiment at 10.5 GeV/c, where the "experimental" value is defined as

$$\langle n_{\mathrm{p}}(\pi^\pm\mathrm{A})\rangle = \langle n_+(\pi^\pm\mathrm{A})\rangle - \langle n_-(\pi^\mp\mathrm{A})\rangle \tag{24}$$

by making use of the isospin invariance. This discrepancy is probably related to a difference in systematic errors for the π^+Ne and π^-Ne measurements.

In Eq. (22), $\langle n_{\mathrm{p}}(\mathrm{hA})\rangle$ includes some "normal" number of fast protons which is obviously equal to that in the hN interaction multiplied by the number of such interactions,

TABLE VI

Estimation of the fast proton number

p[GeV/c]	hA	$\langle n_p(\text{hA})\rangle$ Eq. (22)	$\langle n_p(\text{hA})\rangle$ expt	$\langle n_p(\text{hA})\rangle_{\text{add}}$ Eq. (25) (A)	(B)	$\langle n_p(\text{hA})\rangle_{\text{add}}$ $\xi_s^{(A)} = 1$
21 [28]	pCNO	.89±0.26	1.03±0.06	−0.05±0.26	−0.12±0.26	0.3±0.2
[28]	pEm	1.19±0.26	1.26±0.06	0.18±0.26	0.02±0.26	1.0±0.3
[28]	pAgBr	1.30±0.26	1.34±0.09	0.27±0.26	0.06±0.26	1.2±0.3
28 [31]	pNe	1.35±0.20		0.18±0.20	0.04±0.20	0.7±0.3
300 [37]	pNe	1.47±0.18		0.30±0.18	0.14±0.18	0.6±0.6
10.5 [43]	π^+Ne	0.49±0.04	0.78±0.05	0.07±0.04	−0.05±0.04	0.2±0.2
10.5 [43]	π^-Ne	0.75±0.04	0.46±0.03	0.33±0.04	0.19±0.04	0.1±0.2
25 [45]	π^-Ne	0.85±0.14		0.43±0.14	0.29±0.14	0.4±0.3
40 [46]	π^-C	0.84±0.06		0.43±0.06	0.33±0.06	0.8±0.3
50 [28]	π^-CNO	0.55±0.26		0.32±0.26	0.25±0.26	0.4±0.3
[45]	π^-Ne	0.85±0.14		0.44±0.14	0.29±0.14	0.5±0.3
[28]	π^-Em	1.23±0.26		0.96±0.26	0.82±0.26	1.5±0.4
[28]	π^-AgBr	1.42±0.28		1.14±0.28	0.95±0.28	1.8±0.4

i.e. $\bar{\nu}_{hA}/\bar{\nu}_{qA}$ or $\bar{\nu}_{hA}$ in cases (A) and (B), respectively. As long as the number of slow protons due to an "average" nucleon fragmentation is ~0.3, the normal number of fast protons is $0.2\bar{\nu}_{hA}/\bar{\nu}_{qA}$ in case (A) and $0.2\bar{\nu}_{hA}$ in case (B), plus $\eta(h \to p)$ number due to fragmentation of the incident hadron. Hence, the number of additional fast protons is

$$\langle n_p(\text{hA})\rangle_{\text{add}} = \langle n_s(\text{hA})\rangle - 2\langle n_-(\text{hA})\rangle \mp 1 - 0.2 \times \begin{cases} \bar{\nu}_{hA}/\bar{\nu}_{qA} & \text{(A)} \\ \bar{\nu}_{hA} & \text{(B)} \end{cases} \tag{25}$$

(as before, for the bubble chamber data the 0.2 factor is to be replaced by 0.35). These $\langle n_p(\text{hA})\rangle_{\text{add}}$ values are also given in Table VI. Within errors they are practically the same for (A) and (B), and for comparatively small p_{lab} and A, where the data on $\langle n_-(\text{hA})\rangle$ are available, they are compatible even with $\langle n_p(\text{hA})\rangle_{\text{add}}$ derived from the condition $\xi_s^{(A)} = 1$ and given in the last column of Table VI. An exception is the pp data of Ref. [28], where there is a noticeable discrepancy. At the same time the $\langle n_p(\pi^-\text{A})\rangle_{\text{add}}$ values are systematically higher than $\langle n_p(\text{pA})\rangle_{\text{add}}$ that is obviously related to the $\xi_-(\pi^-\text{A}) < \xi_-$ (pA) inequality. We do not see any physical reason for this phenomenon.

