RELATIVISTIC CALCULATION OF POLARIZED NUCLEAR MATTER

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The binding energy of nuclear matter with excess of neutrons, of spin-up neutrons, and spin-up protons (characterized by the corresponding parameters, $\alpha_\tau = (N-Z)/A$, $\alpha_n = (N\uparrow - N\downarrow)/A$, and $\alpha_p = (Z\uparrow - Z\downarrow)/A$), contains three symmetry energies: the isospin symmetry energy ε_τ, the spin symmetry energy ε_σ, and the spin–isospin symmetry energy $\varepsilon_{\sigma\tau}$. Relativistic correction to the non-relativistic Skyrme effective interaction to order $1/c^2$ is used in order to calculate the relativistic corrections for the binding energy of polarized nuclear matter. The relativistic corrections to ε_τ, ε_σ and $\varepsilon_{\sigma\tau}$ are found to be -2.06, -2.6 and -0.89 MeV respectively. The relativistic correction to the compression modulus is -10.8 MeV.

1. *Introduction*

The ground-state energy of nuclear matter with an excess of neutrons, spin-up neutrons and spin-up protons was considered by Dabrowski and Haensel (DH) [1, 2], using the K-matrix method and applying the Brueckner–Gammel, the Thaler, the Hamada–Johnston and the soft-core Reid nucleon-nucleon potentials.

Another approach to study the properties of nuclear matter is by using effective interactions [3–5]. Dabrowski [4] analyzed the problem of spin instability of nuclear matter with the Skyrme interaction.

A number of relativistic calculations of nuclear matter can be found in the literature. One approach which stems from field theory and the relativistic Bethe–Salpeter equation might be used to describe the scattering of two-fermions. As this equation is very difficult to solve an approximate equation is given by Blankenbecler and Sugar [6] which is tractable. Relativistic correction for the volume energy (ε_{vol}) of nuclear matter has been calculated by Brown, Jackson and Kuo [7], and Richards, Haftel and Tabakin [8] using Blankenbecler–Sugar equation. In their work a modified potential is refitted to the phase shifts and the resulting change in the B.E./A is computed using nonrelativistic Brueckner calculation. Hence, their results can be considered as a special case of phase-shift equivalent potentials [9–11] and are not related to the effects considered in the present paper. Other field theoretic approaches to the study of many-body problems have been attempted by e.g. Chin [12], Moszkowski and Källman [13], and Boguta and Bodmer [14] which are again irrelevant to the present work.

In the literature a second approach using relativistic quantum mechanics has been discussed extensively [15–18]. In this approach one identifies the Hilbert space of a relativistic system as a representation space of the inhomogeneous Lorentz group (IhLG); then the problem of finding a relativistic theory is equivalent to a search for a set of Hermitian operators satisfying the well known commutation relations for the IhLG [17, 19].

In the present work we attempted to calculate relativistic corrections to the binding energy, symmetry energies ε_x and compression modulus by a method [17, 19] which is based on the requirement that the Hamiltonian, together with the correction, remain approximately invariant to the second order in v/c under Lorentz transformation. This method has been used previously in calculating the relativistic correction for the binding energy of the triton [20]. In the next section the theory and method of calculation are presented. Section 3 gives a summary of the results obtained.

2. *Theory*

Coester, Pieper and Serduke [21] gave previously a detailed discussion of the relativistic corrections in nuclear matter calculations with realistic N—N potentials. They used Bruekner theory supplemented by some requirements of relativisitic invariance. In fact a fully relativistic many-body Hamiltonian within their framework is not available and also is not needed. For our purpose we shall use first order perturbation theory with a purely phenomenological effective Skyrme interaction. Equations (4.18) and (4.19) of the above reference will be enough to calculate corrections for the binding energy of polarized nuclear matter up to $(v/c)^2$ terms [17, 19]. This is justified if all velocities are small compared to the speed of light and that the interaction satisfies the separability condition as discussed by Foldy and Krajcik [22]. The Fermi momentum $k_F \approx 1.36\ \text{fm}^{-1}$ is not large in comparison with the nucleon mass $M \approx 4.8\ \text{fm}^{-1}$; therefore, the motion of nucleons below the Fermi surface can be described non-relativistically. But when two nucleons collide relativistic effects on nucleon motion might not be negligible. However, because the total momentum $\boldsymbol{P}$ involved in the calculation is relatively small ($|\boldsymbol{P}| < k_F < M$), an expansion in terms of P/M will be justified.

