ΛNN CORRELATIONS AND THE Λ-PARTICLE BINDING IN NUCLEAR MATTER*

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The Λ particle energy in nuclear matter is calculated with separable S state ΛN and NN potentials of Puff's type. By solving the Bethe-Faddeev equations, the three-body ΛNN cluster energy $E_{\Lambda 3}$ is calculated with the repulsive result $E_{\Lambda 3} \cong 3$–4 MeV, which is less than 10% of the magnitude of the two-body ΛN cluster energy. The result suggests a satisfactory convergence of the reaction matrix method of calculating B_Λ.

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1. Introduction

The binding energy of a Λ-particle in nuclear matter (NM), B_Λ, is a quantity of considerable interest in the phenomenological analysis of the Λ-nucleon interactions $V_{\Lambda N}$ (see, e.g., the review [1]). Most of the existing calculations of B_Λ have used the low-order Brueckner reaction matrix method (LOB). By LOB, we understand a reaction matrix calculation within the two-hole-line approximation, and with the "standard choice" of pure kinetic single particle (s.p.) energies in the intermediate states in the equation for the reaction matrix. The LOB is the first step in the hole-line expansion in which energy diagrams are grouped according to the number of hole-lines, i.e., to the number of interacting particles. In the case of pure NM, it is essential to include three-hole-line diagrams, as was demonstrated by the extensive calculations by Day [2]. In the case of B_Λ, the contribution of the three-hole-line diagrams has never been calculated with a sufficient accuracy. A rough estimate was given in [3], and an approximate calculation for pure

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attractive ΛN and NN interactions was presented in [4]. Only the so called rearrangement term, a special part of the three-body ΛNN contribution to B_Λ, was considered in [5].

In the present paper, we present a calculation of B_Λ, which includes all the two- and three-hole-line diagrams, i.e., two-body ΛN and three-body ΛNN correlations. The contribution of the three-hole-line diagrams is expressed in terms of solutions of the Bethe–Faddeev (BF) equations. To simplify these equations, we assume for both ΛN and NN interactions a separable form of Puff's [6] type. Both interactions are spin-independent, contain a hard shell repulsion and the Yamaguchi [7] type attraction, and act only in the S state.

In case of pure NM with separable spin-independent NN interaction, three-body NNN correlations have been considered by Bhakar and McCarthy [8], whose procedure is followed in the present work. Modification of the procedure for the problem of B_Λ has been outlined in [4].

The paper is organized as follows. In the next Section, we outline our formalism of calculating $-B_\Lambda$ which consists of three parts: the LOB part $E_{\Lambda 2}$, the three-body cluster energy $E_{\Lambda 3}$, and the rearrangement energy E_R. In particular, we write the BF equations for determining $E_{\Lambda 3}$. In Section 3, the formalism is applied to separable S state interactions which introduce drastic simplifications, in particular in calculating $E_{\Lambda 3}$. The two-body NN and ΛN interactions used in our calculations are described in Section 5. Our results are presented and discussed in Section 6. Our notation and kinematical relations are explained in Appendix A, and expressions for two-body t matrices in case of separable two-body interactions are given in Appendix B.

2. *Formalism*

2.1. General scheme

The binding energy B_Λ is defined by

$$-B_\Lambda = E(\Lambda+\text{NM})-E(\text{NM}) = E_\Lambda+E_R, \tag{2.1}$$

where E(NM) and $E(\Lambda+\text{NM})$ are the ground state energies of NM and of the Λ+NM system. We calculate $E(\Lambda+\text{NM})$ and E(NM) with the reaction matrix method and restrict ourselves to two- and three-hole-line diagrams, i.e., to contributions from interactions between two (ΛN and NN) and three (ΛNN and NNN) particles. Contributions to $E(\Lambda+\text{NM})$, which involve only nucleons, cancel the corresponding contributions to E(NM), and we are left with only those contributions to $E(\Lambda+\text{NM})$, denoted by E_Λ, which involve the Λ particle. However, this cancellation is not complete because the NN reaction matrices in pure NM and in the Λ+NM system differ slightly. This difference produces the rearrangement energy E_R [5].

With the help of the three-particle ΛNN reaction matrix T we may write

$$E_\Lambda = \tfrac{1}{2}\sum_{p_1 p_3}^{<k_F} (p_1 p_2 p_3|T(16-4P_{13})\,|p_1 p_2 p_3), \tag{2.2}$$

where particles 1, 3 are nucleons, and particle 2 is the Λ hyperon, P_{13} is the exchange operator of the spatial coordinates of the two nucleons, k_F is the Fermi momentum, and p_i is the momentum of the i-th particle. Obviously, we have $p_2 = 0$. We assume that all interactions are spin- and isospin-independent. In this case, summation over the nucleon spin- and isospin-states in the direct and exchange term leads to the factors 16 and 4 respectively.

We decompose T into

$$T = \sum_{i,j=1,2,3} T^{ij}, \tag{2.3}$$

where T^{ij} is the part of T for which particles j and i are spectators of the first and last interactions respectively. The T^{ij} satisfy the BF equations:

$$T^{ij} = \delta_{ij} t^i + t^i (Q/e) \sum_{k \neq i} T^{kj}, \tag{2.4}$$

where t^i is the two-body reaction matrix for an interaction in which the particle i does not take part, i.e., is a spectator, Q is the exclusion principle operator, i.e., a projection operator onto nucleon states above the Fermi momentum, and $-e$ is the excitation energy.

To specify the off-energy-shell character of t^i in Eq. (2.4) and the value of e, it is convenient to iterate Eq. (2.4) twice. If we do it, and take advantage of the identity of the two nucleons, we get:

$$E_\Lambda = E_{\Lambda 2} + E_{\Lambda 3}, \tag{2.5}$$

where

$$E_{\Lambda 2} = V_\Lambda = -B_{\Lambda 2} = 4 \sum_{p_1}^{<k_F} (p_1 p_2 | t^3 | p_1 p_2), \tag{2.6}$$

$$E_{\Lambda 3} = -B_{\Lambda 3} = 16 E_D - 4 E_x, \tag{2.7}$$

where the direct term

$$\begin{aligned} E_D = \sum_{p_1 p_3}^{<k_F} (p_1 p_2 p_3 | t^1 (Q/e) [2(T^{21} + T^{23} + T^{31} + T^{33})(Q/e) t^2 \\ + (2T^{21} + T^{31} + T^{22})(Q/e) t^3 + (2T^{23} + T^{33} + T^{22})(Q/e) t^1] \\ + t^2 (Q/e)(T^{13} + T^{33})(Q/e) t^2 | p_1 p_2 p_3), \end{aligned} \tag{2.8}$$

and the exchange term E_x is given by the expression which differs from (2.8) only by the appearance of the P_{13} operator.

