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D_5 GROUP IN WEINBERG-SALAM MODEL

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A simple model with four quark flavours is presented. This model is based on the gauge group $SU(2)_L \times U(1)$ supplemented by a discrete group D_5 permitting two values for the Cabibbo angle, both approximately equal to $\vartheta_C \approx \sqrt{\frac{m_d}{m_s}}$.

The question of elementary particle mass spectrum structure seems to be one of the most important and still unsolved problems of contemporary physics. Unified models of particle interactions based on the gauge field theory, which have been developed during the last twelve years did not give an answer. One can look for the reason for this in the equivalence of different quark and lepton generations which couple to intermediate bosons completely in the same way. Due to this symmetry, the mixing angles and masses of fundamental particles remain arbitrary in gauge theories (this symmetry can not be spontaneously broken as it would lead to the appearance of unphysical Goldstone bosons [1], it also can not be gauged as we would get the unobserved interaction [2]). Free parameters should be adjusted to experimental data by the choice of proper values of the renormalized coupling constants in Higgs sector. Thus relations between masses, mixing angles and coupling constants, obtained in the zero order of perturbation calculus, can not be true (they are, in general, modified when the higher orders are taken into account). However, if coupling constants satisfy some symmetry conditions (for instance, when the gauge group is supplemented by a finite group), relations between them will be stable under renormalization.

This approach allows one to express the Cabibbo angle (and/or further mixing angles) in terms of quark mass ratios [3–6] and obtain relations between masses [7, 8]. In most of the papers mentioned above (except [4–7]) the models within Left–Right symmetry (namely those based on the $SU(2)_L \times SU(2)_R \times U(1)$ group) were used. On can summarize the results obtained in them by the equation

$$\vartheta_C \approx \tan^{-1} \sqrt{\frac{m_d}{m_s}}. \tag{1}$$

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Application of discrete groups to minimal $SU(2)_L \times U(1)$ model [9] met some difficulties. It has even been argued that it is impossible [10]. However, Wyler showed [5] that this conclusion is true only under certain unnecessary conditions, namely — all Higgs fields have to form one-dimensional representations of a finite group. If this condition is relaxed the useful relations in the minimal model can be obtained. Wyler proposed a model very similar to one proposed earlier by Pakvasa and Suguwara [4] in which[1]

$$\vartheta_C \approx \tan^{-1} \frac{m_d}{m_s}. \tag{2}$$

In this paper we give a simple four quark model based on the $SU(2)_L \times U(1)$ group. It leads to a relation of the type of Eq. (1) which seems to be in better agreement with experiment than equation (2). According to Ref. [11], the application of a discrete group to $SU(2)_L \times U(1)$ theory involves flavour nonconservation in processes with neutral Higgs particle exchange. In our case it is charm nonconservation. Possible consequences of this fact will be mentioned further.

Let us consider an $SU(2)_L \times U(1)$ model with two left-handed quark doublets $L_1 = \binom{u'}{d'}_L$, $L_2 = \binom{c'}{s'}_L$, four right-handed singlets u'_R, d'_R, c'_R, s'_R (prime indicates that we work with "bare" particles) and three Higgs field doublets ϕ_0, ϕ_1, ϕ_2. Let us assume further, that our model is invariant under a finite group D_5 — dihedral group of order five (for definitions of point groups see Ref. [12]). Quark and Higgs fields transform under D_5 as follows[2]:

$$\begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \sim E_2, \quad \begin{pmatrix} d'_R \\ s'_R \end{pmatrix} \sim E_1, \quad \begin{pmatrix} u'_R \\ c'_R \end{pmatrix} \sim E_2, \quad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \sim E_1, \quad \phi_0 \sim A_1.$$

The Yukawa coupling term is then

$$L_Y = a(\bar{L}_1 c'_R \tilde{\phi}_1 + \bar{L}_2 u'_R \tilde{\phi}_2) + b(\bar{L}_1 u'_R + \bar{L}_2 c'_R)\tilde{\phi}_0 + c(\bar{L}_1 d'_R \phi_1 + \bar{L}_2 s'_R \phi_2) + \text{h.c.} \tag{3}$$

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[1] Pakvasa and Suguwara assumed $m_u = 0$ for the zero order, independent of the symmetry conditions. This seems justifiable since we know that m_u is small, however it would lead to the unstability of the equation for the Cabibbo angle at higher orders. In the model proposed by Wyler there are no such problems and m_u is free. In the limit $m_u = 0$ Wyler's solution approaches the one found in Ref. [4].