5. *Energy dependence of the multiplicity of secondaries in hA collisions*

In the previous sections the multiplicity $\langle n_s(\text{hA})\rangle$ has been shown to be consistent with hypothesis (B) but essentially larger than the value expected in the framework of (A); in the latter case the excess of shower particles increased with energy. This result can be obtained also in a simpler way that does not require an estimate of the slow particle number in hadron-nucleus interactions.

When incident energy varies, the number of secondaries in the nucleus fragmentation region, where the intranuclear cascade is possible, is expected to remain the same. This is supported in particular by constancy of the number of heavy (grey and black) tracks. Therefore one expects the increase of multiplicity with energy to occur entirely due to the central region contribution that is $\bar{\nu}_{hA}/\bar{\nu}_{qA}$ (A) or $\bar{\nu}_{hA}$(B) times larger for collision with a nucleus than for an interaction with hydrogen. By using a phenomenological parametrization for charged multiplicity in a hadron–nucleon interaction,

$$\langle n_{ch}(hN)\rangle = a+b\ln E+c\ln^2 E, \tag{26}$$

one finds

$$\frac{\partial\langle n_s(hA)\rangle}{\partial \ln E} = \frac{\bar{\nu}_{hA}}{\bar{\nu}_{qA}}(b+2c\ln E) \quad \text{(A)}$$

or

$$\frac{\partial\langle n_s(hA)\rangle}{\partial \ln E} = \bar{\nu}_{hA}(b-D_1+2c\ln E)\ \text{(B)} \tag{27}$$

where the term $-D_1$ accounts for energy conservation.

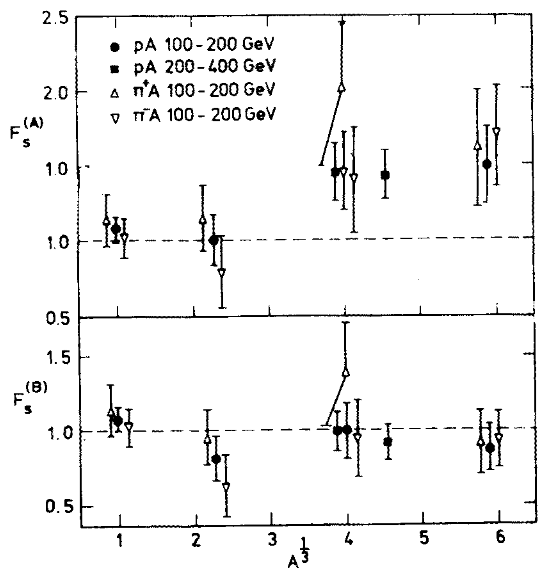

Fig. 5. Rate of the charged multiplicity increasing with energy as a function of A

In Fig. 5 the experimental data on the

$$F_s^{(A)} = \frac{\bar{\nu}_{qA}}{\bar{\nu}_{hA}}\frac{1}{b+2c\ln E}\frac{\partial\langle n_s(hA)\rangle}{\partial \ln E}$$

and

$$F_s^{(B)} = \frac{1}{\bar{\nu}_{hA}}\frac{1}{b-D_1+2c\ln E}\frac{\partial\langle n_s(hA)\rangle}{\partial \ln E} \tag{28}$$

are shown as a function of A for incident protons. π^{+} and π^{-} with $p_{lab} \gg 100$ GeV/c. The values $b = 0.58$ $c = 0.118$ [52] have been used. The hypothesis (A) is generally inconsistent with the data on heavy nuclei though the disagreement is not too strong. The hypothesis (B) is compatible practically with all available data.