2.2. Expression for $E_{\Lambda 2}$

Let us specify the two-body reaction matrices t^i. The NN reaction matrix $t^2 = t_{NN}$ in the relative NN momentum representation satisfies the equation:

$$\begin{aligned} \langle p' | t_{NN}(P_{NN}, z_{NN}) | p \rangle = \langle p' | v_{NN} | p \rangle + (2\pi)^{-3} \int dp'' \langle p' | v_{NN} | p'' \rangle \\ [Q_{NN}(P_{NN}, p'')/(z_{NN} - p''^2/m_N)] \langle p'' | t_{NN}(P_{NN}, z_{NN}) | p \rangle, \end{aligned} \tag{2.9}$$

where $\boldsymbol{P}_{NN}$ is the c.m. momentum of the two nucleons, and

$$Q_{NN}(\boldsymbol{P}, \boldsymbol{p}) = Q_F(\tfrac{1}{2}\boldsymbol{P}+\boldsymbol{p})Q_F(\tfrac{1}{2}\boldsymbol{P}-\boldsymbol{p}), \tag{2.10}$$

where

$$Q_F(x) = \begin{cases} 1 & \text{for} \quad x > k_F \\ 0 & \text{for} \quad x \leqslant k_F \end{cases}. \tag{2.11}$$

The NΛ reaction matrices $t^1 = t^3 = t_{N\Lambda}$ in the relative NΛ momentum representation satisfy the equation:

$$\langle \boldsymbol{p}'|t_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, z_{N\Lambda})|\boldsymbol{p}\rangle = \langle \boldsymbol{p}'|v_{N\Lambda}|\boldsymbol{p}\rangle + (2\pi)^{-3}\int d\boldsymbol{p}''\langle \boldsymbol{p}'|v_{N\Lambda}|\boldsymbol{p}''\rangle$$
$$\times [Q_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, \boldsymbol{p}'')/(z_{N\Lambda}-p''^2/2\mu_{\Lambda N})]\langle \boldsymbol{p}''|t_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, z_{N\Lambda})|\boldsymbol{p}\rangle, \tag{2.12}$$

where $\boldsymbol{P}_{N\Lambda}$ is the c.m. momentum ($\boldsymbol{P}_{N\Lambda} = \boldsymbol{p}_N+\boldsymbol{p}_\Lambda$), $\boldsymbol{p} = (m_\Lambda \boldsymbol{p}_N - m_N \boldsymbol{p}_\Lambda)/(m_N+m_\Lambda)$, $\mu_{\Lambda N} = m_\Lambda m_N/(m_\Lambda+m_N)$, and the exclusion principle operator

$$Q_{N\Lambda}(\boldsymbol{P}, \boldsymbol{p}) = Q_F(\mu_{\Lambda N}\boldsymbol{P}/m_\Lambda+\boldsymbol{p}). \tag{2.13}$$

The t-matrices in Eqs (2.6) and (2.8) describe scattering of two particles in NM with no other particle being excited. Consequently, these reaction matrices are on the energy shell. As the energy arguments in the matrices t^3_{ON} and t^1_{ON} we have:

$$z_{N\Lambda} = V_\Lambda + e_N(p_N) - \boldsymbol{P}^2_{N\Lambda}/2(m_N+m_\Lambda). \tag{2.14}$$

For the single-nucleon spectrum in NM, we assume the effective mass approximation:

$$e_N(p_N) = A_N + p_N^2/2m_N^* \quad (\text{for } p_N < k_F). \tag{2.15}$$

Now, we replace p_N^2 by its average value in the Fermi sea, $\langle p_N^2\rangle = 0.6\,k_F^2$, and $P^2_{N\Lambda}$ by $\langle P^2_{N\Lambda}\rangle = 0.6\,k_F^2$ (notice that $P_{N\Lambda} = p_N$, since $p_\Lambda = 0$), similarly P^2_{NN} by $\langle P^2_{NN}\rangle = 1.2\,k_F^2$. This means we apply the approximation:

$$p_N \cong \sqrt{0.6}\,k_F, \qquad P_{N\Lambda} \cong \sqrt{0.6}\,k_F, \qquad P_{NN} \cong \sqrt{1.2}\,k_F. \tag{2.16}$$

(The accuracy of an average excitation energy in LOB calculation of B_Λ was tested in [9].)

With approximation (2.16), we get

$$z_{N\Lambda} \cong -\gamma_\Lambda = V_\Lambda + A_N + 0.3k_F^2[1/m_N^* - 1/(m_N+m_\Lambda)]. \tag{2.17}$$

Similarly, as the energy argument in the matrix t^2_{ON} with approximations (2.16), we get

$$z_{NN} \cong -\gamma_N = 2A_N + 0.3k_F^2(2/m_N^* - 1/m_N). \tag{2.18}$$

Expression (2.6) for $E_{\Lambda 2}$ takes the final form:

$$E_{\Lambda 2} = V_\Lambda = 4(m_N/\mu_{\Lambda N})^3(2\pi)^{-3}\int d\boldsymbol{p}\langle \boldsymbol{p}|t_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, -\gamma_\Lambda)|\boldsymbol{p}\rangle. \tag{2.18$'$}$$

Since $p_\Lambda = 0$, we have $\boldsymbol{P}_{N\Lambda} = \boldsymbol{p}_N = m_N\boldsymbol{p}/\mu_{\Lambda N}$.

2.3. Expression for $E_{\Lambda 3}$

Expression (2.8) for E_D contains terms of the general form:

$$E_D(i, jk, l) = \sum_{p_1 p_3}^{<k_F} (p_1 p_2 p_3 | t^i (Q/e) T^{jk} (Q/e) t^l | p_1 p_2 p_3). \tag{2.19}$$

By introducing the vectors q_n, k_n ($n = i, j, k, l$) of Eq. (A1), and the notation and relations explained in Appendix A, we get:

$$E_D(i, jk, l) = (M/m_\Lambda)^3 (2\pi)^{-12} \int_{<k_F} dq_1 \int_{<k_F} dq_3 \int db \int dc$$
$$\times \langle k_i | t^i(P_i, -\gamma_i) | \varepsilon_{ij}(b + m_j q_i/M_i) \rangle \{Q_i[P_i, \varepsilon_{ij}(b + m_j q_i/M_i)]/[\gamma_i + (b + m_j q_i/M_i)^2/2\mu_i]\}$$
$$\times T^{jk}[b, -\varepsilon_{ij}(q_i + m_i b/M_j); c, -\varepsilon_{lk}(q_l + m_l c/M_k)]$$
$$\times \{Q_l[P_l, \varepsilon_{lk}(c + m_k q_l/M_l)]/[\gamma_l + (c + m_k q_l/M_l)^2/2\mu_l]\} \langle \varepsilon_{lk}(c + m_k q_l/M_l) | t^l(P_l, -\gamma_l) | k_l \rangle, \tag{2.20}$$

where

$$\mu_i = m_j m_k/(m_j + m_k), \quad \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{31} = 1, \quad \varepsilon_{21} = \varepsilon_{13} = \varepsilon_{32} = -1, \tag{2.21}$$

and

$$k_1 = -\mu_{\Lambda N} q_1/m_\Lambda - q_3, \quad k_3 = q_1 + \mu_{\Lambda N} q_3/m_\Lambda,$$
$$k_2 = -\tfrac{1}{2} q_1 + \tfrac{1}{2} q_3, \quad q_2 = -q_1 - q_3. \tag{2.21'}$$

Expressions similar to (2.20) may be derived for terms

$$E_x(i, jk, l) = \sum_{p_1 p_3}^{<k_F} (p_1 p_2 p_3 | t^i (Q/e) T^{jk} (Q/e) t^l P_{13} | p_1 p_2 p_3), \tag{2.22}$$

contained in expressions for the exchange energy E_x of Eq. (2.7).