[2] D_5 group has two two-dimensional representations E_1 and E_2 which are generated by the matrices

$$E_1: \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^* \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$E_2: \begin{pmatrix} \varepsilon^2 & 0 \\ 0 & \varepsilon^{2*} \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \left(\varepsilon = \exp \frac{2\pi i}{5}\right)$$

and two one-dimensional representations A_1, A_2 generated by the numbers

$$1, \ 1 \text{ and } 1, \ -1$$

appropriately.

Denoting vacuum expectation values (v.e.v.) of ϕ_i by v_i one can easily find the mass matrices

$$M_{ds} = c\begin{pmatrix} v_1 & 0 \\ 0 & v_2 \end{pmatrix}, \tag{4}$$

$$M_{uc} = \begin{pmatrix} b\bar{v}_0 & a\bar{v}_1 \\ a\bar{v}_2 & b\bar{v}_0 \end{pmatrix} \tag{5}$$

and relations between masses

$$\left(\frac{m_d}{m_s}\right)^2 = \left|\frac{v_1}{v_2}\right|^2, \tag{6}$$

$$m_u^2 + m_c^2 = 2|bv_0|^2 + |av_1|^2 + |av_2|^2, \tag{7}$$

$$m_u^2 \cdot m_c^2 = (|bv_0|^2 + |av_1|^2) \cdot (|bv_0|^2 + |av_2|^2) - |ab(v_0\bar{v}_1 + \bar{v}_0 v_2)|^2. \tag{8}$$

Physical quark states diagonalize our matrices and are related to the bare states by the following unitary transformations

$$\begin{pmatrix} d'_{L,R} \\ s'_{L,R} \end{pmatrix} = U_{L,R}\begin{pmatrix} d_{L,R} \\ s_{L,R} \end{pmatrix}, \quad \begin{pmatrix} u'_{L,R} \\ c'_{L,R} \end{pmatrix} = V_{L,R}\begin{pmatrix} u_{L,R} \\ c_{L,R} \end{pmatrix},$$

where U, V are of the form

$$U_L = \begin{pmatrix} e^{i\gamma_L} & 0 \\ 0 & e^{i\delta_L} \end{pmatrix}, \quad U_R = \begin{pmatrix} e^{i\gamma_R} & 0 \\ 0 & e^{i\delta_R} \end{pmatrix}, \tag{9}$$

$$V_L = \begin{pmatrix} \cos\vartheta e^{i\alpha_L} & \sin\vartheta e^{i(\beta_L - \varphi)} \\ -\sin\vartheta e^{i(\alpha_L + \varphi)} & \cos\vartheta e^{i\beta_L} \end{pmatrix}, \quad V_R = \begin{pmatrix} -\sin\vartheta e^{i(\alpha_R - \varphi)} & \cos\vartheta e^{i\beta_R} \\ \cos\vartheta e^{i\alpha_R} & \sin\vartheta e^{i(\beta_R + \varphi)} \end{pmatrix}, \tag{10}$$

with

$$\varphi = \arg(v_0\bar{v}_1 + \bar{v}_0 v_2),$$

$$\tan\vartheta = \frac{2ab|v_0\bar{v}_1 + \bar{v}_0 v_2|}{|av_2|^2 - |av_1|^2 + \sqrt{4a^2b^2|v_0\bar{v}_1 + \bar{v}_0 v_2|^2 + (|av_2|^2 - |av_1|^2)^2}}, \tag{11}$$

$\alpha,\beta,\gamma,\delta$ — free real parameters. One can express the Cabibbo rotation matrix as a function of U and V

$$C = V_L^{-1}U_L = \begin{pmatrix} \cos\vartheta e^{i(\gamma_L - \alpha_L)} & \sin\vartheta e^{i(\delta_L - \alpha_L - \varphi - \pi)} \\ -\sin\vartheta e^{i(\gamma_L - \beta_L + \varphi + \pi)} & \cos\vartheta e^{i(\delta_L - \beta_L)} \end{pmatrix}. \tag{12}$$

