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QCD AND SPECTROSCOPY*

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The bearing of QCD on spectroscopy is illustrated by a discussion of the following topics: the parameters of QCD, QCD Sum Rules, QCD and Current Algebra.

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The theme of these lectures is the inter-relation of QCD and spectroscopy. This is a far reaching subject and what follows is a very brief and selective survey [1]. The topics covered are: The parameters of QCD, QCD Sum Rules, QCD and Current Algebra. Aside from passing references, there is no mention of the very considerable amount of work using potential [2] or bag models [3].

1. *The parameters of QCD*

The deep reasons for favouring QCD have been rehearsed many times. As a candidate for *the theory* of the strong interactions it is clearly the front runner and we are now mainly into the phase of seeking quantitative confirmation. That is not to say that all predictions have been realised at a qualitative level; in particular, much work remains to be done in order to establish the physical properties of the gluons [4]. Here, however, we adopt a book-keeping attitude and ask: What are the *parameters* of QCD; do the various measures of them which are available agree?

For the present phenomenological purpose, a sufficient list of parameters comprises: (a) the quark masses; (b) certain parameters which characterize the QCD vacuum; (c) the running coupling constant $\alpha_s(q^2)$. We begin with a very rapid tour indicating the role and what is known as to the value of these quantities.

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1.2. Quark Masses

(*i*) Mass parameters enter the QCD Lagrangian via the quark mass-matrix

$$H' = \sum_{\text{flavour}} m_i \bar{q}_i q_i, \qquad (i = \text{u, d, s, c, b}\ldots). \tag{1.1}$$

The quantities, m_i, thus defined are the *current quark masses*. They are the sole means of introducing explicit symmetry breaking into QCD, since the strong interactions which couple to colour are supposed to be flavour blind. Eq. (1.1) imposes a very simple underlying pattern of symmetry breaking which it is important to verify in the observed spectrum of hadrons.

(*ii*) Ratios of current quark masses are estimated in terms of those of the light pseudo--scalars using current algebra (cf. also §3), e.g.

$$\frac{m_\text{d}+m_\text{u}}{2m_\text{s}} \simeq \frac{m_\pi^2}{2m_\text{K}^2} \simeq 0.040. \tag{1.2}$$

In this way, and making due allowance for electromagnetic effects, the ratios of m_u to m_d to m_s can be estimated. To fix the scale, Weinberg [5] has proposed the approximate formula

$$M_\text{h} \cong m_0 + m_\text{s} N_\text{hs} \tag{1.3}$$

to express the mass of an ordinary hadron h which contains a number N_hs of strange quarks (these being the only constituents of appreciable bare mass). This yields the estimates [5]

$$m_\text{u} = 4.2 \text{ MeV}, \qquad m_\text{d} = 7.5 \text{ MeV}, \qquad m_\text{s} = 150 \text{ MeV} \tag{1.4}$$

with the universal contribution, m_0, which is attributed to spontaneous symmetry breaking, or equivalently to the effects of confinement, taking a value in the neighbourhood $m_0 \approx 350$ MeV. Similar results have been found by a number of authors [6]. One outcome of this kind of work has been the search for noticeable isospin violation effects stemming from the substantial departure of m_u/m_d from unity [7].

(*iii*) Thus far we have spoken of the current quark masses which feature in the QCD Lagrangian. A secondary, less precise concept is that of constituent quark masses, M_i. These are the masses which have to be attributed to the constituents in the naïve quark model. Since residual effects of binding are supposed to be small, masses are typcially estimated to lie in the ranges

$$\begin{gathered} M_\text{u,d} \sim \tfrac{1}{2} M_\varrho \sim \tfrac{1}{3} M_{\text{N},\Delta} \sim 370 \text{ MeV}, \\ M_\text{s} \sim \tfrac{1}{2} M_\phi \sim 510 \text{ MeV}; \qquad M_\text{c} \sim \tfrac{1}{2} M_\psi \sim 1.55 \text{ GeV}, \\ M_\text{b} \sim \tfrac{1}{2} M_\Upsilon \sim 4.7 \text{ GeV}. \end{gathered} \tag{1.5}$$

Such masses coexist quite well with the observed magnetic moments of baryons, if standard Dirac moments are attributed to the constituent quarks. An alternative approach to the concept of constitutent masses [8] is to tie them to rises in the ratio R ($\sigma(\text{e}^+\text{e}^- \to \text{hadrons}) / \sigma(\text{e}^+\text{e}^- \to \mu^+\mu^-)$) suitably smeared in energy to erase localized resonant structure.