The T^{ij} matrices in expression (2.8) are determined by Eqs (2.4) with off-energy-shell two-body t-matrices, Eqs (A.9–12). With the help of the notations, and relations explained in Appendix A, Eqs (2.4) lead to the following equations for T^{ij} $(q_b k_b; q_a k_a)$:

$$T^{ij}(q_b k_b; q_a k_a) = \delta_{ij} (2\pi)^3 \delta(q_b - q_a) \langle k_b | t^i[q_b] | k_a \rangle - \sum_{k \neq i} (2\pi)^{-3} \int dq$$
$$\times \langle k_b | t^i[q_b] | \varepsilon_{ik}(q + m_k q_b/M_i) \rangle D_{ik}^{-1}(q_b q) T^{kj}(q, -\varepsilon_{ik}(q_b + m_i q/M_k); q_a k_a), \tag{2.23}$$

where

$$D_{ik}(q_b q) = \Gamma + \tfrac{1}{2} [q_b^2/m_i + (q_b + q)^2/(M - m_i - m_k) + q^2/m_k]. \tag{2.24}$$

Equations (2.23) involve two approximations: neglect of the exclusion principle (we drop the Q operators in (2.4), and in equations for $t^i[q_b]$), and use of average nucleon energies in the Fermi sea (in calculating e in (2.4), Eq. (2.24), and in equations for $t^i[q_b]$, Eq. (A. 11)). Notice that we keep the exclusion principle operators in expression (2.8).

Neglecting the exclusion principle is justified by the fact that the T^{ij} matrices in (2.8) describe scattering of particles in excited states where the exclusion principle is less important. The use of the average energies of the occupied states appears reasonable because in calculating $E_{\Lambda 3}$ we have to sum over these states (an analogical approximation in calculating $E_{\Lambda 2}$ was tested in [9]).

2.4. Expression for E_R

A detailed derivation of the expression for the rearrangement energy E_R has been presented in [5] (compare also [9]). Here, we assume that the NN reaction matrix equation contains pure kinetic energies in the intermediate states, and thus only the single-nucleon energies $e_N(p_N)$ of the occupied states are affected by the presence of the Λ particle. Consequently, by applying the simple and accurate approximation of [5], we may write

$$E_R = -\kappa_{NN} E_{\Lambda 2}, \tag{2.25}$$

where κ_{NN} is the wound integral of NM. The general expression for κ_{NN} is given in [9]. In case of spin-independent S-state interaction, we may write it in the form:

$$\kappa_{NN} = \tfrac{3}{4}\,\varrho(2\pi)^{-3}\langle \textstyle\int d\boldsymbol{p} Q_{NN}(\boldsymbol{P}_{NN}, \boldsymbol{p})\,[\langle \boldsymbol{p}| t_{NN}(\boldsymbol{P}_{NN}, -\gamma_N)\,|\boldsymbol{p}\rangle/(\gamma_N + p^2/m_N)]^2\rangle, \tag{2.26}$$

where $\langle\ \rangle$ denotes the average value in the Fermi sea.

In case of the Λ+NM system, we have two wound integrals: κ_{NN} and $\kappa_{\Lambda N}$ given by:

$$\kappa_{\Lambda N} = \tfrac{3}{4}\,\varrho(2\pi)^{-3}\langle \textstyle\int d\boldsymbol{p} Q_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, \boldsymbol{p})\,[\langle \boldsymbol{p}| t_{N\Lambda}(\boldsymbol{P}_{N\Lambda}, -\gamma_\Lambda)\,|\boldsymbol{p}\rangle/(\gamma_\Lambda + p^2/2\mu_{\Lambda N})]^2\rangle. \tag{2.27}$$

3. *Separable interactions*

Now we assume that $v_{\Lambda N}$ and v_{NN} are separable S-state interactions, Eq. (B.1). For the $t^i[q]$ matrices in Eq. (2.23), we have now, Eq. (B.2):

$$\langle \boldsymbol{k}'| t^i[q]\,|\boldsymbol{k}\rangle = \sum_{\lambda\nu} \mathscr{D}^i_{\lambda\nu}[q] g^i_\lambda(k') g^i_\nu(k), \tag{3.1}$$

where $\mathscr{D}^i_{\lambda\nu}[q]$ are determined from Eqs (B.4) with

$$\mathscr{I}^i_{\lambda\nu}[q] = (2\pi)^{-3} \textstyle\int d\boldsymbol{p} g_\lambda(p) g_\nu(p)/[\Gamma + q^2/2\tilde{\mu}_i + p^2/2\mu_i]. \tag{3.2}$$

This means, the $t^i[q]$ matrices are obtained from the general $t^i(P, z)$ matrices of Appendix B by making the special choice of z, Eq. (A.11), and by using the approximation $Q_i \cong 1$.

To solve Eqs (2.23), we make the Ansatz:

$$T^{ij}(\boldsymbol{q}_b \boldsymbol{k}_b;\, \boldsymbol{q}_a \boldsymbol{k}_a) = \sum_{\lambda\nu} g^i_\lambda(k_b) A^{ij}_{\lambda\nu}(\boldsymbol{q}_b \boldsymbol{q}_a) g^i_\nu(k_a), \tag{3.3}$$

and obtain for the functions $A^{ij}_{\lambda\nu}$ the system of integral equations,

$$A^{ij}_{\lambda\nu}(\boldsymbol{ba}) = \delta_{ij}(2\pi)^3\delta(\boldsymbol{b}-\boldsymbol{a})\mathscr{D}^i_{\lambda\nu}[b] - \sum_{\nu'\lambda'} \mathscr{D}^i_{\lambda\nu'}[b] \sum_{k\neq i} (2\pi)^{-3} \textstyle\int d\boldsymbol{q} K^{ik}_{\nu'\lambda'}(\boldsymbol{bq}) A^{kj}_{\lambda'\nu}(\boldsymbol{qa}), \tag{3.4}$$

where

$$K^{ij}_{\lambda\nu}(\boldsymbol{ab}) = g^i_\lambda(|\boldsymbol{b}+m_j\boldsymbol{a}/M_i|)D^{-1}_{ij}(\boldsymbol{ab})g^j_\nu(|\boldsymbol{a}+m_i\boldsymbol{b}/M_j|). \tag{3.5}$$

The kernels of integral equations (3.4) satisfy the general obvious relation:

$$K^{ij}_{\lambda\nu}(\boldsymbol{ab}) = K^{ji}_{\nu\lambda}(\boldsymbol{ba}), \tag{3.6}$$

and the symmetry relations which follow from the identity of the two nucleons (particles 1 and 3):

$$K^{13}_{\lambda\nu}(\boldsymbol{ab}) = K^{31}_{\lambda\nu}(\boldsymbol{ab}) \equiv K^{\mathrm{NN}}_{\lambda\nu}(\boldsymbol{ab}), \tag{3.7}$$

$$K^{23}_{\lambda\nu}(\boldsymbol{ab}) = K^{21}_{\lambda\nu}(\boldsymbol{ab}) \equiv K^{\Lambda\mathrm{N}}_{\lambda\nu}(\boldsymbol{ab}), \tag{3.8}$$

$$K^{12}_{\lambda\nu}(\boldsymbol{ab}) = K^{32}_{\lambda\nu}(\boldsymbol{ab}) \equiv K^{\mathrm{N}\Lambda}_{\lambda\nu}(\boldsymbol{ab}). \tag{3.9}$$

