A NOTE ON GEOMETRY OF THE NEW COSMOLOGY

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The metric of the cosmological model implied by the nonsymmetric unified field theory is transformed to a simpler and more manageable form. Embedding of the metric in a six dimensional flat manifold is given.

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1. In Ref. [1] I showed that the metric of the "new cosmology" or cosmological model implied by the nonsymmetric unified field theory (Ref. [2]) takes the isotropic form

$$ds^2 = f^2(\varrho)dt^2 - g^2(\varrho)(d\varrho^2 + \varrho^2 d\Omega^2), \tag{1}$$

where $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$, and ϱ is given in terms of the original "radial" coordinate by the elliptic relation

$$\varrho = \exp\left[-2^{2/3}\sqrt{r_0}\,\wp^{-1}\left(\frac{(12m^2+r_0^2)r+4mr_0^2}{r-2m},\ 2^{2/3}(\tfrac{1}{3}r_0^2-4m^2),\ -\tfrac{1}{27}r_0(r_0^2+36m^2)\right)\right] \tag{2}$$

with

$$f^2(\varrho) = \left(1-\frac{2m}{r}\right), \quad g(\varrho) = \frac{rr_0}{\varrho\sqrt{r_0^2+r^2}}.$$

This expression renders the study of relevant geometry as well as calculation of relevant astronomical parameters very awkward. On the other hand knowledge of these parameters is essential if a comparison is to be made between the theoretical model and observational data. Accordingly, I want to record the results of some calculations which may throw light on the nature of the proposed geometry.

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2. Let us consider first the problem of writing the cosmological metric in Cartesian coordinates. Clearly this is somewhat ambiguous because the choice of what a local observer should regard as his radial distance, with the observer at the origin, is arbitrary. The most natural choice appears to be

$$\varrho^2 = x^2+y^2+z^2, \tag{3}$$

when of course

$$ds^2 = f^2dt^2-g^2(dx^2+dy^2+dz^2), \tag{4}$$

but the functional form of f and g in terms of $x^2+y^2+z^2$ remains complicated. It is simpler to regard r as the radial coordinate. If

$$r = \sqrt{x^2+y^2+z^2} \tag{5}$$

and

$$\tan\theta = \frac{\sqrt{x^2+y^2}}{z}, \qquad \tan\varphi = y/x,$$

the metric becomes

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2-\frac{r_0^4(xdx+ydy+zdz)^2}{(r_0^2+r^2)^2\left(1-\frac{2m}{r}\right)}-\frac{r_0^2(d\omega_1^2+d\omega_2^2+d\omega_3^2)}{r^2(r_0^2+r^2)}, \tag{6}$$

where

$$d\omega_1 = zdy-ydz, \qquad d\omega_2 = xdz-zdx, \qquad d\omega_3 = ydx-xdy.$$

3. The above expressions are elementary and, as I pointed out, largely arbitrary. Of more interest is the question of embeddability of the space time (1) to (6) in a pseudo--euclidean space of more than four dimensions. Because of similarity of the line element (6) to a Schwarzschild metric we can expect that a six-dimensional flat manifold will be required. This indeed is the case. Let

$$r = \frac{r_0 z}{\sqrt{1-z^2}}, \qquad 0 \leqslant z \leqslant 1, \tag{7}$$

and

$$\xi = z\sin\theta\cos\varphi, \qquad \eta = z\sin\theta\sin\varphi, \qquad \zeta = z\cos\theta.$$

The line element becomes

$$d\sigma^2 = (z-\lambda\sqrt{1-z^2})\frac{d\tau^2}{z}-\frac{z^3+\lambda(1-z^2)^{3/2}}{(1-z^2)(z-\lambda\sqrt{1-z^2})}dz^2-(d\xi^2+d\eta^2+d\zeta^2), \tag{8}$$

where

$$d\sigma = \frac{ds}{r_0}, \quad d\tau = \frac{dt}{r_0} \quad \text{and} \quad \lambda = \frac{2m}{r_0}.$$

Let also

$$x = g(\tau)h(z), \quad y = p(\tau)q(z), \quad w = w(z).$$

Then

$$dx^2 + dy^2 - dw^2 = (z - \lambda\sqrt{1-z^2})\frac{d\tau^2}{z} - \frac{z^3 + \lambda(1-z^2)^{3/2}}{(1-z^2)(z-\lambda\sqrt{1-z^2})}\, dz^2, \tag{9}$$

providing

$$g\dot{g}hh' + p\dot{p}qq' = 0,$$

$$\dot{g}^2h^2 + \dot{p}^2q^2 = \frac{1}{z}(1-\lambda\sqrt{1-z^2}),$$

$$g^2h'^2 + p^2q'^2 - w'^2 = -\frac{z^3+\lambda(1-z^2)^{3/2}}{(1-z^2)(z-\lambda\sqrt{1-z^2})}, \tag{10}$$

dots denoting differentiation with respect to τ and dashes with respect to z. These equations are satisfied only if

$$p = \frac{c}{a}\sin\frac{b}{c}\tau, \quad g = c\cos\frac{b}{c}\tau, \quad h = \frac{1}{b}\sqrt{\frac{z-\lambda\sqrt{1-z^2}}{z}},$$

$$r > 2m, \tag{11}$$

a, b, c arbitrary, nonzero constants. The function w is then given by the differential equation

$$w'^2 = \frac{k^2 + z^3(z^3 + \lambda(1-z^2)^{3/2})}{z^3(1-z^2)(z-\lambda\sqrt{1-z^2})}, \quad k = \frac{\lambda c}{2b}. \tag{12}$$

We can easily check that the 2-space whose metric is given by the right hand side of the equation (9) has a nonvanishing Riemann–Christoffel tensor (unlike the corresponding case of the de Sitter space-time) so that the line element of the embedding flat space

$$ds^2 = dx^2 + dy^2 - dw^2 - d\xi^2 - d\eta^2 - d\zeta^2$$

is irreducible.

4. Finally let us note another substitution which renders the (still irreversible) relation between r and the coordinates ϱ and τ in which the coefficient of $d\tau^2$ is reduced to unity, particularly simple. If we write

$$\frac{\sqrt{2rr_0}}{r_0 - r} = \tan\psi, \quad \varrho = kt + h(r), \quad \tau = t + g(r), \tag{13}$$

the required expression becomes

$$(\sqrt{1+\sin^2 \psi}-\sin \psi)e^{\psi} = e^{\nu(\varrho-k\tau)}, \qquad 14)$$

where

$$\nu = \frac{\lambda \sqrt{2\lambda}}{2mk}.$$

REFERENCES

[1] A. H. Klotz, *Acta Phys. Pol.* **B10**, 295 (1979).
[2] A. H. Klotz, *Acta Phys. Pol.* **B9**, 573 (1978).