(*iv*) Eq. (1.3) gives an explicit expression to the idea of constituent masses being the combined result of confinement and of a bare (current) mass input. In the MIT bag model [9], the corresponding result takes the form

$$M^2 = \frac{\text{const}}{R^2} + (m^{\text{bare}})^2, \tag{1.6}$$

with R the bag radius. Just how far this deviates from the linear formula, Eq. (1.3), depends on the extent to which R adjusts to changes in m^{bare}. If the observed electronic widths of vector mesons ($\Gamma(\mathrm{V} \to \mathrm{e}^+\mathrm{e}^-)$) [10] are taken as a measure of the "concentration" of the corresponding $\bar{\mathrm{q}}\mathrm{q}$ wave functions using the Van Royen–Weisskopf formula [11] ($\Gamma_{\mathrm{e}^+\mathrm{e}^-} =$ known factors $\times\, |\psi(0)|^2/M^2$), then a considerable compensation is indicated. Magnetic moments of baryons (cf. remark below Eq. (1.5)) are readily understood in the bag model [9], since the confined quarks acquire effective moments $g/2M$ with $g \sim 1$.

(*v*) Thus far we have regarded the quark masses as constants; however if they are taken to be the parameters controlling the corresponding quark propagators [12] (for example at large space-like Q^2), then they assume the role of an additional effective coupling constant. As such, their leading power dependence as a function of q is calculable by standard renormalizable group methods. In this way, Politzer [8] derives the formula

$$m_i(Q) \simeq m_i^{\text{bare}}(M_0)\left(\frac{\alpha_s(Q)}{\alpha_s(M_0)}\right)^d + m_i^{\text{spont}}(M_0)\left(\frac{M_0^2}{Q^2}\right)\left(\frac{\alpha_s(Q)}{\alpha_s(M_0)}\right)^{1-d},$$

$$(d = 4/9 \text{ for } N_f = 3). \tag{1.7}$$

Eq. (1.7) encapsulates the whole philosophy of effective masses in QCD. At low Q^2, the spontaneous symmetry breaking contribution m_i^{spont} (which corresponds to $\langle 0|\bar{q}_i q_i|0\rangle \neq 0$) is large and, in the case of the light quarks, dominant; at high Q^2, the bare masses, which correspond to explicit symmetry breaking, take over.

(*vi*) Finally, and to sum up this sub-section, where do the various masses enter sensitively?

(a) As already mentioned, the light current quark masses play an important role in current algebra applications (cf. §3); $m_{\mathrm{d}} - m_{\mathrm{u}}$ feeds isospin violation [7] and is thus a crucial ingredient in formulae for $\eta = 3\pi$ decay (cf. discussion preceding Eq. (3.17)).

(b) Constituent quark masses are obviously key parameters in naïve potential model calculations [13] (which are not discussed here).

(c) For the present discussion, and especially for the QCD sum rules of §2, the most important mass parameter is m_{c}, the charmed quark mass. Operationally, it enters as the mass which is fed into the theoretical formulae which constitute the "left-hand side" of the QCD sum rules. How should it be viewed? As a current quark mass, since it enters quantities calculated perturbatively from the QCD Lagrangian; or, recalling Politzer's definition [8] invoking smoothed $\mathrm{e}^+\mathrm{e}^- \to$ hadron cross-sections, as a constituent quark mass; or just as an "effective" mass which functions in duality relations. The same question comes up in other duality applications involving charmed quarks, for example in discussions of the photo-, neutrino- and hadro-production of charm [14] which tend to favour rather small values of m_c (values as low as $m_c \simeq 0.8$ GeV have been proposed). Probably the most sensitive probe (presumably of the constituent) charmed quark mass, M_c, comes from the study of semi-leptonic charm decays [15]; both the rate and the spectrum shape are well fitted for $M_c = 1.75$ GeV.

1.3. Confinement — Parameters of the QCD Vacuum

Confinement of colour appears to be a fact albeit an unexplained fact [16]. A number of promising mechanisms (not necessarily inequivalent) have been advanced [17]; in particular, as we have heard at this school [18], there has recently been very considerable progress in reproducing confinement in lattice calculations [19]. Thus, fundamental attacks on this problem appear to be prospering.