By combining these symmetry relations with relation (3.6), we get

$$K^{\mathrm{NN}}_{\lambda\nu}(\boldsymbol{ab}) = K^{\mathrm{NN}}_{\nu\lambda}(\boldsymbol{ba}), \tag{3.10}$$

$$K^{\mathrm{N}\Lambda}_{\lambda\nu}(\boldsymbol{ab}) = K^{\Lambda\mathrm{N}}_{\nu\lambda}(\boldsymbol{ba}). \tag{3.11}$$

Because of the above symmetry properties of the kernels, the functions $A^{ij}_{\lambda\nu}$ are not independent. Namely, we have:

$$A^{33}_{\lambda\nu}(\boldsymbol{ab}) = A^{11}_{\lambda\nu}(\boldsymbol{ab}) \equiv A^{\mathrm{NN}}_{\lambda\nu}(\boldsymbol{ab}), \tag{3.12}$$

$$A^{13}_{\lambda\nu}(\boldsymbol{ab}) = A^{31}_{\lambda\nu}(\boldsymbol{ab}) \equiv A^{\mathrm{N}\neq\mathrm{N}}_{\lambda\nu}(\boldsymbol{ab}), \tag{3.13}$$

$$A^{23}_{\lambda\nu}(\boldsymbol{ab}) = A^{21}_{\lambda\nu}(\boldsymbol{ab}) \equiv A^{\Lambda\mathrm{N}}_{\lambda\nu}(\boldsymbol{ab}), \tag{3.14}$$

$$A^{12}_{\lambda\nu}(\boldsymbol{ab}) = A^{32}_{\lambda\nu}(\boldsymbol{ab}) \equiv A^{\mathrm{N}\Lambda}_{\lambda\nu}(\boldsymbol{ab}). \tag{3.15}$$

The functions A^{NN}, $A^{\mathrm{N}\neq\mathrm{N}}$, $A^{\Lambda\mathrm{N}}$, $A^{\mathrm{N}\Lambda}$, and $A^{\Lambda\Lambda} = A^{22}$ are determined by the following integral equations, obtained from Eqs (3.4):

$$A^{(-)}_{\lambda\nu}(\boldsymbol{ba}) = (2\pi)^3\delta(\boldsymbol{b}-\boldsymbol{a})\mathscr{D}^{\mathrm{N}}_{\lambda\nu}[b] + \sum_{\nu'\lambda'} \mathscr{D}^{\mathrm{N}}_{\lambda\nu'}[b]\,(2\pi)^{-3}\int d\boldsymbol{q}K^{\mathrm{NN}}_{\nu'\lambda'}(\boldsymbol{bq})A^{(-)}_{\lambda'\nu}(\boldsymbol{qa}), \tag{3.16}$$

$$A^{(+)}_{\lambda\nu}(\boldsymbol{ba}) = (2\pi)^3\delta(\boldsymbol{b}-\boldsymbol{a})\mathscr{D}^{\mathrm{N}}_{\lambda\nu}[b]$$

$$-\sum_{\nu'\lambda'} \mathscr{D}^{\mathrm{N}}_{\lambda\nu'}[b]\,(2\pi)^{-3}\int d\boldsymbol{q}\{K^{\mathrm{NN}}_{\nu'\lambda'}(\boldsymbol{bq})A^{(+)}_{\lambda'\nu}(\boldsymbol{qa}) + 2K^{\mathrm{N}\Lambda}_{\nu'\lambda'}(\boldsymbol{bq})A^{\Lambda\mathrm{N}}_{\lambda'\nu}(\boldsymbol{qa})\}, \tag{3.17}$$

$$A^{\Lambda\mathrm{N}}_{\lambda\nu}(\boldsymbol{ba}) = -\sum_{\nu'\lambda'} \mathscr{D}^{\Lambda}_{\lambda\nu'}[b]\,(2\pi)^{-3}\int d\boldsymbol{q}K^{\Lambda\mathrm{N}}_{\nu'\lambda'}(\boldsymbol{bq})A^{(+)}_{\lambda'\nu}(\boldsymbol{qa}), \tag{3.18}$$

$$A^{\mathrm{N}\Lambda}_{\lambda\nu}(\boldsymbol{ba}) = -\sum_{\nu'\lambda'} \mathscr{D}^{\mathrm{N}}_{\lambda\nu'}[b]\,(2\pi)^{-3}\int d\boldsymbol{q}\{K^{\mathrm{NN}}_{\nu'\lambda'}(\boldsymbol{bq})A^{\mathrm{N}\Lambda}_{\lambda'\nu}(\boldsymbol{qa}) + K^{\mathrm{N}\Lambda}_{\nu'\lambda'}(\boldsymbol{bq})A^{\Lambda\Lambda}_{\lambda'\nu}(\boldsymbol{qa})\}, \tag{3.19}$$

$$A^{\Lambda\Lambda}_{\lambda\nu}(\boldsymbol{ba}) = (2\pi)^3\delta(\boldsymbol{b}-\boldsymbol{a})\mathscr{D}^{\Lambda}_{\lambda\nu}[b] - 2\sum_{\nu'\lambda'} \mathscr{D}^{\Lambda}_{\lambda\nu'}[b]\,(2\pi)^{-3}\int d\boldsymbol{q}K^{\Lambda\mathrm{N}}_{\nu'\lambda'}(\boldsymbol{bq})A^{\mathrm{N}\Lambda}_{\lambda'\nu}(\boldsymbol{qa}), \tag{3.20}$$

where $\mathscr{D}^{\mathrm{N}} = \mathscr{D}^1 = \mathscr{D}^3$, $\mathscr{D}^{\Lambda} = \mathscr{D}^2$, and

$$A^{(\pm)}_{\lambda\nu}(\boldsymbol{ba}) = A^{\mathrm{NN}}_{\lambda\nu}(\boldsymbol{ba}) \pm A^{\mathrm{N}\neq\mathrm{N}}_{\lambda\nu}(\boldsymbol{ba}). \tag{3.21}$$

Eqs (3.17) and (3.18) are coupled. By expressing $A^{\Lambda N}_{\lambda\nu}$ on the right hand side of (3.17) in terms of $A^{(+)}_{\lambda\nu}$, Eq. (3.18), one obtains equations involving only $A^{(+)}_{\lambda\nu}$, in which part of the kernel is a convolution of the kernels $K^{N\Lambda}$ and $K^{\Lambda N}$. These equations are still coupled with respect to the index λ, i.e., we have two pairs of coupled equations for $A^{(+)}_{1\nu}$, $A^{(+)}_{2\nu}$ ($\nu = 1,2$). After solving these equations for $A^{(+)}_{\lambda\nu}$, we may calculate $A^{\Lambda N}_{\lambda\nu}$ from Eq. (3.18). Eqs (3.19) and (3.20) may be treated similarly. In this way we obtain equations for $A^{N\Lambda}_{\lambda\nu}$. After solving them, we may calculate $A^{\Lambda\Lambda}_{\lambda\nu}$ from Eq. (3.20). This procedure was followed in our calculations.

