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ANOMALOUS DIMENSION AND INFRARED BEHAVIOR FOR HIGH ENERGY QED

By J. Z. Kamiński

Institute of Theoretical Physics, Warsaw University*

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The generalization of the Sudakov results for real photons is obtained. It allows one to calculate the anomalous dimensions for the Coulomb and the electron–electron scattering. The formulas $\Delta E = E\exp(-\eta(E/m))$ for the Coulomb case and $\Delta E = \sqrt{s}\exp(-\eta(s/m^2))$ for the electron–electron case are given. These formulas give anomalous dimensions which depend on the energy. This dependence, which comes from the Sudakov double logarithmic behavior of QED, disappears for semi-inclusive scattering. These results can be extended to other models of QFT like scalar QED or QCD and they partially justify the eikonal approximation.

1. Introduction

Sudakov has shown [1] that in order to obtain the leading logarithms of the vertex function (without vacuum polarization diagrams) it is sufficient to consider only the infrared part of the virtual photon integration. In particular, he has shown that to obtain the leading logarithms one can make the substitution

$$\frac{\not{p}-\sum\limits_{i=1}^{l}\not{k}_i+m}{(p-\sum\limits_{i=1}^{l}k_i)^2-m^2} \to -\frac{\not{p}+m}{2p\sum\limits_{i=1}^{l}k_i}, \tag{1}$$

where p is the electron momentum on the mass shell and k_i are virtual photons momenta. After such a substitution the leading logarithms are obtained from the integration over the infrared region of virtual photons momenta. In this paper I shall extend Sudakov's results to the real photon integration. As an example I shall consider the Coulomb scattering in the Born approximation stressing that my results can be extended to other processes as, for instance, electron–electron scattering. I shall consider the scattering with high momentum transfer. Processes with the scattering angle equal to zero have been considered elsewhere [2]. In the Coulomb case the situation is complicated. In addition to the electron propagator, there is also the energy conservation δ-function and the Fourier transform

* Address: Instytut Fizyki Teoretycznej UW, Hoża 69, 00-681 Warszawa, Poland.

of the Coulomb potential which depend on the momenta of real photons. My statement is that to obtain the leading logarithms one can make the substitution (1) and the replacement

$$\frac{\delta(E-E'-\sum_{i=1}^{n}\omega_i)}{(p-p'-\sum_{i=1}^{n}k_i)^4} \to \frac{\delta(E-E')}{(p-p')^4}. \tag{2}$$

I introduce the photon mass λ as an infrared regulator. Using perturbation theory we obtain the following asymptotic results for semi-inclusive Coulomb scattering in the Born approximation [3]

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{as}} = \left(\frac{d\sigma}{d\Omega}\right)_0^{\text{as}}\left[1+\sum_{i=1}^{\infty}\alpha^i\sum_{j=0}^{i}F_{ij}(\theta)(\ln E/m)^j\right]. \tag{3}$$

The leading logarithms approximation means that we keep only those powers of $\ln E/m$ which are equal to the powers of the coupling constant α, i.e.,

$$\left(\frac{d\sigma}{d\Omega}\right)_L^{\text{as}} = \left(\frac{d\sigma}{d\Omega}\right)_0^{\text{as}}\left[1+\sum_{i=1}^{\infty}F_{ii}(\theta)\alpha^i(\ln E/m)^i\right]. \tag{4}$$

I shall show that to obtain (4) it is sufficient to integrate over the infrared region of real photons momenta, i.e.,

$$\omega \ll E. \tag{5}$$

The relation

$$B \sim A \tag{6}$$

means that the leading logarithms of B are given by A.