For phenomenology, the question is how best to *parameterize* confinement and a number of schemes have been popular:

(a) Potential Models

Much work has been done with a variety of potentials motivated by QCD [20]. This whole approach is well exemplified by the charmonium studies of the Cornell Group [21], whose central potential

$$V(r) = \frac{k}{r} + \frac{r}{a^2} \equiv \overset{\text{coul}}{V}(r) + \overset{\text{conf}}{V}(r) \tag{1.8}$$

comprises two terms: a short range Coulombic part governed by a parameter k ($k = (4/3)\alpha_s$ for the static potential arising from one-gluon exchange); and a linear confining part corresponding to a constant string tension which is sometimes pictured as arising from a colour flux tube of fixed transverse dimensions extending between well separated sources.

(b) The MIT Bag Model]22]

Here, colour sources are supposed to be confined to small island regions (bags) by a uniform isotropic (relativistic) pressure which pervades the vacuum. This is accomplished by confining the field components of QCD (quarks and gluons) to the interior of bags and substituting

$$\mathscr{L}_{\mathrm{QCD}} \to (\mathscr{L}_{\mathrm{QCD}} - B)\theta_{\mathrm{B}}(x), \tag{1.9}$$

where the bag pressure, B, functions as an extra contribution to the stress tensor, $\theta_{\mu\nu}$ (dimensions $[M^4]$).

(c) Spontaneous symmetry breaking [23] — the parameters of the QCD vacuum

We have already encountered the concept of products of field operators having non-vanishing vacuum expectation in connection with the generation of quark masses

$$\Phi_q \equiv \langle m_i \bar{q}_i q_i \rangle_0 \neq 0. \tag{1.10}$$

In the approach to QCD sum rules pioneered by the Moscow (ITEP) group [24] to be discussed in §2, the non-vanishing of Φ_q defined above and especially of the analogous expression for the gluon field

$$\Phi_G \equiv \left\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^{a\mu\nu} \right\rangle_0 \neq 0 \tag{1.11}$$

play a key role in parameterizing confinement. Both Φ_q and Φ_G yield non-vanishing (isotropic) contributions to the stress-energy tensor for the vacuum [25]. In particular, according to the QCD sum rules phenomenology to be described in §2, Φ_G is large (in relation to other effects) and positive and thus reproduces the all pervading vacuum pressure invoked by the MIT bag model [25]. It operates irrespective of flavour and thus for example affords a link between charmonia and light quark compounds. In the derivation of the QCD sum rules (§2), Φ_G emerges in an operator product expansion [26], wherein it is naturally ascribed to long range fluctuations of the gluon field (the G's in (1.11) obviously carry zero momentum). Its authors often refer to the physical phenomenon underlying (1.11) as the "gluon condensate", in analogy with the Bose condensates of low temperature physics. In the language of the operator product expansion, Φ_G (and Φ_q) is the coefficient function of a power correction or "higher twist" [27] addition to the leading perturbative contribution.

Besides being theoretically interesting (it would be good to know its value from lattice calculations), Φ_G and its associated contributions to operator product expansions are phenomenologically very convenient when combined with notions of q^2 duality [28, 76] (§2). Both Φ_G and Φ_q are of course the first of a series of terms involving vacuum quantum number bearing products of fields. In principle, these higher products (yet higher twist) are independent parameters characterizing the vacuum; in applications to date, they have been approximated by saturating the operator products with vacuum intermediate states. The scope for invoking even higher terms in $1/Q^2$ is limited by the onset of short-range fluctuations associated with instantons [29]. It is therefore fortunate [24] that there appears to be an interesting range of energies for which simple controllable power corrections dominate the sum rules.

An important parameter of the QCD vacuum which will not feature in the present discussion is the coefficient θ which controls spontaneous CP-violation [30] $\left(\mathscr{L}_{\text{QCD}} \to \mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{QCD}} - \theta \frac{\alpha_s}{8\pi} G\tilde{G}\right)$. However, the operator,

$$D^0_{\text{anom}} \equiv \frac{\alpha_s}{4\pi} G\tilde{G} \tag{1.12}$$

which $\theta/2$ multiplies has an important role in discussions of the inter-relation of QCD and current algebra (§3). As the notation anticipates, D^0_{anom} is the "anomalous" contribution to the divergence of the singlet axial-vector current. There has recently been much discussion relating various properties of η and η' to the existence of large matrix elements of D^0_{anom} between these states and the vacuum (§3).