There are additional symmetry relations which reduce the number of independent functions. By looking into iterative solutions of Eqs (3.4), or by applying — instead of (2.4) — equivalent equations

$$T^{ij} = \delta_{ij}t^i + \sum_{k\neq i} T^{ik}(Q/e)t^j, \tag{3.22}$$

e may see easily that

$$\begin{aligned} 4^{\Lambda N}_{\lambda\nu}(\boldsymbol{ba}) &= A^{N\Lambda}_{\nu\lambda}(\boldsymbol{ab}),\\ A^{NN}_{\lambda\nu}(\boldsymbol{ba}) &= A^{NN}_{\nu\lambda}(\boldsymbol{ab}),\\ A^{N\neq N}_{\lambda\nu}(\boldsymbol{ba}) &= A^{N\neq N}_{\nu\lambda}(\boldsymbol{ab}),\\ A^{\Lambda\Lambda}_{\lambda\nu}(\boldsymbol{ba}) &= A^{\Lambda\Lambda}_{\nu\lambda}(\boldsymbol{ab}). \end{aligned} \tag{3.23}$$

In the procedure followed in the present paper, relations (3.23) were used as test of the accuracy of our calculations.

Let us mention that with our S-state separable interactions, almost all the exchange terms, Eq. (2.22), are equal to the "corresponding" direct terms, Eq. (2.19), and Eq. (2.7) takes the form:

$$E_{\Lambda 3} = 12E_D + 4\{E_D(1, 31, 3) + E_D(1, 33, 1) - E_x(1, 31, 3) - E_x(1, 33, 1)\}. \tag{3 24}$$

4. *Two-body interactions*

Both NN and ΛN interactions are assumed to be spin-independent S-state separable interactions of rank two, with one ($\nu = 1$) repulsive and one ($\nu = 2$) attractive term. For the repulsive part, we assume the hard shell form [6], and for the attractive part the Yamaguchi [7] form:

$$g^i_\nu(p) = \begin{cases} \sin(pc_{YN})/p, & \nu = 1,\\ 1/(\beta^2_{YN}+p^2), & \nu = 2, \end{cases} \tag{4.1}$$

where Y $= \Lambda$ for $i = 1,3$ and Y $=$ N for $i = 2$. For the strength parameter of the repulsive part ($\nu = 1$) we take the limit $\lambda^i_\nu \to \infty$. This v_{YN} Puff potential has three adjustible parameters: the hard shell radius c_{YN}, the strength parameter $\lambda^i_2 = \lambda_{YN}$ and the range parameter β_{YN} of the attractive part.

For the parameters of the NN interaction, we take the values given by Bhakar and McCarthy [8]:

$$c_{NN} = 0.45 \text{ F}, \quad \beta_{NN} = 2.2785 \text{ F}^{-1}, \quad m_N\lambda_{NN}/(2\pi)^3 = -6.35 \text{ F}^{-3}. \tag{4.2}$$

This NN potential has the hard shell radius c_{NN} of the original Puff [6] potential, and it yields a binding energy per nucleon in NM of $\sim$ 16 MeV at the empirical density. The parameters β_{NN} and λ_{NN} are approximate averages of the corresponding spin dependent parameters of the original Puff potential. The parameters of the single-nucleon spectrum, which correspond to our NN interaction, are [8]:

$$m_N A_N = -2.346 \text{ F}^{-2}, \quad m_N^*/m_N = 0.5373. \tag{4.3}$$

In choosing our ΛN interaction, we start from the singlet and triplet ΛN scattering lengths and effective ranges, a_s, a_t, r_s, r_t. However, these parameters are not well determined. Here, we choose the values:

$$a_s = -2.0 \text{ F}, \quad a_t = -2.2 \text{ F}, \quad r_s = 5.0 \text{ F}, \quad r_t = 3.5 \text{ F}, \tag{4.4}$$

obtained by the Maryland group (Sechi-Zorn et al. [10]). By applying the known relations between $a_{s(t)}$, $r_{s(t)}$ and the parameters of the Puff potential [6], we obtain Puff potentials $v_{\Lambda N,s}$ and $v_{\Lambda N,t}$ in the spin singlet and spin triplet states. We assume the same hard shell radius $c_{\Lambda N}$ in both states and consequently, the whole spin dependence is contained in the attractive Yamaguchi part ($\nu = 2$) of this ΛN interaction. To obtain a spin independent Puff interaction $v_{\Lambda N}$ (with the same hard shell radius $c_{\Lambda N}$), we insist that the relation

$$v_{\Lambda N} = \tfrac{1}{4}\, v_{\Lambda N,s} + \tfrac{3}{4}\, v_{\Lambda N,t} \tag{4.5}$$

holds with a sufficient accuracy. In this way, the parameters $\lambda_{\Lambda N}$ and $\beta_{\Lambda N}$ of $v_{\Lambda N}$ are determined.

TABLE I

Parameters of the ΛN potentials

$v_{\Lambda N}$	$\beta_{\Lambda N}$ (F^{-1})	$2\lambda_{\Lambda N}\mu_{\Lambda N}/(2\pi)^3$ (F^{-3})
$A(0.3)$	1.626	−0.753
$A(0.4)$	1.891	−1.956
$A(0.5)$	2.129	−4.415
$B(0.3)$	1.970	−1.658

We consider three values of $c_{\Lambda N} = 0.3$, 0.4, and 0.5 F, and denote the respective ΛN potentials by $A(0.3)$, $A(0.4)$, and $A(0.5)$. For the respective values of $\lambda_{\Lambda N}$ and $\beta_{\Lambda N}$, we have obtained values given in Table I, for which relation (4.5) holds for the attractive part of $\langle p|v_{\Lambda N}|p'\rangle$ with a better accuracy than 0.25% for $p(p') < 10\, k_F$.

We have considered also another set of ΛN scattering parameters:

$$a_s = -1.96 \text{ F}, \quad a_t = -1.93 \text{ F}, \quad r_s = 3.67 \text{ F}, \quad r_t = 3.27 \text{ F}, \tag{4.6}$$

obtained by Nagels, Rijeken and deSwart [11] in their one-boson-exchange-potential (model D) fit to the Maryland [10] and Rehovoth–Heidelberg [12] Λp scattering data. (The final best fit values of $a_{s(t)}$ and $r_{s(t)}$ given in [11] differ insignificantly from the values in (4.6) of the earlier version of model D.) For $c_{\Lambda N} = 0.3$ F, we have determined the parameters of the ΛN potential, denoted by $B(0.3)$, in the same way as in the case of the potentials $A(c_{\Lambda N})$. The resulting values of $\lambda_{\Lambda N}$ and $\beta_{\Lambda N}$ are given in Table I.

5. *Numerical procedure*

The starting point is the calculation of $E_{\Lambda 2}$, Eq. (2.18), which is very simple. For $t_{N\Lambda}$ we use expressions (B.2), and (B.4) with $\mathscr{I}^2_{\lambda\nu}$ calculated numerically (on the other hand, for $\mathscr{I}^i_{\lambda\nu}[q]$, Eq. (3.2), analytical expressions have been used). Expression (2.18′) involves one-dimensional numerical integration over p. Since expression (2.17) for γ_Λ contains $V_\Lambda = E_{\Lambda 2}$, one has to repeat the calculation of $E_{\Lambda 2}$ at least twice to determine the self--consistent value of $E_{\Lambda 2}$. In calculating $E_{\Lambda 2}$, approximation (4.5) was not used, and Eq. (2.18) was applied with $t_{N\Lambda}$ replaced by $\frac{1}{4}t_{N\Lambda,s}+\frac{3}{4}t_{N\Lambda,t}$, where $t_{N\Lambda,s}$ and $t_{N\Lambda,t}$ are t-matrices obtained from $v_{N\Lambda,s}$ and $v_{N\Lambda,t}$, respectively.

