PADÉ APPROXIMANTS AND SOLUTION OF THE DISPERSION RELATIONS FOR THE PION PHOTOPRODUCTION AMPLITUDES

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Padé approximants are used to solve the dispersion relations for the pion photoproduction amplitudes. The proposed modification of the [0, 1] Padé approximant leads to a satisfactory description of the nonresonant amplitudes. No free parameters are considered in the calculations. Some of the calculated multipole amplitudes are in better agreement with the experimental data than those obtained by other methods of solving the dispersion relations.

A detailed investigation of the process of pion photoproduction on a nucleon is very important because such a study gives valuable and, often, unique information for verification of the particle models and the field theory concepts, for the development of the strong interactions theory. Considerable success in the dynamical description of the process has been achieved with the help of the dispersion relations (DR) method. DR for the photoproduction amplitudes derived in Ref. [1, 2] have been solved by different methods [3–5]. However, due to some features of these methods, a certain ambiguity is involved in results of the calculations. For example, adjustable parameters appear, when DR are solved by the Muskhelishvili–Omnes technique [3]; the omission of imaginary parts of the nonresonant multipoles in dispersion integrals is an important point for the conformal mapping technique [4], however, their contribution to the dispersion integrals can be noticeable [5], etc. It seems to be of interest to apply the method of Padé approximants (PA) for summing up the iterative series for the photoproduction amplitudes obtained on the basis of DR and the unitarity condition. This method gives reasonable results when applied to a number of problems in the strong interactions theory [6]. The absence of free parameters, the possibility of taking into account the contributions of imaginary parts of nonresonant amplitudes to the dispersion integrals enable us to calculate the photoproduction amplitudes in a more definite manner than usually. In this paper two modifications of PA are discussed and the nonresonant multipole photoproduction amplitudes are calcu-

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lated in the low energy region (up to the photon lab energy $E_\gamma \simeq 500$ MeV). Unless otherwise stated, the units used are $\hbar = c = m_\pi = 1$.

DR for the photoproduction amplitudes have the form

$$\mathrm{Re}\, M_i(w) = M_i^{\mathrm{B}}(w) + \frac{1}{\pi} P \int_{M+1}^{\infty} dw' \sum_j K_{ij}(w, w')\, \mathrm{Im}\, M_j(w'), \tag{1}$$

where M_i are the multipole photoproduction amplitudes, M_i^B are their Born parts, K_{ij} are the known kinematical functions, w is the total c.m. energy, P denotes the principal value of the integral, M is the mass of the nucleon.

If the Born amplitudes are chosen as the first approximation for the iterative procedure, the first PA is not in agreement with experimental data [7]. However, our calculations have shown that in this case the solution for the resonant amplitude $M_{1+}^{3/2}$ is similar

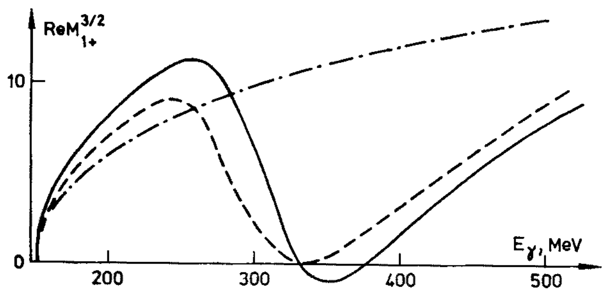

Fig. 1. $\mathrm{Re}M_{1+}^{3/2}$ in units $10^{-3}\, \hbar/m_\pi c$. The solid line is the particular solution of Muskhelishvili–Omnes equation [3]. The dashed line is the prediction of the first PA if $M_{1+}^{3/2\mathrm{B}}$ is chosen as the first term of the iterative procedure. The dash-dotted line is $M_{1+}^{3/2\mathrm{B}}$

to the particular solution of the respective Muskhelishvili–Omnes equation (see Fig. 1) which is a starting point for obtaining a resonant behavior of the amplitude $M_{1+}^{3/2}$ [3].