In our calculations the use of a Skyrme type of interaction makes the nuclear matter calculations quite simple. However, the Skyrme force being an effective two-body interaction, it includes relativistic as well as many-body effects. This does not worry us as we are lumping our ignorance of the correction for the two-body potential into the phenomenological interaction [19, 22]. We shall be interested only in calculating first order corrections of order $1/c^2$. Other terms of order $1/c^2$ will be ignored for the sake of simplicity (e.g. those given by Eqs. (2) and (3) of reference [22]).

The Skyrme interaction [23] can be written as the sum of a two- and three-body terms, namely

$$\begin{aligned} v_{12} = {} & t_0(1+x_0P^\sigma)\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)+\tfrac{1}{2}\,t_1(\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)k^2+k'^2\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)) \\ & +t_2\boldsymbol{k}'\cdot\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}+iw_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{k}'\wedge\delta(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{k}, \end{aligned} \tag{1}$$

where $k = (\nabla_1 - \nabla_2)/2i$ acts on the right while $k' = -(\nabla_1 - \nabla_2)/2i$ acts on the left. The last term represents the two-body spin-orbit force which is irrelevant in nuclear matter. For the three-body term, we shall use the equivalent two-body term introduced by Dabrowski [4] namely:

$$v_{12}^{(3)} = \tfrac{1}{6}\, t_3(1 + x_3 P^\sigma)\,[\varrho + (\varrho_{n\uparrow} + \varrho_{n\downarrow} - \varrho_{p\uparrow} - \varrho_{p\downarrow})T_3$$
$$+(\varrho_{n\uparrow} - \varrho_{n\downarrow} + \varrho_{p\uparrow} - \varrho_{p\downarrow})S_3 + (\varrho_{n\uparrow} - \varrho_{n\downarrow} + \varrho_{p\uparrow} - \varrho_{p\downarrow})Y], \tag{2}$$

where $T_3 = (\boldsymbol{\tau}_1 + \boldsymbol{\tau}_2)_3/2$, $S_3 = (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)_3/2$ and $Y = (\tau_{13}\sigma_{13} + \tau_{23}\sigma_{23})/2$, $\varrho_{n\uparrow}$, $\varrho_{n\downarrow}$, $\varrho_{p\uparrow}$ and $\varrho_{p\downarrow}$ represent the densities of neutrons with spin up and down and protons with spin up and down respectively. Relativistic effects may be included in a straight forward manner (to second order in v/c) by using the relations [17].

(*i*) the relativistic correction to the kinetic energy is given by

$$\Delta T^{\mathrm{R}} = -\frac{\hbar^4 k_i^4}{8M^3c^2} \tag{3}$$

$\hbar k_i$ is the momentum of particle i,

(*ii*) the relativistic correction to the two-body interaction can be written in the form:

$$\Delta V^{\mathrm{R}} = -\frac{\hbar^2}{4M^2c^2}\left[K^2 + \tfrac{1}{2}(\boldsymbol{k}\cdot\boldsymbol{K})\left(\boldsymbol{K}\cdot\frac{\partial}{\partial \boldsymbol{K}}\right) + \tfrac{1}{2}(\boldsymbol{k}'\cdot\boldsymbol{K})\left(\boldsymbol{K}\cdot\frac{\partial}{\partial \boldsymbol{k}'}\right)\right]\langle \boldsymbol{k}|V|\boldsymbol{k}'\rangle, \tag{4}$$

where V is the two-body potential, $\boldsymbol{k}$ and $\boldsymbol{K}$ are the relative and centre of mass momenta (in units of $\hbar$) respectively.

Polarized nuclear matter

The expression of the binding energy of nuclear matter composed of $N\uparrow(N\downarrow)$ neutrons with spin-up (down) and $Z\uparrow(Z\downarrow)$ protons with spin-up (down), with corresponding Fermi momenta $k_n(\lambda_n)$ and $k_p(\lambda_p)$ can be written in the form [4]

$$E/A = \varepsilon_{\mathrm{vol}} + \tfrac{1}{2}(\varepsilon_\tau \alpha_\tau^2 + \varepsilon_\sigma \alpha_\sigma^2 + \varepsilon_{\sigma\tau}\alpha_{\sigma\tau}^2), \tag{5}$$

where α_τ is the isospin excess parameter, α_σ is the spin excess parameter, and $\alpha_{\sigma\tau}$ is the spin–isospin excess parameter,

$$\alpha_\tau = (N-Z)/A, \quad \alpha_\sigma = (N\uparrow + Z\uparrow - N\downarrow - Z\downarrow)/A$$
$$\alpha_{\sigma\tau} = (N\uparrow + Z\downarrow - N\downarrow - Z\uparrow)/A. \tag{6}$$

Terms higher than quadratic in α_x ($x = \tau, \sigma, \sigma\tau$) are neglected in (5). $\varepsilon_{\mathrm{vol}}$ is the volume energy.

