QCD JET FRAGMENTATION FUNCTIONS

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Analytic solutions for the jet fragmentation functions in the leading log approximation for x close to 1 are found and compared with exact numerical results.

In this paper we find approximate solutions of the Q^2 evolution equations for quark and gluon jets in the region of x close to 1. As seen from the figures our analytic expressions give a good approximation to the exact numerical solutions of the evolution equations over a wide range of x and Q^2.

We use master equations proposed by Wosiek and Zalewski [1]. After integration over x the equations read:

$$\frac{d}{dt}\int\limits_{1-2\varepsilon}^{1} q_i(x,t)dx = \int\limits_{1-2\varepsilon}^{1} dx\left\{g(x,t)\int\limits_{\frac{1-2\varepsilon}{x}}^{1} K_{qg}(z)dz - q_i(x,t)\int\limits_{0}^{\frac{1-2\varepsilon}{x}} K_{qq}(z)dz\right\}, \tag{1}$$

$$\frac{d}{dt}\int\limits_{1-2\varepsilon}^{1} g(x,t)dx = \int\limits_{1-2\varepsilon}^{1} dx\left\{\sum_{j=1}^{2f} q_j(x,t)\int\limits_{\frac{1-2\varepsilon}{x}}^{1} K_{gq}(z)dz\right.$$

$$\left. -g(x,t)\left[\int\limits_{1/2}^{\frac{1-2\varepsilon}{x}} K_{gg}(z)dz + f\int\limits_{0}^{1} K_{gg}(z)dz\right]\right\}, \tag{2}$$

where $q_i(x, t)$ is the distribution of quarks of i-th flavour, $g(x, t)$ is the distribution of gluons, f is the number of flavours, ε is a small parameter.

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$K_{ab}(z)$ are Born transition probabilities [2]:

$$K_{qq}(z) = K_{gq}(1-z) = \tfrac{4}{3}\frac{1+z^2}{1-z},$$

$$K_{gg}(z) = 6\left[\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right],$$

$$K_{qg}(z) = \tfrac{1}{2}\left[z^2+(1-z)^2\right]. \tag{3}$$

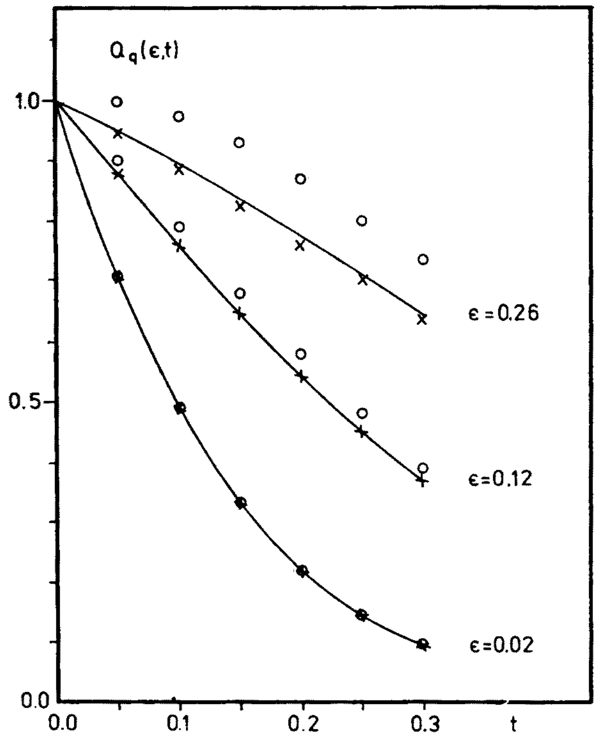

Fig. 1. $Q_q(\varepsilon, t)$ – quark distribution in the quark jet

Parameter t is related to the strong coupling constant in the following way:

$$t = -\int_{Q_0^2}^{Q^2} \frac{dQ^2}{Q^2}\frac{\alpha(Q^2)}{2\pi}. \tag{4}$$

Our aim is to calculate probabilities (denoted by $Q(\varepsilon, t)$, $G(\varepsilon, t)$) that the energy carried by quarks (gluons) which fall into calorimeter of an opening angle 2δ is greater than a $(1-2\varepsilon)$ fraction of their initial energy. Nb. $Q(\varepsilon, t)+G(\varepsilon, t)$ is the probability that in e^+e^- collision all but a fraction ε of the total energy is emitted into a pair of oppositely directed calorimeters one of which covers solid angle 2π and the other $\pi\delta^2$. These probabilities are given by:

