ON THE THERMAL PROPERTIES OF NUCLEAR MATTER WITH NEUTRON EXCESS

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The schematic model of nuclear matter proposed by Gomes, Walecka and Weisskopf which was generalized to finite temperatures including interacting Fermi particle aspects is extended here to include nuclear matter with neutron excess. We calculated the level density parameter as a function of neutron excess. We also calculated the temperature dependence of the equilibrium Fermi momentum.

1. Introduction

Thermal properties of nuclear matter have aroused great interest in recent years. This is mainly because of a) the importance of studying excited nuclei in nuclear reactions [1, 2], b) the study of "hot" neutron stars in astrophysics [3].

These properties can be determined by extension of Thomas–Fermi approach to ground-state nuclei to finite temeratures $T > 0$ [4]. If one keeps in mind the simplicity of this model, it reproduces for example the nuclear masses and details of the nuclear surface surprisingly well. Therefore its generalization to excited states appeared very promising. Many authors have tried to study these properties [5–8] using different approaches.

In this work we are particularly interested in the generalization of Stocker's [6] model. In his work Stocker generalized the schematic model of nuclear matter proposed by Gomes, Walecka and Weisskopf to finite temperatures including interacting Fermi particle aspects. We generalize this model to describe nuclear matter with neutron excess using the results of Dąbrowski and Hassan [9] and of Hassan and Montasser [10] who have generalized the model by Gomes, Walecka and Weisskopf to asymmetric nuclear matter. We also calculated the symmetry energy as a function of temperature and the level density parameter as a function of the neutron excess $\alpha(= (N-Z)/A)$.

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In Section 2 we discuss the theory where we got expressions for some of the parameters of the thermal properties of nuclear matter with neutron excess as a function of α. In Section 3 we give the results of the calculations of these parameters.

2. Theory

2.1. The equation of state

The binding energy of nuclear matter with pure hard core interaction and with neutron excess is given by [9, 10]

$$E_R/A = \tfrac{3}{5}\frac{\hbar^2 k_F^2}{2M}\left(\frac{\gamma_n^5+\gamma_p^5}{2}\right)+\frac{1}{\pi}\frac{\hbar^2 k_F^2}{M}(k_F r_c)\,(1-\tfrac{1}{3}\alpha^2)+\frac{1}{\pi^2}\frac{\hbar^2 k_F^2}{M}(k_F r_c)^2$$

$$[\tfrac{1}{35}(11-2\ln 2)(\gamma_n^7+\gamma_p^7)-3\{(\tfrac{23}{420}(\gamma_n^7+\gamma_p^7)-\tfrac{1}{10}(\gamma_n^5\gamma_p^2+\gamma_n^2\gamma_p^5)$$

$$+\tfrac{1}{12}(\gamma_n^4\gamma_p^3+\gamma_n^3\gamma_p^4))\ln(\gamma_n+\gamma_p)-\tfrac{4}{105}(\gamma_n^7\ln\gamma_n+\gamma_p^7\ln\gamma_p)$$

$$+(-\tfrac{1}{60}(\gamma_n^7+\gamma_p^7)+\tfrac{1}{10}(\gamma_n^5\gamma_p^2+\gamma_n^2\gamma_p^5)-\tfrac{1}{21}(\gamma_n^4\gamma_p^3+\gamma_n^3\gamma_p^4))\ln|\gamma_n-\gamma_p|$$

$$-\tfrac{1}{14}(\gamma_n^6\gamma_p+\gamma_n\gamma_p^6)-\tfrac{11}{70}(\gamma_n^4\gamma_p^3+\gamma_n^3\gamma_p^4)+\tfrac{2}{105}(\gamma_n^5\gamma_p^2+\gamma_n^2\gamma_p^5)\}]$$

$$+\frac{\hbar^2 k_F^2}{2M}(k_F r_c)^3\left[\frac{1}{30\pi}\{(\gamma_n^8+\gamma_p^8)+2\gamma_n^3\gamma_p^3(\gamma_n^2+\gamma_p^2)\}+0.192+0.032\alpha^2\right.$$

$$\left.+\frac{3}{10\pi}\{\gamma_n^8+\gamma_p^8+\tfrac{2}{3}\gamma_n^3\gamma_p^3(\gamma_n^2+\gamma_p^2)\}\right], \tag{1}$$

where $\gamma_n = (1+\alpha)^{1/3}$, $\gamma_p = (1-\alpha)^{1/3}$, r_c is the hard core radius.