To calculate E_R, Eq. (2.25), we have to calculate the wound integral κ_{NN} by calculating numerically the p-integral in expression (2.26). Similarly, we calculate $\kappa_{\Lambda N}$, Eq. (2.27).

In all these calculations, as well as in calculating $E_{D(x)}(i, jk, l)$, Eq. (2.20), we use for $P_{N\Lambda}$ and P_{NN} their average values (2.16).

To solve the BF equations, Eqs (3.4), we write:

$$A^{ij}_{\lambda\nu}(\boldsymbol{ba}) = \delta_{ij}(2\pi)^3\delta(\boldsymbol{b}-\boldsymbol{a})\mathscr{D}^i_{\lambda\nu}[b]+\mathscr{A}^{ij}_{\lambda\nu}(\boldsymbol{ba}), \tag{5.1}$$

and obtain for $\mathscr{A}^{ij}_{\lambda\nu}$ integral equations

$$\mathscr{A}^{ij}_{\lambda\nu}(\boldsymbol{ba}) = (\delta_{ij}-1)\sum_{\lambda'\nu'}\mathscr{D}^i_{\lambda\nu'}[b]K^{ij}_{\nu'\lambda'}(\boldsymbol{ba})\mathscr{D}^j_{\lambda'\nu}[a]$$
$$-\sum_{\nu'\lambda'}\mathscr{D}^i_{\lambda\nu'}[b]\sum_{k\neq i}(2\pi)^{-3}\int d\boldsymbol{q}K^{ik}_{\nu'\lambda'}(\boldsymbol{bq})\mathscr{A}^{kj}_{\lambda'\nu}(\boldsymbol{qa}), \tag{5.2}$$

whose inhomogeneous terms are regular in contradistinction to the δ-type terms in equations (3.4). All the symmetry properties of the A-functions remain valid also for the $\mathscr{A}$-functions, and we are left with the problem of solving integral equations for $\mathscr{A}^{NN}$, $\mathscr{A}^{N\neq N}$, $\mathscr{A}^{\Lambda N}$, $\mathscr{A}^{N\Lambda}$ and $\mathscr{A}^{\Lambda\Lambda}$, i.e., equations analogous to (3.16–20). Actually, we eliminated $\mathscr{A}^{\Lambda N}$ and $\mathscr{A}^{\Lambda\Lambda}$ from these equations (as explained in Section 3), and have solved separate integral equations for $\mathscr{A}^{(+)}$, $\mathscr{A}^{(-)}$, and $\mathscr{A}^{N\Lambda}$.

The first δ-type term of the decomposition of A^{ij} in (5.1) leads to that part of $E_{\Lambda 3}$, which is of third order in the t matrices, and which we denote by $E_{\Lambda 3}(\sim t^3)$.

For the kernels of the BF equations, Eqs (3.5), we apply the angle-average approximation:

$$K^{ij}_{\lambda\nu}(\boldsymbol{ab}) \cong K^{ij}_{\lambda\nu}(ab) = \overline{g^i_\lambda(|\boldsymbol{b}+m_j\boldsymbol{a}/M_i|)}\,\overline{D^{-1}_{ij}(\boldsymbol{ab})}\,\overline{g^j_\nu(|\boldsymbol{a}+m_i\boldsymbol{b}/M_j|)}, \tag{5.3}$$

where the bars denote angle averages, according to the general definition:

$$\overline{f(ab)} = \tfrac{1}{2} \int_{-1}^{1} dx f(ab), \tag{5.4}$$

where x is the cosine of the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. The averages appearing in (5.3) have been calculated analytically. Approximation (5.3) implies that

$$\mathscr{A}^{ij}_{\lambda\nu}(\boldsymbol{ba}) = \mathscr{A}^{ij}_{\lambda\nu}(ba), \tag{5.5}$$

and integral equations (5.2) for $\mathscr{A}^{ij}_{\lambda\nu}$, and consequently the integral equations for $\mathscr{A}^{(+)}$, $\mathscr{A}^{(-)}$, and $\mathscr{A}^{\mathrm{NA}}$ become one-dimensional integral equations. These one-dimensional integral equations were transformed into linear algebraic equations by approximating the integrals over q by sums (with the help of Gauss–Laguerre quadrature method, with 18 points, and with an upper limit cutoff at 10 k_{F}). The linear equations were solved with the Gauss method. Functions $\mathscr{A}(b\, a)$ have been obtained from these solutions by applying the Lagrange multiple interpolation method [13].

The integrals over $\boldsymbol{b}$ and $\boldsymbol{c}$ in expression (2.20) for $E_{\mathrm{D}}(i, jk, l)$ were reduced to integrals over b and c by applying averaging over the angles between $\boldsymbol{b}$ and $\boldsymbol{c}$, similar to that described above (see Eq. (5.3)). The Q_i operators in Eq. (2.20) have been replaced by the average operators $\bar{Q}_i$ of Appendix B. The integrations (over b, c, q_1, q_3, and $\hat{\boldsymbol{q}}_1\hat{\boldsymbol{q}}_3$) in (2.20) have been performed numerically.

6. *Results and discussion*

Our results obtained for $k_{\mathrm{F}} = 1.35$ F with the NN and ΛN interactions described in Section 3, are given in Table II.

First of all we notice that the term of third order in the reaction matrices, $E_{\Lambda 3}(\sim t^3)$, is much bigger than the total value of $E_{\Lambda 3}$. This illustrates the known fact that the t^3-approximation would be misleading, and that summation of all three-hole-line diagrams (by solving the BF equations) is necessary to calculate the total correction $E_{\Lambda 3}$ to $E_{\Lambda 2}$.

TABLE II

Contributions (in MeV) to $-B_\Lambda$, and values of $\kappa_{\Lambda\mathrm{N}}$

$v_{\Lambda\mathrm{N}}$	$E_{\Lambda 2}$	$E_{\Lambda 3}(\sim t^3)$	$E_{\Lambda 3}$	$\lvert E_{\Lambda 3}/E_{\Lambda 2}\rvert$	E_{R}	$-B_\Lambda$	$\kappa_{\Lambda\mathrm{N}}$
$A(0.3)$	−43.7	4.5	1.7	0.04	4.6	−37.4	0.09
$A(0.4)$	−40.2	8.0	3.0	0.07	4.2	−33.0	0.11
$A(0.5)$	−37.3	12.7	4.1	0.11	3.9	−29.3	0.13
$B(0.3)$	−62.2	5.0	1.9	0.03	6.5	−53.8	0.09

It appears [9] that $E_{\Lambda 3}(\sim t^3)$ is a reasonable approximation of $E_{\Lambda 3}$ only for purely attractive interactions, in which case $E_{\Lambda 3}$ turns out to be negative [4]. (The negative sign of $E_{\Lambda 3}$ for attractive forces follows immediately from the structure of the expression for $E_{\Lambda 3}(\sim t^3)$, Eq. (2.8) with $T^{ij} \cong t^i \delta_{ij}$.) The big positive values of $E_{\Lambda 3}(\sim t^3)$ in Table II, compared to the smaller values of $E_{\Lambda 3}$, demonstrate a strong cancellation of the t^3-contributions by the higher order terms produced predominantly by the short range repulsion.