$$Q(\varepsilon, t) = \int_{1-2\varepsilon}^{1} \sum_{i=1}^{2f} q_i(x, t)dx, \qquad G(\varepsilon, t) = \int_{1-2\varepsilon}^{1} g(x, t)dx \tag{5}$$

for $Q^2 = \delta^2 Q_0^2$.

In the following we will find the solutions for $Q(\varepsilon, t)$ and $G(\varepsilon, t)$.[1]

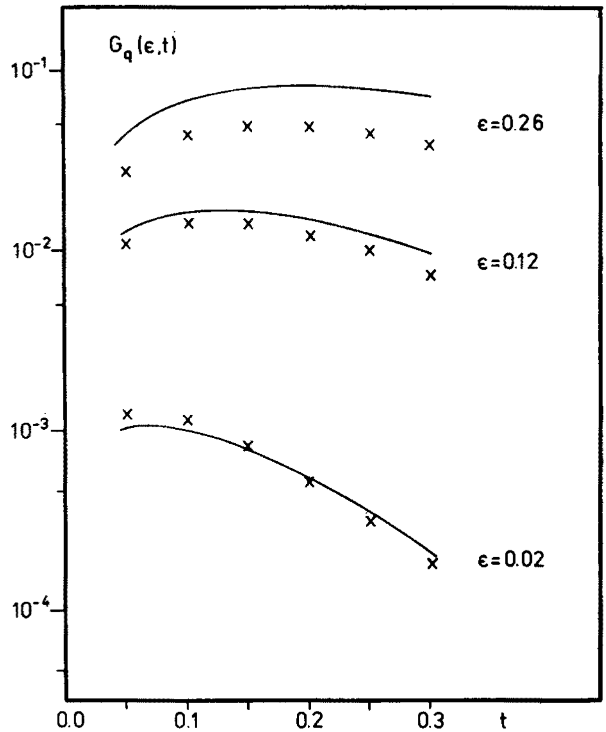

Fig. 2. $G_q(\varepsilon, t)$ — gluon distribution in the quark jet

The integrals of probabilities $K_{ab}(z)$ entering equations (1) and (2) are regular and can be easily calculated in analytical form. Nevertheless, in order to solve the equations, we must expand these integrals for small ε. Thus, we find the quark and gluon distributions for large x (i.e. for small ε). For a quark initiated jet the solutions are as follows:

$$Q_q(\varepsilon, t) = \frac{e^{(3/4-\gamma)A_F}}{\Gamma(A_F+1)} (2\varepsilon)^{A_F} \left(1 - A_F \frac{2-A_F}{1+A_F} \varepsilon + O(\varepsilon^2)\right),$$

$$G_q(\varepsilon, t) = \frac{4}{3} \frac{e^{(b/6-\gamma)A_G}}{\Gamma(A_G+2)} (2\varepsilon)^{A_G+1} \int_0^t \frac{\Gamma(6t'+2)}{\Gamma(\frac{8}{3}t'+2)} e^{-(b+2-c\gamma)t'} (2\varepsilon)^{-ct'} dt'(1+O(\varepsilon)).$$

[1] Given these functions one can of course obtain $\sum_{i=1}^{2f} q_i(x, t)$ and $g(x, t)$ by differentiation over 2ε.