6. *Discussion*

Strictly speaking, both hypotheses (A) and (B) have been shown to disagree with experiment.

In case (A) some additional mechanism in necessary, giving rise to a significant and increasing with energy number of "additional" fast protons but not of π^{-} (and therefore π^{+}) mesons. At a first sight, an intranuclear cascade might be just a mechanism of the kind. So let us consider this possibility in more detail.

An interaction of a nuclear nucleon leads to disintegration of only one of its three constituent quarks. The other two quarks remain spectators and constitute a part of one or two hadrons in the fragmentation region of the nucleus, carrying $x \simeq 2/3$ or $x \simeq 1/3$ in the rest frame of projectile. In the former case the hadron is a baryon with laboratory momentum $p_{lab} \simeq 0.5$ GeV/c, which obviously cannot produce an additional fast proton. If there are two hadrons with $x \simeq 1/3$ in the rest frame of the projectile, they can be either mesons with $p_{lab} \simeq 0.5$ GeV/c or baryons with $p_{lab} \simeq 1.5$–2 GeV/c. Only in the latter case production of a single fast proton is possible, however, its probability is too small ($\lesssim 1/6$ per one interaction [53, 54]) to give a required number of additional protons.

As to the interactions of comparatively slow (in the lab. system) secondaries they should result in an increase of both positive and negative particles, due say, to a process $\pi N \to \pi\pi N$. Besides, in all cases there seems to be no possibility to obtain $\langle n_p(hA)\rangle_{add}$ rising with the incident energy as required by Eq. (20).

Another possibility could be the emission of fast proton spectators from the nucleus due to their Fermi motion. The number of such protons can be estimated similarly as in Ref. [55] in terms of the known multiplicity of so-called cumulative protons. It turns out to be not larger than a few tenths even for the heaviest nuclei. In this case, the energy dependence observed would be absent as well.

Furthermore, there is a direct experimental evidence against additional protons whose number increases with incident energy. It appears plausible that a growing number of fast protons must be accompanied also by a growing number of slower protons usually referred to as grey particles. (In inelastic collisions, the number of "shower" and "grey" protons are approximately equal.) On the other hand, as seen in Fig. 6a taken from Ref. [56], the multiplicity of grey particles depends quite weakly on energy and is rather a decreasing function of p_{lab}. (More recent data of Ref. [41] are also shown in Fig. 6a.)

Measurements of the energy spectra of fast protons in collisions of incident protons with nucleons and photoemulsion nuclei were made in Ref. [57]. The results found are shown in Fig. 6b as a function of the secondary momentum provided it is larger than 1 GeV/c. The ratio of the spectra is seen to be independent of momentum and within the errors consistent with $\bar{\nu}_{pA} \simeq 2.5$.

Thus we see no way to reconcile hypothesis (A) with experiment and it appears to be unlikely.

The alternative hypothesis (B) agrees with the data in all cases except that of the π^-A interaction where the number of negative secondaries is less than required and they are somehow replaced by the positive hadrons. A question arises on a possible pion absorption

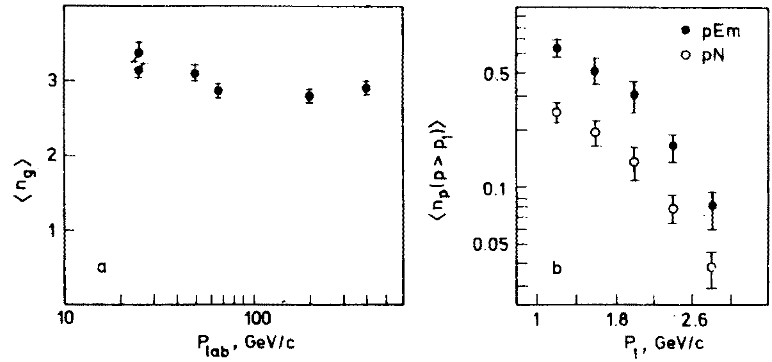

Fig. 6. (a) Multiplicity of the "grey" particles, mainly protons with momenta less than 1 GeV/c, in pEm interactions as a function of the incident energy [41, 56]. (b) Distribution in the fast proton momenta in the pN → pX and pEm → pX processes at 21 GeV/c [58]