It should be noted, that in the case of the amplitudes $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ which describe the transitions to the resonant P_{33} state, contributions of the non-resonant amplitudes to the dispersion integral can be neglected (since these contributions are less than 1% of the Born multipole amplitudes $M_{1+}^{3/2\mathrm{B}}$ or $E_{1+}^{3/2\mathrm{B}}$, respectively). Therefore DR for these amplitudes can be separated from the rest of the system and can be written as

$$\mathrm{Re}\, M_{1+}^{3/2}(w) = M_{1+}^{3/2\mathrm{B}}(w) + \frac{1}{\pi} P \int_{M+1}^{\infty} dw' [K_{MM}(w, w')\, \mathrm{Im}\, M_{1+}^{3/2}(w') + K_{ME}(w, w')\, \mathrm{Im}\, E_{1+}^{3/2}(w')],$$

$$\mathrm{Re}\, E_{1+}^{3/2}(w) = E_{1+}^{3/2\mathrm{B}}(w) + \frac{1}{\pi} P \int_{M+1}^{\infty} dw' [K_{EE}(w, w')\, \mathrm{Im}\, E_{1+}^{3/2}(w') + K_{EM}(w, w')\, \mathrm{Im}\, M_{1+}^{3/2}(w')]. \tag{2}$$

At present these amplitudes are well known from multipole analyses [8–13] at energies up to $E_\gamma = 800$ MeV. Substitution of the experimental data on $\mathrm{Im}\, M_{1+}^{3/2}$ and $\mathrm{Im}\, E_{1+}^{3/2}$ into the dispersion integral shows that the sum of the Born term and the integral is very close to the experimental values of $\mathrm{Re}\, M_{1+}^{3/2}$ and $\mathrm{Re}\, E_{1+}^{3/2}$, respectively. In particular, the double crossing of the energy axis for $\mathrm{Re}\, M_{1+}^{3/2}$ (see Fig. 2) and the double zero of $\mathrm{Re}\, E_{1+}^{3/2}$ at the resonance energy are reproduced. This shows that a CDD-zero of the amplitude

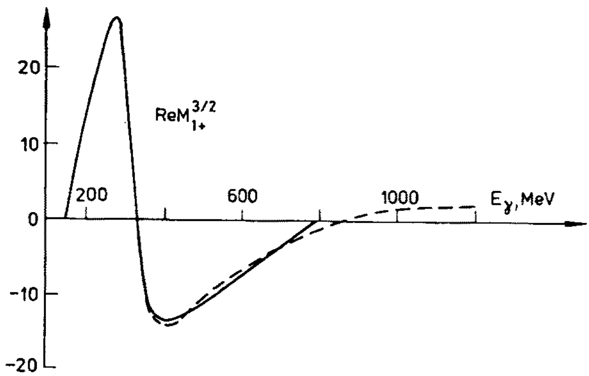

Fig. 2. Re $M_{1+}^{3/2}$ in units 10^{-3} $\hbar/m_\pi c$. The solid line is experimental data interpolation, the dashed line is the result of substitution of the experimental data to the right hand side of expression (2)

$M_{1+}^{3/2}$ occurs at $E_\gamma \simeq 800$ MeV. Its location is important for DR theory [3, 14]. As follows from the calculations the CDD-zero of the amplitude $E_{1+}^{3/2}$ should be located at an energy higher than 800 MeV.

Thus, the solution of (2) can be considered as known.

Now let us consider the nonresonant amplitudes. As noted above, the Born multipole amplitudes are not an adequate first approximation for the iterative procedure. This is due to the appreciable contribution of $\mathrm{Im}\, M_{1+}^{3/2}$ to the dispersion integral. Therefore, it is reasonable to choose the first approximation as $M_i^1 = M_i^{\mathrm{B}} + M_i\,(33)$, where $M_i\,(33)$ is the contribution of $\mathrm{Im}\, M_{1+}^{3/2}$ and $\mathrm{Im}\, E_{1+}^{3/2}$ to the dispersion integral.