1.4. The running coupling constant $\alpha_s(q^2)$ [31]

(a) Definitions and formalism

It is no accident that by far the most discussed parameter of QCD is the running coupling constant, $\alpha_s(q^2)$. Properly speaking it is not a parameter at all (indeed to the extent that all quark masses approximate either zero or infinity the theory has no parameters) but rather an expression of how the full interacting theory evolves as one adjusts external momenta; and it is universal because one is in a gauge theory with an irreducible symmetry group [31]. In so far as one can perform realistic perturbation calculations in QCD, it controls all (leading) departures from elementary parton model predictions for high energy processes (scaling-violation), all violations of the OZI rule (e.g. hadronic decays of J/ψ) and all interactions attributable to gluon exchange or emission.

The primary occurrence of the strong coupling constant, g, in the QCD Lagrangian

$$\mathscr{L}_{\text{QCD}} = -\tfrac{1}{4}\, G^a_{\mu\nu}G^{\mu\nu a} + i\sum_q \bar{q}_i\gamma D q_i + \underset{\substack{\text{mass}\\ \text{terms}}}{\Delta_{\text{QCD}}} + \text{gauge terms} \tag{1.13}$$

is straightforward, since the field tensor, $G^a_{\mu\nu}$, and the covariant derivative D_μ are given by the formulae

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \underset{\substack{\uparrow\\ g}}{g f_{abc}} A^b_\mu(x)A^c_\nu(x) \tag{1.14}$$

and

$$D_\mu \equiv \partial_\mu - \frac{i}{2}\, \overset{\substack{g\\ \downarrow}}{g} A^a_\mu \lambda^a \tag{1.15}$$

with f_{abc} the structure constants given by

$$[\lambda^a, \lambda^b] = 2i f_{abc}\lambda^c. \tag{1.16}$$

Feynman rules [32], in particular the expressions for the elementary vertices of the theory (Fig. 1.1), are thus controlled by g. Furthermore, because we are working in a (strong) field theory, all Green's functions and in particular three point vertices get (appreciably) modified by higher quantum corrections (Fig. 1.2) and this is calculable because the theory is renormalizable [33]. The effective strength of a vertex thus depends on the values of the external momenta; to leading order, it just depends on the big momentum involved

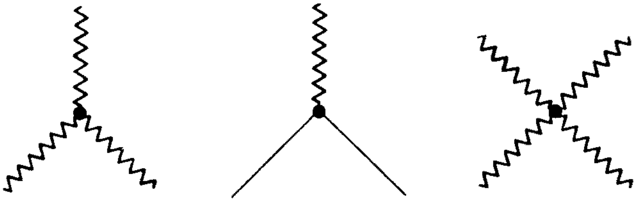

Fig. 1.1. Basic vertices of QCD (ignoring ghosts)

$$\alpha_s \equiv \frac{\bar{g}^2}{4\pi} \equiv \alpha_s(q^2). \tag{1.17}$$

The nature of the theory makes the variation with scale of α_s peculiarly susceptible to a renormalization group equation (RGE) analysis [34] which yields the following explicit formula [35] useful at small and moderate $\bar{g}$'s:

$$\frac{d\bar{g}}{d(\ln q)} = \beta(\bar{g}) = -\frac{\beta_0 \bar{g}^3}{16\pi^2} - \frac{\beta_1 \bar{g}^5}{(16\pi^2)^2} + \dots \tag{1.18}$$

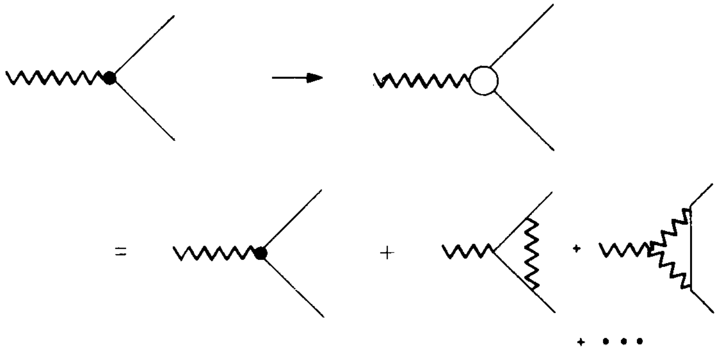

Fig. 1.2. Modification of $\bar{q}qg$ vertex by higher quantum corrections

For all practical purposes, the coefficients β_0 and β_1 are gauge and renormalization scheme independent [36], the explicit values being

$$\beta_0 = 11 - \tfrac{2}{3} N_f, \tag{1.19a}$$

$$\beta_1 = 102 - \tfrac{38}{3} N_f, \tag{1.19b}$$

with N_f the number of (active) flavours. The scale parameter, Λ, is conventionally introduced as an integration constant [37]

$$\int_0^{\bar{g}^2} \frac{d\bar{g}^2}{\bar{g}\beta(\bar{g})} = \ln(Q^2/\Lambda^2). \tag{1.20}$$