mechanism in nuclei. However we do not see any reasonable mechanism of the kind. The charge-exchange processes like $\pi^-p \leftrightarrow \pi^0 n$ and $\pi^+ n \leftrightarrow \pi^0 p$ hardly represent the absorption mechanism in question. First, they can go in both directions. Second, if they worked some "additional" π^0 mesons would have appeared instead of π^- that is not supported by the data (Table V).

In principle, absorption of π^- by many-nucleon clusters is also possible, say $\pi^- + (pp)_{\text{nucleus}} \to p + n$, however both this and a similar $\pi^+ + (pn)_{\text{nucleus}} \to p + p$ processes would have lead to an increase of the number of fast protons in contradiction with the data in Fig. 6.

Of course, an intermediate case between the (A) and (B) is also possible where the number of secondaries in a quark-nucleus interaction increases with A but slower than in Eq. (7) due to some essential interaction of partons in the final state. In order to describe the data on $\langle n_s(\text{hA})\rangle$ in this case it is also necessary to introduce a number of "additional" fast protons, which however can be significantly smaller than in Eq. (20). To distinguish between all the possibilities direct measurements of the positive and negative hadron numbers separately, as well as of the fast proton multiplicity are highly desirable at high enough energies and on targets with different and not small A.

Let us now discuss briefly the theoretical accuracy of the above analysis. To compute N_q we have made use of the formulae of Appendix A which include the quark-nucleon inelastic cross section assumed to be $\sigma_{qN}^{\text{inel}} \simeq \frac{1}{3}\sigma_{NN}^{\text{inel}} \simeq \frac{1}{2}\,\sigma_{\pi N}^{\text{inel}} \simeq 10$ mb. In fact, a part of $\sigma_{NN}^{\text{inel}}$ and $\sigma_{\pi N}^{\text{inel}}$ is contributed by the diffraction processes that proceed owing to elastic scattering of constituent quarks [58]. Therefore the $\sigma_{qN}^{\text{inel}}$ value should be reduced by 1–1.5 mb. On the other hand we did not take into account the fact of $\sigma_{NN}^{\text{inel}}$ and $\sigma_{\pi N}^{\text{inel}}$ increase with energy.

Altogether, these corrections can result in a decrease of $\xi_s^{(A)}$ and $\xi_-^{(A)}$ by 5–7% for $A \simeq 60$, which is too small to make hypothesis (A) consistent with the data in Fig. 2. For hypothesis (B), the analogous correction is not larger than 2–3%.

Errors in our estimates of the secondary numbers in the projectile fragmentation region are also possible but they influence the results rather weakly. For instance variation of the $\langle n_{ch}^{f}(pN)\rangle$ by 0.1 even at as small energy as 21 GeV/c leads to the variation of $\xi_s^{(A)}$ at $A = 60$ by 3–4% only.

The $D(\bar{v}_{qA}, E)$ correction for the energy losses in multiple interactions is the most important at low energy. Say for pEm collisions replacing $x = 1/2$ in Eqs. (B.6) and (B.7) by $x = 1/3$ increases $\xi_s^{(B)}$ by 11% at 21 GeV/c and only by 5% at 400 GeV/c. Besides one cannot exclude some energy dependence of the $\langle n_{ch}^{f}(hN)\rangle$ and $\langle n_-^{f}(hN)\rangle$ as well as the x parameter of Eqs. (B.6), (B.7) when going from low (10–20 GeV) to high ($\geqslant 100$ GeV) energies. In this respect the high energy data are more reliable.