The attractive part of the potential is taken in the form:

$$V_A = \begin{cases} -\tfrac{1}{2}(1+P^M)V_0 & \text{for} \quad r_c < r < r_c+b, \\ 0 & \text{for} \quad r > r_c+b, \end{cases} \tag{2}$$

P^M is the Majorana exchange operator. The intrinsic range b is determined from the relation [9]:

$$2r_c+b = 2.7\ \text{fm}.$$

From high energy scattering experiments r_c can be taken equal to 0.45 fm. Therefore $b = 1.8$ fm. The potential depth is fixed by the relation [9]:

$$\frac{MV_0}{\hbar^2}b^2 = \frac{\pi^2}{4}s, \tag{3}$$

where the parameter s is equal to one if the two-body system has a bound state at zero energy, which we considered to be the case. The attractive part contribution to the binding energy taking into account only the first order term in the perturbation expansion is

$$E_A/A = -\frac{3V_0}{4}\left[\frac{k_F^3}{2.}\{(b+r_c)^3 - r_c^3\}(\gamma_n^6 + 4\gamma_n^3\gamma_p^3 + \gamma_p^6) + k_F(\gamma_n^4 I_{nn} + 4\gamma_n^2\gamma_p^2 I_{np} + \gamma_p^4 I_{pp})\right], \tag{4}$$

where

$$I_{np} = \int_{r_c}^{r_c+b} dr j_1(\kappa r) j_1(\lambda r), \quad \kappa = \gamma_n k_F; \quad \lambda = \gamma_p k_F,$$

$$j_1(x) = (\sin x - x\cos x)/x^2.$$

The above model of nuclear matter has been generalized to finite temperatures by Stocker [6] using methods of statistical mechanics of interacting Fermi systems at low temperatures [11]. For nuclear matter with neutron excess we used the method discussed by Kupper et al. [7] and keeping terms up to T^2 and α^2 we get for the entropy per particle:

$$s(T, n) = \frac{\pi^2}{2}\frac{M}{\hbar^2 k_F^2} k_B^2 T(\gamma_n + \gamma_p) = \pi^2 \frac{M}{\hbar^2 k_F^2} k_B^2 T\left(1 - \frac{\alpha^2}{9}\right) \tag{5}$$

and the internal energy per nucleon:

$$e(T, n) = e(T = 0, n) + \frac{1}{n}\int_0^T dT' C_v(T'), \tag{6}$$

where the specific heat C_v is given by

$$C_v(T) = \frac{M}{3\hbar^2}(\kappa + \lambda) k_B^2 T. \tag{7}$$

The free energy per particle is

$$f(T, n) = e(T, n) - Ts. \tag{8}$$

The pressure p is given by

$$p = n^2\left(\frac{\partial f}{\partial n}\right)_T, \quad n = \frac{2k_F^3}{3\pi^2}. \tag{9}$$

Expanding the entropy and the internal energy up to second order in α we get:

$$s = s_v + \alpha^2 s_s, \tag{10a}$$

where

$$s_v = \pi^2 \frac{M}{\hbar^2 k_F^2} k_B^2 T, \tag{10b}$$

$$s_s = -\frac{\pi^2}{9}\frac{M}{\hbar^2 k_F^2} k_B^2 T \tag{10c}$$

and

$$e(T, n) = e_v(T, n) + \alpha^2 e_s(T, n), \tag{11a}$$

$$e_v(T, n) = e_v(T = 0, n) + \frac{T}{2} s_v, \tag{11b}$$

$$e_s(T, n) = e_s(T = 0, n) + \frac{T}{2} s_s, \tag{11c}$$

where

$$e(T = 0, n) = (E_R + E_A)/A = e_v(T = 0, n) + \alpha^2 e_s(T = 0, n). \tag{12}$$

The pressure as a function of density at different temperatures can be calculated from Eq. (9) by inserting the internal energy Eq. (11a) into the free energy expression Eq. (8). As we will see from the next section, the $p = p(v, T)$ curves can be used to calculate the excitation energy from which we can get the level density parameter.