My statement is true for one real photon [4]. In order to prove it for an arbitrary number of photons, I have to estimate the integrals

$$C_{n,l}^{s,r} = \int \frac{d^3k}{\omega}|\vec{p}'|dE' \frac{\delta(E-E'-\sum_{i=1}^{r}\omega_i-\omega)}{(p-p'-\sum_{i=1}^{r}k_i-k)^4}\frac{E^{2-s}\omega^{2(n+l-2)+s}}{(2p'k+\lambda^2)^n(-2pk+\lambda^2)^l}, \tag{7}$$

where all k and k_i are on the mass shell and

$$0 \leqslant s \leqslant 2(n+l+2). \tag{8}$$

It is shown in the Appendix that only $C_{1,1}^{0,r}$ gives the leading logarithm and that

$$C_{1,1}^{0,r} \sim \int_0^{\omega_0} \frac{d^3k}{\omega}|\vec{p}'|dE' \frac{\delta(E-E'-\sum_{i=1}^{r}\omega_i)}{(p-p'-\sum_{i=1}^{r}k_i)^4}\frac{E^2}{(2p'k+\lambda^2)(-2pk+\lambda^2)}, \tag{9}$$

where $\omega_0 \ll E$.

2. *Purely inelastic diagrams*

In previous papers [3, 4] I have studied the semi-inclusive scattering at high energy in QED. I mentioned there that these processes are more suitable to study the high energy scattering than those with a very small resolution energy because their cross sections are relativistically invariant (not for the Coulomb case where one deals with external fields) and positive in every order of perturbation theory. I have conjectured there that if we want

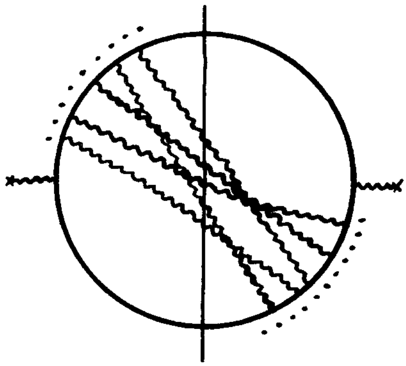

Fig. 1

to calculate the leading logarithms we may treat the real photons as the soft ones. At the beginning I prove this for purely inelastic processes. Let me take as an example the diagram in Fig. 1. The diagramatic representations of the transition probabilities are drawn according to rules given in [5, 6]. This diagram has the following contribution to the transition probability

$$D_n = \sum_\sigma \int \prod_{i=1}^n \frac{d^3k_i}{\omega_i} |\vec{p}'| dE' \frac{\delta(E-E'-\sum_{i=1}^n \omega_i)}{(p-p'-\sum_{i=1}^n k_i)^4} \mathrm{Tr}\Bigg(\gamma^0(\not{p}'+m)$$

$$\times \gamma^{\mu_1} \frac{\not{p}'+\not{k}_1+m}{(p'+k_1)^2-m^2} \gamma^{\mu_2} \dots \gamma^{\mu_n} \frac{\not{p}'+\sum_{i=1}^n \not{k}_i+m}{(p'+\sum_{i=1}^n k_i)^2-m^2} \gamma^0(\not{p}+m)$$

$$\times \gamma_{\mu_{\sigma(1)}} \frac{\not{p}-\not{k}_{\sigma(1)}+m}{(p-k_{\sigma(1)})^2-m^2} \gamma_{\mu_{\sigma(2)}} \dots \gamma_{\mu_{\sigma(n)}} \frac{\not{p}-\sum_{i=1}^n \not{k}_{\sigma(i)}+m}{(p-\sum_{i=1}^n k_{\sigma(i)})^2-m^2}\Bigg), \tag{10}$$

where the summation is over all the possible permutations ot the set $\{1, 2, \dots, n\}$. We have the estimate

$$D_n \leqslant Z_n E^{2n+1} d_n, \tag{11}$$

with

$$d_n = \sum_{\sigma} \int \sum_{i=1}^{n} \frac{d^3k_i}{\omega_i} |\vec{p}'| dE' \frac{\delta(E-E'-\sum_{i=1}^{n}\omega_i)}{(p-p'-\sum_{i=1}^{n}k_i)^4} \frac{1}{(p'+k_1)^2-m^2}$$

$$\cdots \frac{1}{(p'+\sum_{i=1}^{n}k_i)^2-m^2} \frac{1}{(p-k_{\sigma(1)})^2-m^2} \cdots \frac{1}{(p-\sum_{i=1}^{n}k_{\sigma(i)})^2-m^2}, \tag{12}$$

and Z_n is an n-dependent number.