Our present results for $E_{\Lambda 3}$ are consistent with our previous simple estimate [3], based on the method applied by Moszkowski [14] in pure NM. The result of the simple estimate was $E_{\Lambda 3} \sim 2$ MeV for local ΛN potentials, adjusted to Λp scattering and to binding energies of light hypernuclei, with hard core radius 0.45–0.5 F (an improvement of the ΛN correlation functions leads to an increase in $E_{\Lambda 3}$ [15]).

In applying the reaction matrix method in calculating B_Λ, we follow the systematic approach in terms of the number of hole-lines, worked out for pure NM. Simple consideration suggests (see, e.g., [2]) that by introducing into a diagram an additional independent hole line, we change its contribution to the energy of NM by a factor of the order of κ_{NN}, Eq. (2.26), which plays the role of the smallness parameter in the hole-line expansion method. The same considerations applied to diagrams which contribute to B_Λ, suggest that by introducing an additional nucleon hole-line we change its contribution to B_Λ by a factor of the order of κ_{NN} or $\kappa_{\Lambda N}$, Eq. (2.27), depending on the location of the additional hole-line. Consequently, we expect that the order of magnitude of $E_{\Lambda 3}/E_{\Lambda 2}$ should be determined by the two wound integrals κ_{NN} and $\kappa_{\Lambda N}$. For the Puff NN interaction, Eq. (4.2), we have $\kappa_{NN} = 0.105$, and the values of $\kappa_{\Lambda N}$ for our ΛN interactions are given in Table II. As expected, κ_{NN} and $\kappa_{\Lambda N}$ are of the order of magnitude of $|E_{\Lambda 3}/E_{\Lambda 2}|$. Actually, they are bigger than $|E_{\Lambda 3}/E_{\Lambda 2}|$, and approximately agree with $|E_{\Lambda 3}/E_{\Lambda 2}|$ for the ΛN interaction $A\,(0.5)$ with the biggest hard shell radius $c_{\Lambda N}$. This is in accordance with the considerations suggesting that the wound integrals are the smallness parameters of the hole-line expansion. These considerations are of a qualitative character, and are most convincing for the hard core part of the interaction [2].

Our results show a reasonable convergence of the reaction matrix method of calculating B_Λ. For a reasonable size of the ΛN repulsion ($c_{\Lambda N} \cong 0.4-0.5$ F), similar to the NN repulsion, we get a repulsive three-body contribution $E_{\Lambda 3} \cong 3-4$ MeV. Our simplified model of S state ΛN and NN interactions appears justified in calculating $E_{\Lambda 3}$ which is dominated by the short range repulsion acting predominantly in the S state.

On the other hand, the ΛN P state contribution to $E_{\Lambda 2}$ turns out to be important (see e.g., [9]), and thus the values of $E_{\Lambda 2}$ in Table II are not realistic. The same criticism appears to apply also to our values of E_R which are proportional to $E_{\Lambda 2}$, Eq. (2.25). However, more realistic estimates of E_R [9], [16] lead to similar results, namely to $E_R \cong 4-5$ MeV. Consequently, we expect a total repulsive correction to $E_{\Lambda 2}$, $E_{\Lambda 3}+E_R \cong 7-9$ MeV.

A repulsive contribution of this magnitude (together with the important effect of ΛΣ conversion (see, e.g., [16])) is sufficient to solve the Λ overbinding problem. On the other hand, it increases the discrepancy between the values of B_Λ calculated with the reaction matrix method, and the much bigger values of B_Λ obtained with the variational method (see, e.g., [17–19]).

APPENDIX A

Notation and kinematics

By p_1, p_2, p_3, we denote the momenta (in units of $\hbar$) of the three particles. Their respective masses divided by $\hbar^2$ are denoted by $m_1 = m_3 = m_N$, $m_2 = m_\Lambda (m_N = 0.02412\ \text{MeV}^{-1}\text{F}^{-2}$, $m_\Lambda = 0.02865\ \text{MeV}^{-1}\text{F}^{-2})$. For the momentum states normalized in the periodicity box of volume Ω, we use the notation $|p)$, i.e., $(r|p) = \exp(pr)/\sqrt{\Omega}$. By $|p\rangle$, we denote momentum states with the normalization $\langle p'|p\rangle = (2\pi)^3\delta(p'-p)$, i.e., $\langle r|p\rangle = \exp(pr)$.

Instead of p_1, p_2, p_3, it is convenient to use other vectors to label the states of the three particles. As one of them, we choose the total momentum $P = p_1 + p_2 + p_3$, which remains constant during all interactions. Consequently, we shall work in the subspace of our three-particle system with a fixed P. In this space, two vectors are necessary to label the states of the three particles. Following Faddeev [20], we introduce three such pairs of vectors:

$$q_i = [m_i(p_j+p_k) - M_i p_i]/M = m_i P/M - p_i, \quad k_i = (m_j p_k - m_k p_j)/M_i, \tag{A.1}$$

where the indices i, j, k form one of the cyclic permutations of 1, 2, 3, $M = m_1+m_2+m_3$ is the total mass, and $M_i = M - m_i = m_j + m_k$ is the mass of the particles j and k which we shall call the i-th pair.

Each pair of the vectors q_i, k_i, according to the equations:

$$q_i|q'k'\rangle_i = q'|q'k'\rangle_i, \quad k_i|q'k'\rangle_i = k'|q'k'\rangle_i, \tag{A.2}$$

labels the momentum states of three different bases $|qk\rangle_i$ (i = 1, 2, 3). The interchange between these three bases is determined by

$$\begin{aligned} {}_1\langle qk|q'k'\rangle_2 = {}_2\langle q'k'|qk\rangle_1 &= (2\pi)^6\delta(q'+m_2 q/M_1 - k)\delta(k'+m_3 Mq/(M_1M_2)+m_1 k/M_2) \\ &= (2\pi)^6\delta(q+m_1 q'/M_2+k')\delta(k-m_3 Mq'/(M_1M_2)+m_2 k'/M_1), \end{aligned} \tag{A.3}$$

and equations obtained from Eq. (A.3) by the replacements: $1 \to 2$, $2 \to 3$, and $1 \to 3$, $2 \to 1$.

Notice the following relations:

$$P_{13}|qk\rangle_1 = |q-k\rangle_3, \quad P_{13}|qk\rangle_2 = |q-k\rangle_2. \tag{A.4}$$

The action of the exclusion principle operator Q in the three bases is described by:

$$Q|qk\rangle_i = Q_i(qk)\,|qk\rangle_i \tag{A.5}$$

where

$$Q_1(qk) = Q_{N\Lambda}(P_1 k), \quad Q_3(qk) = Q_{N\Lambda}(P_3 - k), \quad Q_2(qk) = Q_{NN}(P_2 k), \tag{A.6}$$

where $P_i = P - p_i$ is the c.m. momentum of the i-th pair, and Q_{NN}, $Q_{N\Lambda}$ are defined in Eqs (2.10), (2.13).