Solving to lowest order (i.e. dropping β_1 and all higher terms in (1.18)) gives the familiar lowest order approximation [38]

$$\alpha_s^{\mathrm{LO}} = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}. \tag{1.21}$$

The corresponding second order approximation is most conveniently expressed as an implicit relation for α_s [39]

$$\frac{4\pi}{\beta_0 \alpha_s} - \frac{\beta_1}{\beta_0^2} \ln\left(1 + \frac{4\pi\beta_0}{\beta_1 \alpha_s}\right) = \ln(Q^2/\Lambda^2). \tag{1.22}$$

The resulting dependence of α on Q^2/Λ^2 from (1.21) and (1.22) is shown in Fig. 1.3 for $N_f = 3,4$ and 5.

Since α_s is presumed not to be very small at presently accessible energies (just how small or large is currently a matter of debate, as will be discussed), it is important to exercise the options left in its definition so as to optimise perturbation expansions for various physical quantities. The options in question are principally to do with which renormalization scheme is adopted ("minimal subtraction" (MS), "modified minimal subtraction" ($\overline{\text{MS}}$) and "momentum subtraction" (MOM) being popular, practicable schemes [40]); and there is also gauge dependence within these schemes (such dependence is rather slight for reasonable choices of gauge). Since to the order we are working, $\beta(\bar{g})$ of Eqs. (1.18) and (1.20) is independent of scheme, all adjustments concentrate into a re-scaling of Λ [41]:

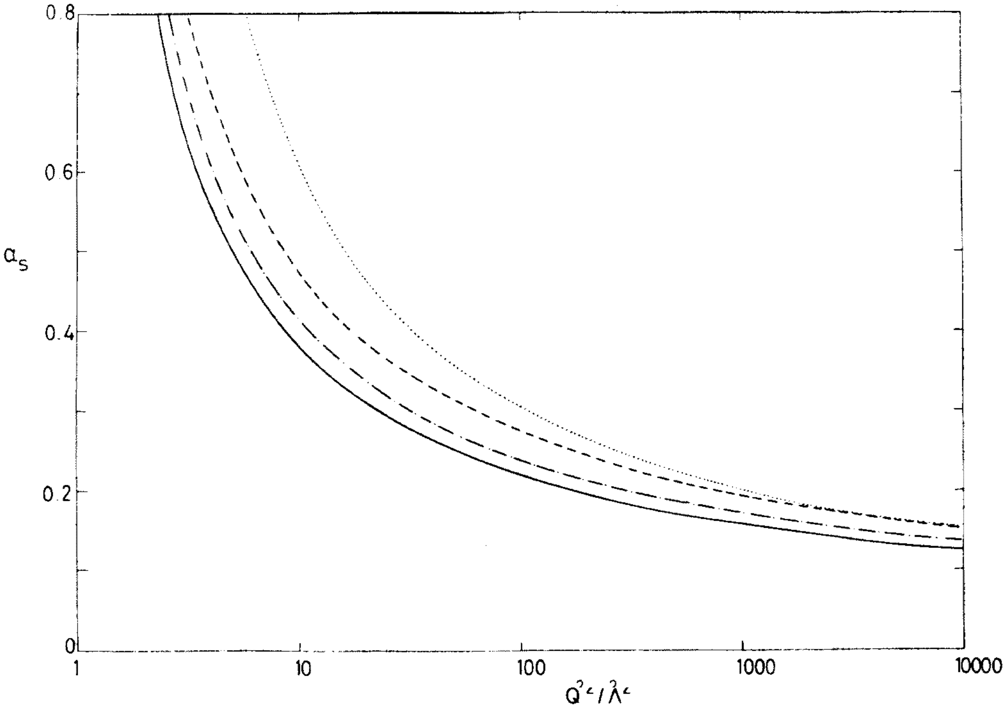

Fig. 1.3. Functional dependence of the strong coupling constant $\alpha_s(Q^2)$: according to second order formula (1.22) with $N_f = 3$ (———), $N_f = 4$ (—·—·) and $N_f = 5$ (— — —), and according to the first order approximation (1.21) (· · · ·)

$$\Lambda_{\overline{\text{MS}}} = 0.46\Lambda_{\text{MOM}} = 2.66\Lambda_{\text{MS}}. \tag{1.23}$$

Note however that this simple conversion formula does not suffice to translate *determinations* of α_s from data according to one (truncated) perturbation scheme into those according to another.

