LETTERS TO THE EDITOR

USE OF BERNSTEIN MOMENTS IN QCD TESTS

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It is pointed out that when comparing QCD predictions with experimental data for deep inelastic structure functions it is advisable to use Bernstein moments.

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The purpose of this letter is to point out that Bernstein moments

$$B_M^N(Q^2) = \int_0^1 P_M^N(x)W(x, Q^2)dx, \tag{1}$$

where the Bernstein polynomials

$$P_M^N(x) = \frac{(M+N+1)!}{N!M!} x^M(1-x)^{N-M}; \quad M \leqslant N, \tag{2}$$

are particularly convenient for the comparison with experiment of QCD predictions for the Q^2 evolution of structure functions.

Two approaches for this comparison have been popular (cf. e.g. review [1]). One is to study the moments

$$M_N(Q^2) = \int_0^1 x^N W(x, Q^2)dx. \tag{3}$$

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The advantage is that simple closed QCD formulae for the Q^2 dependence of these moments are available. The disadvantage is that within this approach it is not possible to cut out the x-region close to $x = 1$, where the present version of perturbative QCD predictions is probably unreliable (cf. e.g. Ref. [2]), or any other x-region, where the data would be poor, or nonexistent. Moreover, except for the first few, all the moments are dominated by the contributions from the $x \approx 1$ region. Thus their errors are strongly correlated and the confidence levels of the fits are unclear.

The other approach is, to invert the formulae for the moments and thus to obtain predictions for the Q^2 evolution of the structure functions themselves. Actually, in this context the Bernstein moments have already successfully been used [3]. The inversion approach makes any desired cuts in x easy to apply, but the inversion has to be performed numerically and no closed analytic expression results.

The Bernstein moments combine the advantages of both approaches. Since the moments (1) are known linear combinations of the moments (3), close analytic formulae for their Q^2 evolution are available. On the other hand, a Bernstein polynomial $P_M^N(x)$ has a maximum at $x = M/N$ and a width decreasing with increasing N at fixed M/N. Therefore, provided N is large enough, one can very well approximate a cut in x by rejecting some moments. For instance, rejecting the moments with $M = N$, one circumvents the controversial problem: whether or not the elastic contribution should be included. Bernstein polynomials for $M = 0, 1, \ldots, N$ are independent linear combinations of the monomials x^M for $M = 0, 1, \ldots, N$. Thus, using all the Bernstein polynomials with a given N corresponds to the use of standard moments up to N. Since, however, the Bernstein polynomials differing in M peak in different regions of the x axis, the errors on the moments (1) are much less correlated than those for the moments (3).

Our formulae have been written down for Coleman Norton moments. The application to Nachtmann moments is, however, also possible. Work on application of these ideas to data analysis is in progress.

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