The expressions of ε_x are given before by Dabrowski [4], therefore we shall give here only their relativistic corrections:

$$\Delta\varepsilon_{\mathrm{vol}}^{\mathrm{R}} = -\tfrac{3}{56}\frac{\hbar^4 k_{\mathrm{F}}^4}{M^3c^2} - \tfrac{9}{80}\frac{\hbar^2 k_{\mathrm{F}}^2}{M^2c^2}\varrho t_0$$
$$-\tfrac{27}{560}\frac{\hbar^2 k_{\mathrm{F}}^4}{M^2c^2}\varrho(\tfrac{3}{5}t_1 - t_2) - \tfrac{3}{160}\frac{\hbar^2 k_{\mathrm{F}}^2}{M^2c^2}\varrho^2 t_3 \tag{7}$$

and

$$\Delta\varepsilon_x^{\mathrm{R}} = -\tfrac{1}{6}\frac{\hbar^4 k_{\mathrm{F}}^4}{M^3c^2} - \tfrac{1}{24}\frac{\hbar^2 k_{\mathrm{F}}^4}{M^2c^2}\,\varrho(\tfrac{7}{5}\,t_1 - 5t_2)$$

$$+\tfrac{1}{4}\frac{\hbar^2 k_{\mathrm{F}}^2}{M^2c^2}\,\varrho t_0 x_0 \begin{Bmatrix} +1 \\ -1 \\ 0 \end{Bmatrix} + \frac{\hbar^2 k_{\mathrm{F}}^2}{M^2c^2}\,\varrho^2 t_3 \begin{Bmatrix} (x_3+4)/120 \\ -(9x_3+4)/120 \\ \frac{1}{30} \end{Bmatrix} \tag{8}$$

for

$$x = \begin{Bmatrix} \tau \\ \sigma \\ \sigma\tau \end{Bmatrix}.$$

Due to the unitarity of the Wigner rotation matrices [21] the spin traces appearnig in the ground state energy expressions of unpolarized nuclear matter are not affected. However, in the case of polarized nuclear matter there is no effect of these matrices on the spin traces performed in a given reference frame in a spin one and/or spin zero subspaces. This can be seen from the properties of the Wigner rotation matrices [18, 21, 24]. Also using Eqs. (2.8), (4.18) and (4.19) of Ref. [21] we can see that such partial traces due to the Wigner matrices are not needed to be calculated in the case of polarized nuclear matter since they will only add a little contribution comparing with the main terms calculated according to Eq. (4). One final remark is that in the presence of spin and/or isospin dependent two-body potential, the relativistic (order $1/c^2$) interaction must generally include three-body as well as two-body terms in order to have relativistic invariance as well as separability to the indicated order [22]. We only calculated the two-body terms given by equation one of the above reference which is identical to those given by references [17] and [19] and neglecting those corrections given by Eqs. (2) and (3) of that reference. In fact as pointed out by Foldy and Krajcik [22] the theory of relativistic corrections for the many-body system is still suffering from a substantial degree of arbitrariness and the need for further physical input is clearly needed before any truly reliable calculations can be carried out.

3. *Results and discussions*

The parameters of the Skyrme force are taken to be (SIII force) [25]: $t_0 = -1128.75$ MeV fm^3, $t_1 = 395.0$ MeV fm^5, $t_2 = -95.0$ MeV fm^5, $t_3 = 14000$ MeV fm^6 and $x_0 = 0.45$. This SIII force leads to saturation of nuclear matter at $k_{\mathrm{F}} = 1.29$ fm^{-1}. The values of $\varepsilon_{\mathrm{vol}}$, ε_τ, ε_σ and $\varepsilon_{\sigma\tau}$ are respectively -15.9, 56, 81 and 69 MeV and their relativistic corrections are 0.39, -2.06, -2.60, -0.89 MeV.

To our knowledge the relativistic corrections to ε_x have not been previously calculated. However, it is clear from these calculations that the relativistic corrections are small and this justifies the use of perturbation theory to calculate these parameters.

The relativistic correction for the compression modulus is -10.8 MeV. The relativistic correction to the compression modulus given by Coester et al. [21] changes in magnitude as well as sign. This shows that the corrections to the compression modulus depend strongly on the phenomenological potential chosen to describe the N–N interaction.

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