2.2. The temperature and neutron excess dependence of the equilibrium density of nuclear matter

The equilibrium Fermi momentum was discussed before by several authors [12] for zero temperature nuclear matter with neutron excess. We will now generalize this discussion to the case of non-zero temperature. The equilibrium Fermi momentum for a fixed neutron excess α and finite temperature T is trated as the solution of the equation

$$\frac{\partial f(\alpha, T, k_F)}{\partial k_F} = 0. \tag{13}$$

Now, we assume the knowledge of the solution to Eq. (13) in the case $\alpha = 0$ and $T = 0$, and we denote this solution as k_{F0}, where k_{F0} satisfies the condition

$$\left.\frac{\partial f(\alpha = 0, T = 0, k_F)}{\partial k_F}\right|_{k_{F0}} = 0. \tag{14}$$

Then we expand the derivative in Eq. (14) in power series about k_{F0} neglecting terms of higher order than first. We obtain:

$$\frac{\partial f(\alpha, T, k_F)}{\partial k_F} = \left.\frac{\partial f(\alpha, T, k_F)}{\partial k_F}\right|_{k_{F0}} + \Delta k_F \left.\frac{\partial^2 f(\alpha, T, k_F)}{\partial k_F^2}\right|_{k_{F0}} = 0. \tag{15}$$

To solve this equation we have to expand the derivatives in power series of α and T. Neglecting terms of higher order than α^2 and T^2 and making use of equations (8), (10) and (11) we get

$$\Delta k_F = -\frac{k_F^2}{K}\left(\alpha^2 \frac{\partial e_s(0, n_0)}{\partial k_F} + \frac{\pi^2 M}{\hbar^2 k_F^3} T^2 k_B^2\right), \tag{16}$$

where K is the compressibility and n_0 is the equilibrium density.

3. *Results and discussion*

The equation of state $p = p(V, T)$ following for nuclear matter in this formalism can be calculated. It consists of a term representing the free nucleon gas part and some correlation parts which take into account the nuclear attraction and the short range repulsion correlation. In Fig. 1 the isotherms following from the equation of state (9) are displayed for $\alpha = 0$. The difference between these isotherms and those done by Stocker is due to the fact that the hard core radius r_c in our calculations is fixed to the value 0.45 fm and the

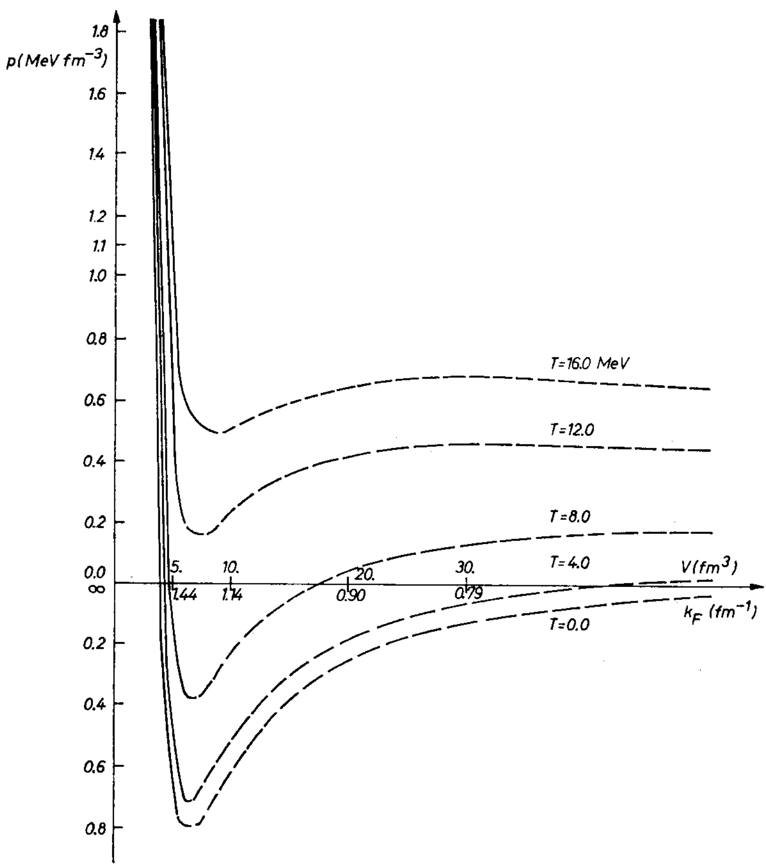

Fig. 1. Nuclear matter isotherms in the v, p diagram $r_c = 0.45$ fm, $m^*/m = 1, \alpha = 0$

effective mass is taken to be equal to the free nucleon mass. This choice of the effective mass is justified by the fact that the effective mass at the Fermi surface in nuclear matter may be well approximated by the free nucleon mass. This is in agreement with the results of investigations of heavy nuclei [6]. In Stocker's calculations he fitted three parameters (C, B and d_0 in his paper) and took r_c to be a free parameter.