To construct the iterative series

$$M_i = M_i^1 + M_i^2 + \ldots \tag{3}$$

the unitarity condition is used to find the imaginary part $(\mathrm{Im}\, M_i)^2$ of the second term in (3)

$$(\mathrm{Im}\, M_i^{1/2})^2 = (\mathrm{Re}\, M_i^{1/2})^1 \,\mathrm{tg}\, \delta_i^1, \quad (\mathrm{Im}\, M_i^{3/2})^2 = (\mathrm{Re}\, M_i^{3/2})^1 \,\mathrm{tg}\, \delta_i^3,$$
$$(\mathrm{Im}\, M_i^0)^2 = (\mathrm{Re}\, M_i^0)^1 \,\mathrm{tg}\, \delta_i^1, \tag{4}$$

while the real part of this term is calculated from DR

$$\mathrm{Re}\, M_i^2(w) = \frac{1}{\pi} P \int_{M+1}^{A} dw' \sum_j{}' K_{ij}(w, w')\, \mathrm{Im}\, M_j^2(w') \tag{5}$$

and so on for other terms. The prime above $\sum$ in expression (5) indicates that the terms with Im $M^{3/2}_{1+}$ and Im $E^{3/2}_{1+}$ are omitted, Λ is the upper limit of integration needed for numerical calculations. In expression (4) δ^1_i and δ^3_i are the πN-scattering phase shifts with the isospin 1/2 and 3/2 and with appropriate angular momentum which were taken from the phase shift analysis [15]. At energies higher than the threshold for pion pair photoproduction the relations (4) become approximate. However, up to $E_\gamma \simeq 700$ MeV this approximation is good enough because the respective inelasticity coefficients are close to unity. At higher energies contributions to the dispersion integrals are suppressed by the kinematical factors K_{ij}. Therefore for calculations at the energy region discussed it is reasonable to neglect the inelasticity and to cut the integration (5) at $\Lambda = 1800$ MeV.

We used the first [0, 1] PA to sum up approximately the iterative series (3)

$$M_{i[0,1]} = \frac{(M^1_i)^2}{M^1_i - M^2_i}\,. \tag{6}$$

Since at the energies considered only lower partial waves are important, we calculated the multipoles with the orbital angular momentum $l \leqslant 2$: $E_{0+}, E_{1+}, M_{1+}, M_{1-}, E_{2-}, M_{2-}, E_{2+}, M_{2+}$. While calculating, the contributions of imaginary parts of all these multipoles to the dispersion integrals are taken into consideration (unlike calculations [3, 4]).

Numerical results regarding the amplitudes $E^{3/2}_{0+}$ and M^0_{1-} are shown in Figs. 3 and 4. As can be seen from the figures, the results of our calculations are in satisfactory agreement with the experimental data [8–11, 16, 17][1]. Our calculations of the amplitude $E^{3/2}_{0+}$ (see Fig. 3) at the energies $E_\gamma < 350$ MeV give a good description of the results of pheno-

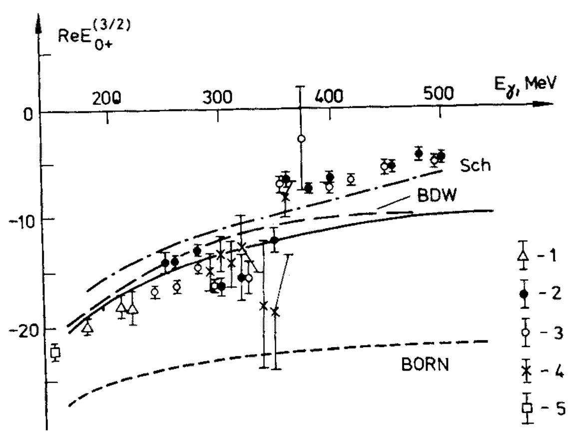

Fig. 3. Re $E^{3/2}_{0+}$ in units 10^{-3} $\hbar/m_\pi c$. *1* — the result of analysis [8], *2* — [9], *3* — [10], *4* — [11], *5* — the point of [16, 17]. The solid line is the prediction of the present work, the dash-dotted line is the calculation of work [3], the long dashed line is the calculation of work [4], the short dashed line is the Born approximation

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[1] We do not compare our predictions with the results of the multipole analysis [12] because the use of pseudodata causes an underestimation of errors [9].

menological analyses. At $E_\gamma > 350$ MeV the experimental points fall systematically above the theoretical curves. Some ambiguities of phenomenological analyses at the resonance energy may be responsible for this descrepancy.