Let me divide the integration region in (10) on R and $\tilde{R}$. R is such a region where for every electron propagator the following expansion

$$\frac{1}{2p'\sum_{i=1}^{m}k_i+(\sum_{i=1}^{m}k_i)^2} = \frac{1}{2p'\sum_{i=1}^{m}k_i}\left(1-\frac{(\sum_{i=1}^{m}k_i)^2}{2p'\sum_{i=1}^{m}k_i}+\ldots\right) \tag{13}$$

is allowed (for instance, for $\omega_i \ll E$, $1 \leqslant i \leqslant n$). Therefore, we have

$$d_n^R = \sum_{r=0}^{\infty} d_n^r = \sum_{r=0}^{\infty} (-1)^n \sum_{\sigma} \sum_{\sum_{i=1}^{\infty}(l_i+l'_i)=0} \prod_{j=1}^{n} \int_R \prod_{i=1}^{n} \frac{d^3k_i}{\omega_i} |\vec{p}'| dE'$$

$$\times \frac{\delta(E-E'-\sum_{i=1}^{n}\omega_i)}{(p-p'-\sum_{i=1}^{n}k_i)^4} \frac{1}{P_{\sigma(j)}P'_j} \left(\frac{K_{\sigma(j)}}{P_{\sigma(j)}}\right)^{l_i} \left(-\frac{K_j}{P'_j}\right)^{l'_i}, \tag{14}$$

where

$$P_{\sigma(m)} = 2p\sum_{i=1}^{m}k_{\sigma(i)}, \tag{15}$$

$$P'_m = 2p'\sum_{i=1}^{m}k_i, \tag{16}$$

$$K_{\sigma(m)} = (\sum_{i=1}^{m}k_{\sigma(i)})^2, \tag{17}$$

$$K_m = (\sum_{i=1}^{m}k_i)^2. \tag{18}$$

I have the following estimate

$$|d_n^1| \leqslant \frac{2n^2}{n!}\left|\int_R \prod_{i=1}^{n} \frac{d^3k_i}{\omega_i}|\vec{p}'|dE' \frac{\delta(E-E'-\sum_{i=1}^{n}\omega_i)}{(p-p'-\sum_{i=1}^{n}k_i)^4}\left(\prod_{i=1}^{n}\frac{1}{(2p'k_i)(2pk_i)}\right)\right.$$

$$\left.\times\left(\sum_{i=1}^{n}\frac{\omega_i^2}{2p'k_i}+\sum_{i=1}^{n}\frac{\omega_i^2}{2pk_i}\right)\right|. \tag{19}$$

In a similar manner one can estimate d_n^2, d_n^3 etc. In order to obtain (17) I have used the formulas (see the Appendix)

$$\left|\frac{(\sum_{i=1}^{m}k_i)^2}{2p'\sum_{i=1}^{m}k_i}\right| \leqslant 2m^2\sum_{i=1}^{m}\frac{\omega_i^2}{2p'k_i}, \tag{20}$$

and [1]

$$\sum_{\sigma}\frac{1}{a_{\sigma(1)}(a_{\sigma(1)}+a_{\sigma(2)})\ldots(a_{\sigma(1)}+\ldots+a_{\sigma(n)})}=\prod_{i=1}^{n}\frac{1}{a_i}. \tag{21}$$

Therefore, from (7) and (9)

$$d_n^R \sim d_n^0, \tag{22}$$

since in d_n^1, d_n^2 and so on, there are integrals such as $C_{n,l}^{s,r}$ with $n>1$ or $l>1$.

Using formula

$$\int_a^b f(x)g(x)dx \leqslant f(\tilde{x})\int_a^b g(x)dx, \tag{23}$$

where $a \leqslant \tilde{x} \leqslant b$ and $f(x)$, $g(x)$ are bounded, monotonic and continuous functions of x, and assuming that at least one ω_j does not satisfy condition (5), I have from (9) that

$$d_n^0 \leqslant \frac{1}{n!}E^{-2n-3}F_n^0(\theta, \ln E/\lambda)(\ln E/m)^{n-1}. \tag{24}$$

This means that all ω_i have to fulfil (5).