For the on-shell t-matrices in Eqs (2.6), (2.8), we have:

$${}_i\langle q'k'|t^i_{ON}|qk\rangle_i = (2\pi)^3\delta(q'-q)\,\langle k'|t^i(P_i, -\gamma_i)\,|k\rangle, \tag{A.7}$$

where, with approximations (2.16), we have $\gamma_1 = \gamma_3 = \gamma_\Lambda$, and $\gamma_2 = \gamma_N$. In the notation of Section 2.2, we have

$$\langle \boldsymbol{k}'|t^1(\boldsymbol{P}_1, -\gamma_\Lambda)|\boldsymbol{k}\rangle = \langle \boldsymbol{k}'|t_{N\Lambda}(\boldsymbol{P}_1, -\gamma_\Lambda)|\boldsymbol{k}\rangle,$$
$$\langle \boldsymbol{k}'|t^3(\boldsymbol{P}_3, -\gamma_\Lambda)|\boldsymbol{k}\rangle = \langle -\boldsymbol{k}'|t_{N\Lambda}(\boldsymbol{P}_3, -\gamma_\Lambda)|-\boldsymbol{k}\rangle,$$
$$\langle \boldsymbol{k}'|t^2(\boldsymbol{P}_2, -\gamma_N)|\boldsymbol{k}\rangle = \langle \boldsymbol{k}'|t_{NN}(\boldsymbol{P}_2, -\gamma_N)|\boldsymbol{k}\rangle. \tag{A.8}$$

The T^{ij} matrix of Eq. (2.8) satisfies Eq. (2.4) with an off-energy-shell two-body reaction matrix t^i_{OFF} which describes the scattering of the i-th pair (particles j and k) in NM after the i-th particle has been excited already. Consequently, at least one of the particles j or k has been excited also before it undergoes the scattering described by t^i_{OFF}. In this situation, the exclusion principle appears to be less important and we neglect it. This means, we drop the exclusion principle operator Q from Eqs (2.9) and (2.12). Consequently, our t^i_{OFF} does not depend on the c.m. momentum of the i-th pair, and we have

$${}_i\langle \boldsymbol{q}'\boldsymbol{k}|t^i_{\text{OFF}}|\boldsymbol{q}\boldsymbol{k}\rangle_i = (2\pi)^3\delta(\boldsymbol{q}'-\boldsymbol{q})\langle \boldsymbol{k}'|t^i(z_{\Lambda NN})|\boldsymbol{k}\rangle, \tag{A.9}$$

where

$$z_{\Lambda NN} = V_\Lambda + e_N(p_1) + e_N(p_2) - P^2/2M - q^2/2\tilde{\mu}_i, \tag{A.10}$$

where $\tilde{\mu}_i = m_i M_i/M$. With the help of approximations (2.16), we get

$$z_{\Lambda NN} \cong -\Gamma - q^2/2\tilde{\mu}_i = V_\Lambda + 2A_N + 0.6k_F^2(1/m_N^* - 1/M) - q^2/2\tilde{\mu}_i, \tag{A.11}$$

For our approximate t^i_{OFF} matrices, we shall use the notation

$$\langle \boldsymbol{k}'|t^i(-\Gamma - q^2/2\tilde{\mu}_i)|\boldsymbol{k}\rangle = \langle \boldsymbol{k}'|t^i[q]|\boldsymbol{k}\rangle. \tag{A.12}$$

For the matrices T^{ij}, we use the notation:

$${}_i\langle \boldsymbol{q}'\boldsymbol{k}'|T^{ij}|\boldsymbol{q}\boldsymbol{k}\rangle_j = T^{ij}(\boldsymbol{q}'\boldsymbol{k}'; \boldsymbol{q}\boldsymbol{k}). \tag{A.13}$$

APPENDIX B

t matrices for separable interactions

Here we assume v^i ($v^1 = v^3 = v_{N\Lambda}$, $v^2 = v_{NN}$) to be of a general separable S-state form

$$\langle \boldsymbol{p}'|v^i|\boldsymbol{p}\rangle = \sum_\nu \lambda^i_\nu g^i_\nu(p')g^i_\nu(p), \tag{B.1}$$

where the number of ν-terms determines the rank of the interaction.

The solution of the t^i-equation (Eq. (2.9) for $t^2 = t_{NN}$, Eq. (2 12) for $t^1 = t^3 = t_{N\Lambda}$) is:

$$\langle \boldsymbol{p}'|t^i(\boldsymbol{P}_i, z)|\boldsymbol{p}\rangle = \sum_{\lambda\nu} \mathscr{D}^i_{\lambda\nu}(P_i, z)g^i_\lambda(p')g^i_\nu(p), \tag{B.2}$$

where the coefficients $\mathscr{D}^i_{\lambda\nu} = \mathscr{D}^i_{\nu\lambda}$ are solutions of the system of linear algebraic equations:

$$\sum_\mu [\delta_{\nu\mu}/\lambda^i_\nu + \mathscr{I}^i_{\nu\mu}(P_i z)]\mathscr{D}^i_{\mu\lambda}(P_i z) = \delta_{\nu\lambda}, \tag{B.3}$$

where

$$\mathcal{I}^i_{\nu\lambda}(Pz) = \mathcal{I}^i_{\lambda\nu}(Pz) = -(2\pi)^{-3} \int d\boldsymbol{p} Q_i(Pp) g^i_\lambda(p) g^i_\nu(p) / [z - p^2/2\mu_i]. \quad \text{(B.4)}$$

Our S-state interactions lead to the appearance of the angle-averaged Pauli principle operators $Q_1(Pp) = Q_3(Pp) = \bar{Q}_{N\Lambda}(Pp)$ and $Q_2(Pp) = \bar{Q}_{NN}(Pp)$, where

$$\bar{Q}_{NN}(P, p) = (4\pi)^{-1} \int d\hat{\boldsymbol{P}} Q_{NN}(\boldsymbol{P}, \boldsymbol{p}) = \begin{cases} 0 & \text{for} \quad p < (k_F^2 - \frac{1}{4} P^2)^{1/2}, \\ 1 & \text{for} \quad p > \frac{1}{2} P + k_F, \\ (\frac{1}{4} P^2 + p^2 - k_F^2)/Pp & \text{otherwise}, \end{cases} \quad \text{(B.5)}$$

$$\bar{Q}_{N\Lambda}(P, p) = (4\pi)^{-1} \int d\hat{\boldsymbol{P}} Q_{N\Lambda}(\boldsymbol{P}, \boldsymbol{p}) = \begin{cases} 0 & \text{for} \quad p < k_F - \mu_{\Lambda N} P/m_\Lambda, \\ 1 & \text{for} \quad p > k_F + \mu_{\Lambda N} P/m_\Lambda, \\ [(p + \mu_{\Lambda N} P/m_\Lambda)^2 - k_F^2]/(4\mu_{\Lambda N} Pp/m_\Lambda) & \text{otherwise.} \end{cases}$$

(B.6)

Notice that our t^i matrices do not depend on the directions of $\boldsymbol{P}_i$, $\boldsymbol{p}$, $\boldsymbol{p}'$.

For the Puff potential, i.e., for the rank two potential with $\lambda^i_1 \to \infty$, the solution of the system of equations (B.3) has the form:

$$\mathcal{D}^i_{11} = (\mathcal{I}^i_{22} + 1/\lambda^i_2)/\Delta_i, \quad \mathcal{D}^i_{22} = \mathcal{I}^i_{11}/\Delta_i, \quad \mathcal{D}^i_{12} = \mathcal{D}^i_{21} = -\mathcal{I}^i_{12}/\Delta_i, \quad \text{(B.7)}$$

where

$$\Delta_i = \mathcal{I}^i_{11}(\mathcal{I}^i_{22} + 1/\lambda^i_2) - (\mathcal{I}^i_{12})^2. \quad \text{(B.8)}$$

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