The calculated s-wave amplitude $E_{0+}^{1/2}$ coincides practically with that of Ref. [3] and as well as for $E_{0+}^{3/2}$, agrees with the experimental data at $E_\gamma < 350$ MeV.

The amplitude E_{0+}^{0} is poorly defined in multipole analyses. The [0, 1] PA gives for E_{0+}^{0} a result close to the Born approximation, which is in reasonable agreement with the experimental data. It should be mentioned that the threshold values of the s-wave amplitudes are well known [16, 17] and the calculated values are in good agreement with the experimental ones (see Table I).

TABLE I

Threshold values of the amplitudes of pion photoproduction in units $10^{-3}\ \hbar/m_\pi c$

The amplitude	$E_{0+}\ (\gamma p \to \pi^+ n)$	$E_{0+}\ (\gamma n \to \pi^- p)$	$E_{0+}\ (\gamma p \to \pi^0 p)$
The experimental data [16, 17]	28.5 ± 0.45	-31.5 ± 1.5	-2.2 ± 0.2
The results of present calculations	28.4	-32.5	-1.8

The values of the p-wave amplitude $M_{1-}^{1/2}$ obtained in different analyses vary considerably and at present it is impossible to prefer any of the theoretical approaches which in turn also give different results.

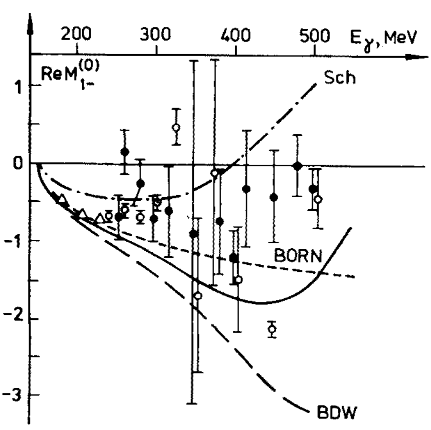

Fig. 4. Re M_{1-}^{0}. The units and symbols are as in Fig. 3

As can be seen from Fig. 4, PA (6) gives for the amplitude M_{1-}^{0} better agreement with experiment than the calculations of Ref. [4]. At the same time PA does not improve the agreement of DR theory predictions for the amplitude $M_{1-}^{3/2}$ with experimental data.

For the amplitude $M_{1+}^{1/2}$ our calculations describe the experiment better than the calculations of Refs. [3, 4] do. To draw, however, more definite conclusions it is necessary to improve the experimental data.

Finally, the amplitudes M_{1+}^{0}, $E_{1+}^{1/2}$, E_{1+}^{0} derived from Eq. (6) coincide practically with the Born approximation and do not contradict the experimental values.

Data on the *d*-wave photoproduction amplitudes at low energies are not available at the moment. Our calculations show that the predicted *d*-wave amplitudes differ only slightly from the Born approximation (up to 10–15%).

Thus, once an appropriate first approximation in the iterative procedure is chosen, satisfactory agreement with the experiment for the nonresonant amplitudes (apart from $M_{1-}^{3/2}$) can be achieved even by the first [0, 1] PA.

It is worth noting that in this method there are no adjustable parameters and that the contributions of nonresonant multipoles to dispersion integrals are taken into consideration. Agreement with the results of multipole analyses is improved for some multipole amplitudes and *s*-wave threshold amplitudes are well described. To compare further the theory with experiment it is necessary to improve the multipole analyses. It would be of interest to use a higher PA to calculate the pion photoproduction amplitudes and also to carry out calculations in the energy region of photoexcitation of the second and the third nucleon resonances.

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