$\tilde{R}$ is such a region where, for at least one term, say $((\sum_{i=1}^{m}k_i)^2+2p'\sum_{i=1}^{m}k_i)^{-1}$, the expansion (13) is not allowed. This means that

$$\left|\frac{2p'\sum_{i=1}^{m}k_i}{(\sum_{i=1}^{m}k_i)^2}\right| < 1. \tag{25}$$

Therefore, there is at least one k_j, $i \leqslant j \leqslant m$, that condition (5) is not fulfilled. In the region $\tilde{R}$ I have the following convergent expansion:

$$\left(\left(\sum_{i=1}^{m} k_i\right)^2 + 2p' \sum_{i=1}^{m} k_i\right)^{-1} = \frac{1}{(\sum\limits_{i=1}^{m} k_i)^2}\left(1 - \frac{2p' \sum\limits_{i=1}^{m} k_i}{(\sum\limits_{i=1}^{m} k_i)^2} + \ldots\right), \tag{26}$$

and from (25)

$$|\tilde{d}_n^0| \leqslant \frac{1}{n!}\left|\int\limits_{\tilde{R}} \prod_{i=1}^{n} \frac{d^3k_i}{\omega_i} |\vec{p}'| dE' \frac{\delta(E-E'-\sum\limits_{i=1}^{n}\omega_i)}{(p-p'-\sum\limits_{i=1}^{n} k_i)^4} \prod_{i=1}^{n} \frac{1}{(2p'k_i)(2pk_i)}\right|. \tag{27}$$

Since the integration over ω_j is not from zero to $\omega_0 \ll E$, therefore $|\tilde{d}_n^0|$ is estimated in the same manner as in Eq. (24). In a similar way one can study $\tilde{d}_n^1$, $\tilde{d}_n^2$ and so on, and obtain the same result. This means that the integration over $\tilde{R}$ does not contribute to the leading logarithms, which ends the proof of my statement for diagrams with only real photons. Of course, other types of diagrams with the only real photons may be treated similarly.

3. *Diagrams with virtual photons*

The fact that one can obtain the leading logarithms treating the real photons as soft ones also means that in a diagram which gives a contribution to the leading logarithms a real photon has to be emitted by the fermion on the mass shell. For instance, the diagram

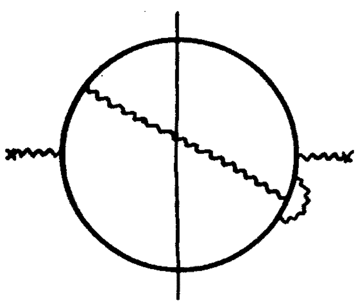

Fig. 2

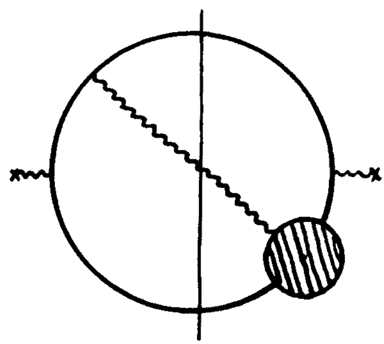

Fig. 3

of Fig. 2 is not leading. Let me take as an example the diagram of Fig. 3. Its contribution to the full transition probability is

$$\int \frac{d^3k}{\omega} |\vec{p}'| dE' \frac{\delta(E-E'-\omega)}{(p-p'-k)^4} \operatorname{Tr}\left(\gamma_0(\not{p}'+m)\gamma^\mu \right.$$

$$\left. \times \frac{\not{p}'+\not{k}+m}{(p'+k)^2-m^2}\gamma^0(\not{p}+m)\Gamma_\mu(p, p-k, k)\frac{\not{p}-\not{k}+m}{(p-k)^2-m^2}\right), \tag{28}$$

where $\Gamma_\mu(p, p-k, k)$ is the renormalized vertex function. The renormalization condition shows that

$$\lim_{k\to 0} \Gamma_\mu(p, p-k, k) = \gamma_\mu. \tag{29}$$

The renormalized vertex function fulfils the renormalization group equation [7, 8]

