LETTERS TO THE EDITOR

THE HAGEDORN TEMPERATURE AND DIRAC'S LARGE NUMBERS HYPOTHESIS

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New relations $N_5 \approx 10^{40}$ and $N_6 \approx (10^{40})^2$ are in excellent agreement with Dirac's theory predictions $G \sim t^{-1}$ and $N_3 \sim t^2$.

The fine structure constant $\hbar c/e^2 \approx 137$ and the ratio between the masses of the proton and the electron $m_p/m_e \approx 1840$ (where h is the Planck constant, m_p is the proton mass, m_e is the electron mass, c is the velocity of light, e is the elementary charge) are not really explained, but some physicists argue that with the progress of physics an explanation will be found.

The ratio of the electric to the gravitational forces between protons and electrons is

$$N_1 = e^2/Gm_em_p \approx 10^{40}, \tag{1}$$

where G is the gravitational constant. The order of the age of the Universe H^{-1} (H—Hubble's constant) expressed in terms of atomic units of time is also approximately equal to 10^{40}. It is denoted usually [1] by N_4. The radius of the Universe R expressed in atomic units is approximately

$$N_2 = H^{-1}e^{-2}m_ec^3 \approx 10^{40}. \tag{2}$$

In 1937 Dirac pointed out that the ratios $N_1:N_2:N_3^{\frac{1}{2}}:N_4$ are of the order of unity, where

$$N_3 \approx M_u/m_p \approx 10^{80}, \tag{3}$$

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and M_u is the mass of the universe. The Dirac large numbers hypothesis (LNH) (a new principle) states that the values of these large numbers vary with atomic time but the relation between them does not change. One of the most important consequences of the LNH is the following relation

$$G \sim R^{-1} \sim t^{-1}, \tag{4}$$

which means that the gravitational constant G changes with the cosmic time t at the rate

$$\frac{\dot{G}}{G} \approx -8.33 \times 10^{-11} \mathrm{yr}^{-1}. \tag{5}$$

Van Flandern [2] argued that observations are consistent with G decreasing at the rate given by (5).

LNH says that we do not need a new theory to determine numbers such as 10^{40}; all large numbers of the order 10^{40} are just equal to t. In the Dirac theory the fine structure constant and the ratio m_p/m_e are equal to t^0.

Dirac suggested [3–11] that the proper way to unterstand the LNH is to consider of two metrics: the atomic ds_A and Einstein ds_E metrics. The element of distance defined by

$$ds_E^2 = g_{ik} dx^i dx^k$$

is not the same as the element of distance in terms of atomic units, but differs from it by a certain factor; this factor must be a scalar function of position. According to Canuto, Adams, Hsieh, Tsiang, Owen and Lodenquai [1] and [12–21] the equations describing physical phenomena should be independent not only of coordinates but also of the system of units. Within the gauge covariant theory of gravitation [1, 13–16, 21] the gravitational equations given by General Relativity are valid in gravitational (= Einstein), and not electromagnetic (= atomic) units. The relation between the atomic and Einstein metrics [1, 3–21] is the following:

$$ds_A/ds_E = t^{-1}.$$

The G-varying gravitation was considered e.g. in [22–28].

We present new dimensional coincidences and also demonstrate that they are consistent with the LNH. These relations are obtained by adding to the set of universal constants the Boltzmann constant k and the Hagedorn universal temperature T_H, which were not included in Dirac's relations built from gravitational, electromagnetic and cosmological constants. We have the first number of the order 10^{40}

$$N_5 = h m_p^{-5/2} k^{\frac{1}{2}} T_H^{\frac{1}{2}} G^{-1} \approx 10^{40}. \tag{6}$$

The formula (6) and Dirac's fundamental dimensionless number N_2 leads to the gravitational constant G as the familiar function of the cosmic time

$$G \sim t^{-1}.$$

From relations (3), (4) and (6) it follows that $N_3 \sim G^{-2} \sim t^2$.

The second new coincidence relates the constants c, G, k, T_H and H:

$$N_6 = c^5 H^{-1} G^{-1} k T_H \approx (10^{40})^2. \tag{7}$$

From relations (7) and (1) and from relations (7) and (2) we obtain again $G \sim t^{-1}$. From equations (7), (3) and (4) we obtain Dirac's relation

$$N_3 \sim t^2.$$

The relation (4) is fundamental for the scale-covariant theory of gravitation [1, 3–21] and can be taken to support the idea that any a priori assumption about the strength of different forces is unjustifiable. This idea is connected with the recent success of gauge fields with broken symmetries.

We conclude that relations (6) and (7) are in agreement with Dirac's theory predictions $G \sim t^{-1}$ and $N_3 \sim t^2$; a priori one would expect that the new coincidences with the new constants (fundamental for strong interactions) might change Dirac's hypothesis based on N_1, N_2, N_3, and N_4 large numbers. This indicates that there is some relation between gravitational and strong interactions (not only between gravitational and electromagnetic interactions as it was suggested by Dirac, Eddington, Weyl, Canuto et al.).

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