As shown in Fig. 1, there are often two different densities for given p and T, corresponding to the liquid and vapour phases. The dashed parts of the isotherms where p

decreases with increasing density of homogeneous nuclear matter represent a region of thermodynamic instability. The zero pressure configurations of homogeneous nuclear matter, corresponding to the right-hand side intersections of the isotherms with $p = 0$ line, correspond to metastable configurations with negative compressibility.

Neglecting nuclear curvature effects, excited nuclei correspond to thermodynamical systems with vanishing external pressure. Thus in the plot of isotherms of Fig. 1, excited nuclear matter corresponds to the first isotherm intersection with V-axis, this represents a state of minimal free energy. The excitation energy per particle as a function of nuclear temperature:

$$e^*(T, \alpha) = e(T, n_0(T, \alpha)) - e(0, n_0(0, \alpha)) \tag{17}$$

($n_0(T, \alpha)$ is the equilibrium density at temperature T and neutron excess α) may be approximated by

$$e^*(T, \alpha) = a(\alpha) k_B^2 T^2. \tag{18}$$

$n_0(T)$, can be calculated from the first intersection with the V-axis of the isotherms which is calculated at different neutron excess α. We can calculated the level density parameter $a(\alpha)$ using these isotherms and Eq. (18). The results are shown in Fig. 2. It is worth mention-

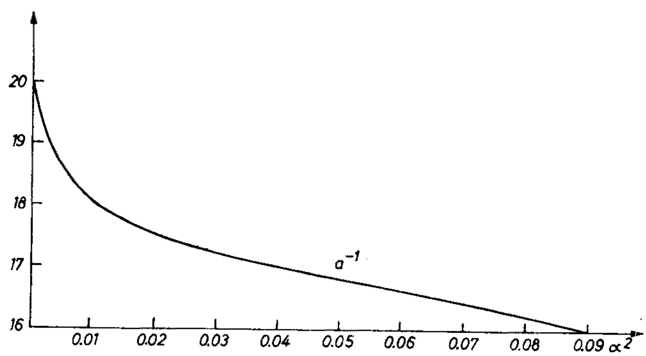

Fig. 2. The reciprocal of the nuclear level density parameter in MeV as a function of α

ing that when α increases the model approaches the finite nucleus case. Then it is to be expected that the value of a^{-1} decreases with the increase of α. This effect is reproduced in Fig. 2. Of course, the inclusion of curvature effects will improve the model much further near to the finite nuclei [5].

We also calculated the volume and symmetry energy at equilibrium density $n_0(T, \alpha)$ as a function of temperature up to T^2 and we got the result:

$$E_{vol}(T) = -14.3 + 0.053 k_B^2 T^2, \tag{19}$$

$$E_{sym}(T) = 46.11 - 5.9 \times 10^{-3} T^2 k_B^2. \tag{20}$$

These expressions were calculated by Sauer et al. [8] using Skyrme interaction and they got the value 0.055 for the coefficient of T^2 in the volume energy and the value 6×10^{-3}

for the symmetry energy, but with positive sign. The value of the symmetry energy at $T = 0$ is larger than the experimental value which is around 30 MeV. This may be due to the choice of the attractive part of the potential and neglecting higher order terms in the perturbation expansion.

The temperature dependence of the equilibrium Fermi momenta can be obtained from Eq. (16) (for $\alpha = 0$)

$$k_{\mathrm{Feq}}(T) = k_{\mathrm{Feq}}(T = 0) - \frac{\pi^2 M}{\hbar^2 k_{\mathrm{F}} K} k_{\mathrm{B}}^2 T^2, \tag{21}$$

where we calculated the compressibility K and we got the value 149 MeV which is in agreement with Stocker's calculations. Eq. (21) can then be written as:

$$k_{\mathrm{Feq}}(T) = k_{\mathrm{Feq}}(0) - 1.06 \times 10^{-3} k_{\mathrm{B}}^2 T^2. \tag{22}$$

It is worth mentioning that the choice of the interaction and the T^2 approximation need further investigation. This investigation is under progress.

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