For a gluon initiated jet we obtain:

$$G_g(\varepsilon, t) = \frac{e^{(b/6-\gamma)A_G}}{\Gamma(A_G+1)} (2\varepsilon)^{A_G} \left(1 - A_G \frac{2-A_G}{1+A_G} \varepsilon + O(\varepsilon^2)\right),$$

$$Q_g(\varepsilon, t) = f \frac{e^{(3/4-\gamma)A_F}}{\Gamma(A_F+2)} (2\varepsilon)^{A_F+1} \int\limits_0^t \frac{\Gamma(\frac{8}{3}t'+2)}{\Gamma(6t'+2)} e^{(b-2-c\gamma)t'} (2\varepsilon)^{ct'} dt' (1+O(\varepsilon)).$$

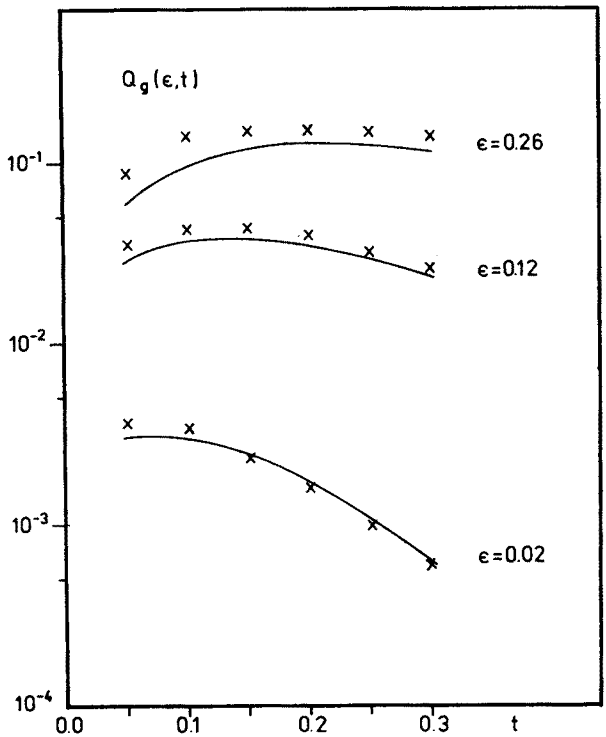

Fig. 3. $Q_g(\varepsilon, t)$ — quark distribution in the gluon jet

Here $A_F = \frac{8}{3}t$, $A_G = 6t$, $b = \frac{11}{2} - \frac{f}{3}$, $c = 10/3$ and $\gamma = 0.5772157$ is Euler's constant.

This result improves by one order in ε the result obtained by Wosiek and Zalewski [3]. Due to the higher accuracy in ε mixing between the quark and gluon distributions appears, which results in a non-zero probability for finding a quark (gluon) in the gluon (quark) initiated jet.

In the following figures we show the comparison of our result with the exact numerical calculation [4] for $f = 4$. Smooth lines are taken from Ref. [4]. Circles correspond to the result with ε^0 accuracy and crosses to the result with ε accuracy. There are no circles in Fig. 2 and Fig. 3 because in these cases the respective solutions are identically equal to zero.

It turns out that the approximation is quite good even for x not very close to 1.

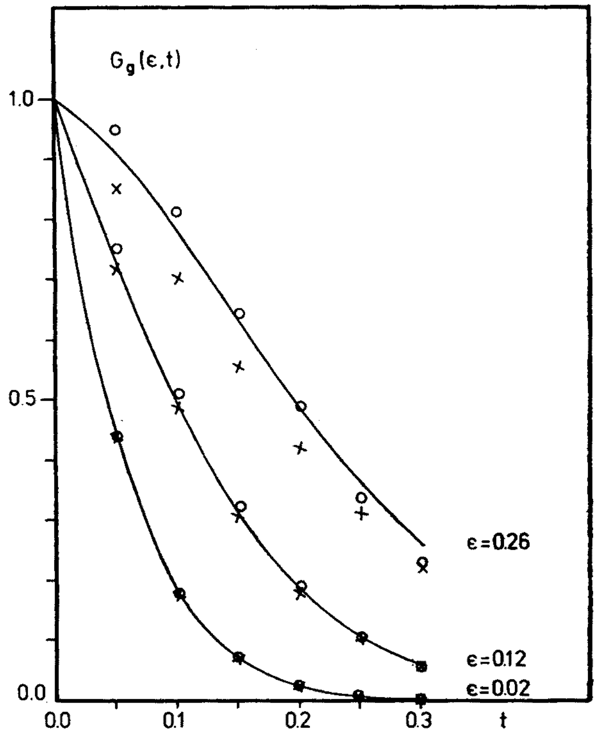

Fig. 4. $G_g(\varepsilon, t)$ — gluon distribution in the gluon jet

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