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APPENDIX A

Average number of quarks interacting in a nucleus

Consider a proton interacting with a nucleus. The probabilities to interact with the nuclear matter for one, two or all three constituent quarks are as follows [1]

$$V_1^p = \frac{3}{\sigma_{pA}^{prod}} \int d^2b e^{-2\sigma_q T(b)}[1-e^{-\sigma_q \cdot T(b)}],$$

$$V_2^p = \frac{3}{\sigma_{pA}^{prod}} \int d^2b e^{-\sigma_q T(b)}[1-e^{-\sigma_q \cdot T(b)}]^2,$$

$$V_3^p = \frac{1}{\sigma_{pA}^{prod}} \int d^2b [1-e^{-\sigma_q \cdot T(b)}]^3, \tag{A1}$$

where the profile function $T(b)$ is determined by the nuclear density of nucleons, $\varrho(r)$,

$$T(b) = \int_{-\infty}^{\infty} \varrho(\sqrt{b^2+z^2})dz, \tag{A2}$$

$\sigma_q = \sigma_{qN}^{inel} \simeq 10$ mb and the cross section

$$\sigma_{pA}^{prod} = \int d^2b[1-e^{-3\sigma_q T(b)}] = \int d^2b[1-e^{-\sigma_{pN}^{inel} \cdot T(b)}] \tag{A3}$$

results from the normalization condition $V_1^p + V_2^p + V_3^p = 1$. The meaning of Eqs. (A1) is obvious if one notes that the $e^{-\sigma_q \cdot T(b)}$ factor stands for the probability for a quark with the impact parameter b to pass through entire nucleus without interacting.

Now, the average number of quarks interacting inelastically in the nucleus is obviously

$$N_q^{pA} = V_1^p + 2V_2^p + 3V_3^p = \frac{3}{\sigma_{pA}^{prod}} \int d^2b[1 - e^{-\sigma_q \cdot T(b)}] = 3\,\frac{\sigma_{qA}^{prod}}{\sigma_{pA}^{prod}}. \tag{A4}$$

After introducing $\bar{\nu}_{pA}$ and $\bar{\nu}_{qA}$ according to Eqs. (3) and (4) and utilizing the $3\sigma_q = \sigma_{pN}^{inel}$ equality one finds

$$N_q^{pA} = \bar{\nu}_{pA}/\bar{\nu}_{qA}. \tag{A5}$$

A similar equation can be also obtained for incident pions instead of protons. In the incident K meson case the strange (s) and nonstrange (q) quarks have generally different cross section so that

$$N_q^{KA} = \frac{\sigma_{qA}^{prod} + \sigma_{sA}^{prod}}{\sigma_{KA}^{prod}}. \tag{A6}$$

APPENDIX B

Correction for energy losses in multiple interactions

When a quark interacts with several nucleons one has to account for the energy losses [59, 10]. There is a problem here how the incident energy is shared between a number of interactions. For instance one may assume the energy loss is partitioned among them equally [10] or in a geometrical progression. If the incident energy is not too small both

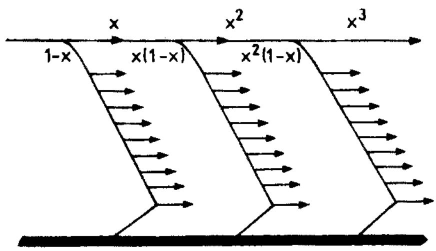

Fig. 7. A constituent quark interacting with several nucleons in a nucleus

distributions lead to approximately equal corrections. In what follows we adopt the geometrical progression distribution and suppose a scattered quark to carry out an x share of energy while the $1\text{-}x$ share is spent for the secondary production, say in a multiperipheral ladder shown in Fig. 7.