Since phases in the above formula are unmeasurable we can set them equal to zero [13] and choose for further simplicity

$$\alpha_L = \gamma_L = 0, \quad \delta_L = \beta_L = \pi + \varphi. \tag{13}$$

Then

$$\vartheta = \vartheta_C. \tag{14}$$

It can be easily checked that the relations (6), (7), (8) are not sufficient for expressing the Cabibbo angle through the quark masses. The formula for the Cabibbo angle obtained from these equations depends additionally on the Higgs field v.e.v. phases (by the factor $e^{i(2\chi_0-\chi_1-\chi_2)}$). However, this dependence may be eliminated due to the scalar fields potential properties. The most general invariant form of it is:

$$\begin{aligned} V = {} & \mu_0^2\bar{\Phi}_0\phi_0+\mu_1^2(\bar{\Phi}_1\phi_1+\bar{\Phi}_2\phi_2)+A(\bar{\Phi}_0\phi_0)^2+B(\bar{\Phi}_0\phi_0)(\bar{\Phi}_1\phi_1+\bar{\Phi}_2\phi_2) \\ & +C(\bar{\Phi}_1\phi_1+\bar{\Phi}_2\phi_2)^2+D(\bar{\Phi}_1\phi_1-\bar{\Phi}_2\phi_2)^2+G(\bar{\Phi}_2\phi_1)(\bar{\Phi}_1\phi_2) \\ & +F[(\bar{\Phi}_1\phi_0)(\bar{\Phi}_0\phi_1)+(\bar{\Phi}_2\phi_0)(\bar{\Phi}_0\phi_2)]+F'[(\bar{\Phi}_0\phi_1)(\bar{\Phi}_0\phi_2)+(\bar{\Phi}_2\phi_0)(\bar{\Phi}_1\phi_0)]. \end{aligned} \tag{15}$$

This is the same potential as in Ref. [5]. Parametrizing v.e.v. as follows

$$v_0 = r_0e^{i\chi_0}, \quad v_1 = r_1\cos\alpha e^{i\chi_1}, \quad v_2 = r_1\sin\alpha e^{i\chi_2}, \tag{16}$$

we find one of the minimalization conditions

$$\sin(2\chi_0-\chi_1-\chi_2) = 0, \tag{17}$$

i.e.

$$e^{i(2\chi_0-\chi_1-\chi_2)} = 1 \quad \text{or} \quad e^{i(2\chi_0-\chi_1-\chi_2)} = -1.$$

Only the first relation allows one to express the Cabibbo angle as a function of masses. Thus, we can choose without the loss of generality

$$\chi_0 = \chi_1 = \chi_2 = 0. \tag{18}$$

According to Ref. [5] the potential V has a stable minimum for real v.e.v. if

$$\sin 2\alpha = \frac{2F'}{4D-G}\left(\frac{r_0}{r_1}\right)^2 \tag{19}$$

(some relations between constants $A, \ldots, F'$ should be satisfied). On the other hand

$$|\tan\alpha| = \frac{m_s}{m_d}.$$

In order to make this true we ought to choose properly only one constant — F'. Anyhow, it can have repercussions on r_0 and r_1 values. But by inspecting the formulas (7) and (8) one establishes that v.e.v. are multiplied in them by the free parameters a, b. Thus, the quark masses in our model also remain free.

Expressing the Cabibbo angle through the quark mass ratios we obtain two solutions

$$\tan 2\vartheta_C = \frac{2\sqrt{\dfrac{m_d}{m_s}\pm\left(1+\dfrac{m_d}{m_s}\right)^2\dfrac{\dfrac{m_u}{m_c}}{\left(1\mp\dfrac{m_u}{m_c}\right)^2}}}{1-\dfrac{m_d}{m_s}}. \tag{20}$$

For the typical values of $\left(\frac{m_d}{m_s}\right)^2$ and $\frac{m_u}{m_c}$ satisfying

$$\left(\frac{m_d}{m_s}\right)^2 \sim \frac{m_u}{m_c} \ll 1$$

both solutions of (20) are approximately equal

$$\tan 2\vartheta_C \approx 2\sqrt{\frac{m_d}{m_s}}\,,$$

or with the same accuracy

$$\vartheta_C \approx \sqrt{\frac{m_d}{m_s}}\,. \tag{21}$$

Assuming that constants a, b, c are positive one gets relations between bare and physical quark states

$$\begin{pmatrix} d'_L \\ s'_L \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} d_L \\ s_L \end{pmatrix}, \quad \begin{pmatrix} d'_R \\ s'_R \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}, \tag{22}$$