$$\left(\kappa\frac{\partial}{\partial\kappa} - \beta(\alpha)\alpha\frac{\partial}{\partial\alpha} + (1+\gamma_m(\alpha))m\frac{\partial}{\partial m} + (1+\gamma_\lambda(\alpha))\lambda\frac{\partial}{\partial\lambda} + \delta(\alpha)\right)$$

$$\times\Gamma_\mu(\kappa E_0, \kappa\omega_0, \vec{n}, \vec{n}_k, \alpha, m, \lambda, \mu) = 0, \tag{30}$$

where $\vec{n}$ and $\vec{n}_k$ are momenta directions of electron and photon, respectively. κ is the scaling parameter and $E = \kappa E_0$, $\omega = \kappa\omega_0$, E_0 and ω_0 are arbitrary parameters of the mass dimension. Moreover, from perturbation theory we have

$$\begin{bmatrix} \beta(\alpha) \\ \gamma_m(\alpha) \\ \gamma_\lambda(\alpha) \\ \delta(\alpha) \end{bmatrix} = \sum_{i=1}^{\infty} \alpha^i \begin{bmatrix} b_i \\ g_i \\ h_i \\ d_i \end{bmatrix}. \tag{31}$$

From these equations it follows that asymptotically for a large κ, the vertex function has the form of (at the end we put $\kappa = E/m$)

$$\Gamma_\mu(p, p-k, k) = \gamma_\mu + \sum_{l=1}^{\infty}\sum_{n\leq l} \Delta_\mu^{n,l}(\vec{n}, \vec{n}_k, \omega/E, \ln E/\lambda)\alpha^l(\ln E/m)^n. \tag{32}$$

Since we use the photon mass λ as the infrared regulator, $\Delta_\mu^{n,l}$, as functions of ω, are bounded. Moreover, from the renormalization condition (23) it follows that

$$\Delta_\mu^{n,l}(\vec{n}, \vec{n}_k, 0, \ln E/\lambda) = 0. \tag{33}$$

From the explicit calculations I have

$$|\Delta_\mu^{0,1}(\vec{n}, \vec{n}_k, \omega/E, \ln E/\lambda)| \leq \frac{\omega}{E}\tilde{\Delta}_\mu^{0,1}(\vec{n}, \vec{n}_k, \ln E/\lambda), \tag{34}$$

$$|\Delta_\mu^{1,1}(\vec{n}, \vec{n}_k, \omega/E, \ln E/\lambda)| \leq \frac{\omega}{E}\tilde{\Delta}_\mu^{1,1}(\vec{n}, \vec{n}_k, \ln E/\lambda), \tag{35}$$

and $\tilde{\Delta}_\mu^{0,1}$, $\tilde{\Delta}_\mu^{1,1}$ are bounded functions of $\vec{n}$, $\vec{n}_k$. From the renormalization group equation (30) it follows that for all functions $\Delta_\mu^{n,l}$ conditions like (34) and (35) are fulfilled. Therefore, from (7) I obtain that only the bare vertex γ_μ contributes to the leading terms. In a similar way one can show that the diagrams of Figs. 4 and 5 are not leading in higher orders too. The conclusion is that only diagrams shown in Fig. 6 contribute to the leading logarithms of the Coulomb scattering in the Born approximation.