Experimental data on charged multiplicity in pp collisions are described well by a phenomenological polynomial $a + b\ln E + \ln^2 E$ of Eq. (26) with the parameters [52]

$$a = 1.25 \pm 0.12, \quad b = 0.58 \pm 0.06, \quad c = 0.118 \pm 0.006. \tag{B1}$$

The secondary number in the central region that is present in the denominator of Eq. (9) is nearly the same for different incident hadrons and differs from the above polynomial only by a redefinition of the constant a,

$$\langle n_{\rm ch}^{\rm c.r.}({\rm hN})\rangle = \langle n_{\rm s}({\rm hN_A})\rangle - \langle n_{\rm ch}^{\rm f}({\rm hN})\rangle \approx \hat{a} + b\ln E + c\ln^2 E. \tag{B2}$$

If a quark interacts with ν nucleons in a nucleus the secondary multiplicity is seen from Fig. 7 to be

$$\sum_{k=0}^{\nu-1}\left[\hat{a} + b\ln(E\cdot x^k) + c\ln^2(E\cdot x^k)\right]$$

$$= \nu\langle n_{\rm ch}^{\rm c.r.}({\rm hN})\rangle + \frac{\nu(\nu-1)}{2}(b+2c\ln E)\cdot\ln x + c\,\frac{\nu(\nu-1)(2\nu-1)}{6}\ln^2 x. \tag{B3}$$

By assuming a simplest Poisson distribution in $\nu-1$ with an average $\bar{\nu}_{\rm qA}-1$, i.e.

$$P_{\bar{\nu}_{\rm qA}-1}(\nu-1) = e^{-\bar{\nu}({\rm qA}-1)}\frac{(\bar{\nu}_{\rm qA}-1)^{\nu-1}}{(\nu-1)!} \tag{B4}$$

and averaging Eq. (B3) with it we find[2]

$$\bar{\nu}_{\rm qA}[\langle n_{\rm ch}^{\rm c.r.}({\rm hN})\rangle - D(\bar{\nu}_{\rm qA}, E)], \tag{B5}$$

where

$$D(\bar{\nu}_{\rm qA}, E) = D_1(\bar{\nu}_{\rm qA})\cdot\ln E + D_2(\bar{\nu}_{\rm qA}),$$

$$D_1(\bar{\nu}_{\rm qA}) = c\,\frac{\bar{\nu}_{\rm qA}^2-1}{\bar{\nu}_{\rm qA}}\ln\frac{1}{x},$$

$$D_2(\bar{\nu}_{\rm qA}) = \frac{\bar{\nu}_{\rm qA}-1}{2\bar{\nu}_{\rm qA}}\left[b(\bar{\nu}_{\rm qA}+1) - \frac{c}{3}(2\bar{\nu}_{\rm qA}^2+5\bar{\nu}_{\rm qA}-1)\ln\frac{1}{x}\right]\ln\frac{1}{x}. \tag{B6}$$

The D_1 and D_2 parameters are given in Table II under the simplest assumption of

$$x = 1/2. \tag{B7}$$

The expression in square brackets in Eq. (B5) is an average multiplicity in a quark--nucleon interaction, corrected for energy conservation. By introducing thus estimated correction in Eqs. (9) or (14) one obtains Eqs. (16) and (17).

Note added in proofs:

We believe it is worth making a remark on the identification of the entire excess of positives among relativistic secondaries with fast protons, implied in Sections 3 and 4. There is no mechanism seen for transmission, after averaging over all channels possible, of the positive charge from nuclear protons to particles of other kind, with the only exception of some relatively rare transitions into strange hadrons like $p \to \Lambda K^+$. Say, for not very heavy nuclei rates of the $p \to n\pi^+$ and $n \to p\pi^-$ fragmentation processes

[2] A calculation of the probability distribution in ν corresponding to the exp $(\sigma_{\rm q}\,T(b))-1$ expansion with the Saxon-Woods density of nuclear matter leads to a distribution formally different from (B4) but the numerical value of the correction $D = D_1\cdot\ln E + D_2$ varies quite insignificantly.

are practically the same. Therefore they give equal contributions into π^+ and π^- multiplicities and on the average do not alternate the number of protons. On the other hand, such fragmentation enhances the momentum transfer to a secondary proton making it faster in the lab. system. For heaviest nuclei, the $n \rightarrow p\pi^-$ transition is more frequent than the $p \rightarrow n\pi^+$ one, giving rise to some additional increase of $\langle n_p(hA)\rangle$ in comparison with Eq. (22). A direct measurement of the fast proton number would be of great interest.

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