$$\begin{pmatrix} u'_L \\ c'_L \end{pmatrix} = \begin{pmatrix} \cos\vartheta_C & -\sin\vartheta_C \\ -\sin\vartheta_C & -\cos\vartheta_C \end{pmatrix} \begin{pmatrix} u_L \\ c_L \end{pmatrix},$$

$$\begin{pmatrix} u'_R \\ c'_R \end{pmatrix} = \begin{pmatrix} \pm\sin\vartheta_C & -\cos\vartheta_C \\ \mp\cos\vartheta_C & -\sin\vartheta_C \end{pmatrix} \begin{pmatrix} u_R \\ c_R \end{pmatrix}. \tag{23}$$

Formulas (22), (23) permit us to rewrite the Yukawa coupling in terms of physical states

$$\begin{aligned} L_Y = c\Bigg[& \bar{d}_L d_R \Phi_1^0 + \bar{s}_L s_R \phi_2^0 + (\bar{u}, \bar{c})_L \begin{pmatrix} \cos\vartheta_C & 0 \\ -\sin\vartheta_C & 0 \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_R \phi_1^+ \\ & + (\bar{u}, \bar{c})_L \begin{pmatrix} 0 & \sin\vartheta_C \\ 0 & \cos\vartheta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}_R \phi_2^+ + \text{h.c.} \Bigg] \\ + b\Bigg[& (\bar{u}, \bar{c})_L \begin{pmatrix} \pm\sin 2\vartheta_C & -\cos 2\vartheta_C \\ \pm\cos 2\vartheta_C & \sin 2\vartheta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \bar{\Phi}_0^0 \\ & + (\bar{d}, \bar{s})_L \begin{pmatrix} \mp\sin\vartheta_C & \cos\vartheta_C \\ \mp\cos\vartheta_C & -\sin\vartheta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \phi_0^- + \text{h.c.} \Bigg] \\ + a\Bigg[& (\bar{u}, \bar{c})_L \begin{pmatrix} \mp\cos^2\vartheta_C & -\frac{1}{2}\sin 2\vartheta_C \\ \pm\frac{1}{2}\sin 2\vartheta_C & \sin^2\vartheta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \bar{\Phi}_1^0 + (\bar{d}, \bar{s})_L \begin{pmatrix} \pm\cos\vartheta_C & \sin\vartheta_C \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \phi_1^- \\ & + (\bar{u}, \bar{c})_L \begin{pmatrix} \mp\sin^2\vartheta_C & \frac{1}{2}\sin 2\vartheta_C \\ \mp\frac{1}{2}\sin 2\vartheta_C & \cos^2\vartheta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \bar{\Phi}_2^0 \\ & + (\bar{d}, \bar{s})_L \begin{pmatrix} 0 & 0 \\ \pm\sin\vartheta_C & -\cos\vartheta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}_R \phi_2^- + \text{h.c.} \Bigg]. \end{aligned} \tag{24}$$

We see that in our model there are no strangeness changing interactions with neutral Higgs particle exchange, but there are similar processes with $\Delta c = 1$ or $\Delta c = 2$. It would lead, for example, to the abnormal $D^0 - \overline{D^0}$ mixing which in principle is measurable. However, such mixing can be suppressed by choosing Higgs field masses large enough. Anyhow, the presence of such interactions is not in conflict with the known properties of charmed particles.

Note

One can suppose that the choice of a discrete group is unrestricted. This, however, is not true. Wyler showed that such a group should have at least one two-dimensional representation under which two of the three Higgs fields have to transform. If this condition is satisfied there are two further classes of possible cases:

— two right-handed quark fields built a doublet and two other — different singlets of a finite group;
— right-handed quark fields are assembled into two different doublets satisfying some other conditions.

In both cases left-handed fields transform as a doublet.

The models presented by Pakvasa and Suguwara [4] and Wyler [5] belong to the first class. The simplest model of the second kind is proposed in this paper.

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Note added in proof. Recently several six-quark models based on the $SU(2)_L \times U(1)$ gauge group have been proposed (see Ref. [6, 7] and D. Wyler, *Phys. Rev.* **D19**, 3369 (1979); E. Derman, H. S. Tsao, *Phys. Rev.* **D20**, 1207 (1979). However, the model presented above remains the only one with four quarks leading to the proper relation for the Cabibbo angle.

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