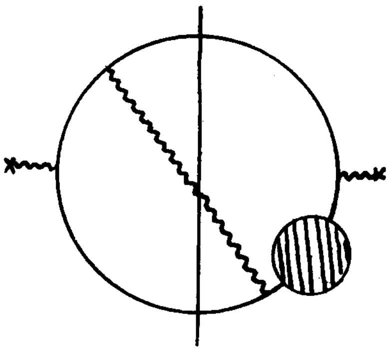

Fig. 4

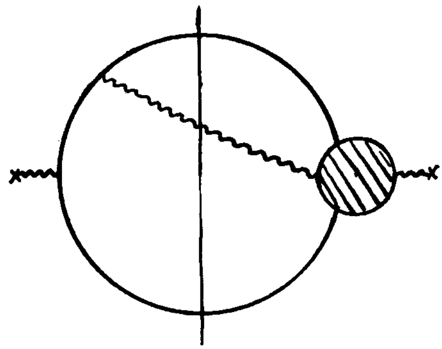

Fig. 5

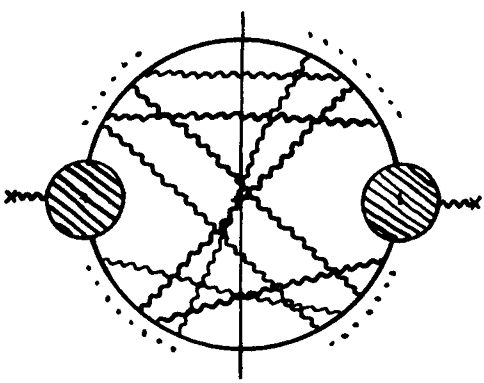

Fig. 6

4. *Conclusions and discussion*

I have broadened Sudakov's results to the case of real photons for the Coulomb scattering. In a similar manner one can do it for other processes like the electron–electron or electron–positon scattering.

My results are also valid for processes where the total energy of final photons is restricted by the resolution energy of detectors used in the experiment. Let me assume that the resolution energy for high energy experiments depends on the energy of particles. Therefore, for the high energy Coulomb scattering

$$\Delta E = E \exp\left(-\eta(E/m)\right), \tag{36}$$

where $\eta(E/m) > 0$ and can be, for instance, a constant function. This formula, for suitably large η, also contains the condition $\Delta E \ll m$. With this choice of the resolution energy the cross section for the Coulomb scattering is asymptotically equal to [3]

$$\left(\frac{d\sigma}{d\Omega}\right)^{as} = \left(\frac{d\sigma}{d\Omega}\right)_0^{as} \exp\left[\frac{\alpha}{\pi}(5-4\eta)E/m))\ln E/m\right], \tag{37}$$

and the anomalous dimension is equal to

$$\gamma(\alpha, E/m) = -\frac{\alpha}{\pi}(5-4\eta(E/m))+O(\alpha^2). \tag{38}$$

One can ask what is the ΔE from the relativistic point of view. I am not able to answer that, but I can assume that ΔE is a Lorentz scalar. For the electron–electron scattering

$$\Delta E = \sqrt{s}\exp(-\eta(s/m^2)), \tag{39}$$

where $\sqrt{s}$ is the total energy of the initial electrons in the centre of mass system. The cross section is equal to [3]

$$\left(\frac{d\sigma}{dt}\right)^{as} = \left(\frac{d\sigma}{dt}\right)_0^{as} \exp\left[\frac{8\alpha}{\pi}(1-\eta(s/m^2))\ln s/m^2\right], \tag{40}$$

with the anomalous dimension

$$\gamma(\alpha, s/m^2) = -\frac{16\alpha}{\pi}(1-\eta(s/m^2))+O(\alpha^2). \tag{41}$$

The results obtained in this paper are correct for other models of QFT-like scalar QED or QCD. Moreover, these results partially justify the eikonal approximation. This approximation consists in the recipe

$$(p^2+2p\sum_{i=1}^{n} k_i+(\sum_{i=1}^{n} k_i)^2-m^2)^{-1} \rightarrow (p^2+2p\sum_{i=1}^{n} k_i-m^2)^{-1}. \tag{42}$$

We see that this recipe is justified in the leading logarithm approximation.

It is known [9] that the generalized optical theorem (Mueller's theorem) interconnects real and virtual photons. Therefore, one can assume that my statement for real photons can be obtained through the Mueller theorem from the Sudakov results. But it should be

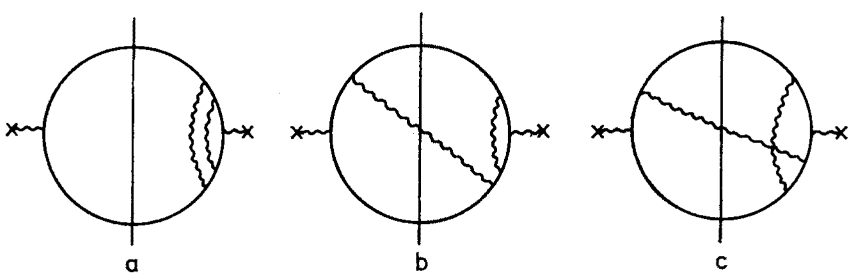

Fig. 7

done cerefully. Let me consider the diagrams of Fig. 7. Cutting the virtual line diagrams 7b and 7c can be obtained from 7a. This means that from the leading diagram it is possible to obtain, through the cutting procedure, a non-leading one. Moreover, my results are more general than those which could be obtained from the Mueller theorem, because they are valid also for processes with the resolution energy $\Delta E < E-m$, whereas the Mueller theorem treats only the semi-inclusive scattering.

I would like to thank Professor I. Białynicki-Birula for critical discussions.

APPENDIX

In this Appendix I show that only $C_{1,1}^{0,r}$ is leading. Using the formula

$$\frac{1}{a^{\alpha}b^{\beta}} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 dx \, \frac{x^{\alpha-1}(1-x)^{\beta-1}}{(xa+(1-x)b)^{\alpha+\beta}} \tag{A1}$$

and performing the integration over photon angles I obtain

$$C_{n,l}^{s,r} = \frac{2\pi\Gamma(n+l+3)}{\Gamma(n)\Gamma(l)\Gamma(4)} E^{2-s}(E-m)^{2(l+n)-3+s}$$

$$\times \int |\vec{p}'|dE' \int_0^1 dxdydz\delta(E-E'-\sum_{i=1}^{l}\omega_i - z(E-m))z^{2(n+l)-3+s}x^{l-1}(1-x)^{n-1}y^{n+l-1}(1-y)^3$$

$$\times \frac{(2q_0+Q+2|\vec{q}|)^{n+l+3}-(2q_0+Q-2|\vec{q}|)^{n+l+3}}{2|\vec{q}|\,(4q^2+4q_0Q+Q^2)^{n+l+3}}, \tag{A2}$$

where

$$Q = (1-y)\,(p-p'-\sum_{i=1}^{r} k_i)^2+\lambda^2, \tag{A3}$$

$$q = (E-m)z((1-y-x)p+(y-x)p'+(1-y)\sum_{i=1}^{r} k_i). \tag{A4}$$

To calculate these integrals asymptotically in the limit $k = E/m \to \infty$ it is convenient to make the Mellin transformation of C. We have

$$\bar{C}(\zeta) = \int_0^{\infty} \kappa^{-\zeta}C(\kappa)d\kappa. \tag{A5}$$

The behavior of $\bar{C}(\zeta)$ at $\zeta \sim 0$ is related to the behavior of $C(\kappa)$ for $\kappa \to \infty$. Specifically, the term ζ^{-n} in $\bar{C}(\zeta)$, $n > 0$, corresponds to the term $[(n-1)!]^{-1}\,\kappa^{-1}(\ln\kappa)^{n-1}$ in $C(\kappa)$. Therefore obtaining the asymptotic form of $C(\kappa)$ for high energies is equivalent to obtaining all the pole terms in $\bar{C}(\zeta)$ at $\zeta = 0$ [2]. The pole terms of $\bar{C}(\zeta)$ come from the integration over the parameters x, y and z [2]. The integration over y is divergent if we put $\lambda = 0$. Therefore, from the y integration we obtain $\ln E/\lambda$. To obtain the second order pole we must have two Feynman parameters. This pole we obtain if the integration in (A2) is over the regions $x \approx 0$, $z \approx 0$ or $x \approx 1$, $z \approx 0$ and $n = l = 1$.

To obtain (20) let me assume that ω_j = maximum $\{\omega_i\}$, therefore,

$$\frac{1}{2p' \sum_{i=1}^{m} k_i} \leqslant \frac{1}{2p'k_j}, \tag{A6}$$

and

$$|(\sum_{i=1}^{m} k_i)^2| \leqslant 2m^2\omega_j^2. \tag{A7}$$

This means that

$$\left| \frac{(\sum_{i=1}^{m} k_i)^2}{2p' \sum_{i=1}^{m} k_i} \right| \leqslant 2m^2 \frac{\omega_j^2}{2p'k_j} \leqslant 2m^2 \sum_{i=1}^{m} \frac{\omega_i^2}{2p'k_i}. \tag{A8}$$

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