Comparisons and codifications of the effective coupling in QCD in various physical situations are bedevilled by the choices and ambiguities inherent in a strong coupling regime. The scale parameter, Λ, which has been a popular vehicle for phenomenology, exposes all these problems in magnified form. To lowest order, it is inherently ambiguous [42], since changes of scale ($\Lambda' = c\Lambda$) only modify higher order corrections which are in any case being ignored. This in no way prevents the use of formula (1.21) as a rough approximation over a limited energy range; however, it is difficult to achieve any great precision since the defining formula involves an exponential

$$\Lambda^{\mathrm{LO}} = Q \exp\left[-2\pi/\beta_0\alpha_s^{\mathrm{LO}}(Q^2)\right]. \tag{1.24}$$

Finally, there is the scheme dependence exemplified in Eq. (1.23), but this complication is shared by $\alpha_s(q^2)$ itself.

(b) Determinations of α_s briefly reviewed

Most of the phenomenological determinations of α_s to be touched on in the following brief survey do not expose the complications alluded to above. One simply implants a fixed coupling constant into the perturbation calculations which is supposed to serve for the energy domain in question (see for example Eq. (1.26) below). To date, the most important exception is the analysis of deep inelastic lepton scattering (DIL) [43] where the range of momentum transfer (q^2) investigated and the precision of the data makes a more sophisticated approach necessary and worthwhile.
That said we now proceed to list examples:

(*i*) Charmonium and related studies [44]

(a) Spectrum: Extensive potential model analyses have been made, notably by the Cornell group [45]. The form of potential which they use $V(r) = -\dfrac{k}{r} + \dfrac{r^2}{a^2}$ has already been mentioned (Eq. (1.8) above). If the coefficient of the Coulomb piece is interpreted as arising from one gluon exchange in static approximation, then the value emerging from the phenomenology corresponds to an effective coupling

$$\text{``}\alpha_s(\psi)\text{''} = \tfrac{3}{4}\, k \approx 0.4. \tag{1.25}$$

(b) Hadronic transitions: Motivated by the prospect of $\alpha_s(\psi)$ being relatively small in accord with motions of asymptotic freedom [38], Appelquist and Politzer [46] proposed at a very early stage that OZI violating decays of ($\bar{c}$c) compounds such as the J/ψ be computed perturbatively as is done for positronium decay. Hadronic decays are conceived as proceeding via an intermediate stage comprising the minimal number of gluons (3 for a 1^- colourless compound, 2 for a 0^- initial state). The initial wave function dependence (the $|\psi(0)|^2$ factor) is eliminated by taking the ratio with the leptonic width to yield the famous formula [46]

$$\frac{\Gamma(\mathrm{J}/\psi \to \text{hadrons})}{\Gamma(\mathrm{J}/\psi \to \mathrm{e}^+\mathrm{e}^-)} = \frac{\Gamma(\mathrm{J}/\psi \to 3\mathrm{g})}{\Gamma(\mathrm{J}/\psi \to \mathrm{e}^+\mathrm{e}^-)} = \frac{10(\pi^2-9)}{81\pi e_Q^2}\,\frac{\alpha_s^3}{\alpha^2},$$

$$(e_Q = \tfrac{2}{3},\ \alpha^{-1} \approx 137), \tag{1.26}$$

which on inserting the observed experimental branching ratios leads to the estimate

$$\alpha_s(\psi) = .19 \pm .02. \tag{1.27}$$

The hadronic width entering the above formula is understood to have all non-hadronic processes, including J/ψ → intermediate "γ" → hadrons, removed

$$\begin{aligned} \Gamma_{\text{had}} &= \Gamma_{\text{tot}} - N_l \Gamma_{\mu\mu} - \Gamma_{\text{“}\gamma\text{”hadrons}} - \Gamma_{\text{em}} \\ &= \Gamma_{\text{tot}}\{1 - (N_l + R_{\text{continuum}}(\psi))B_\mu\} - \Gamma_{\text{em}}. \end{aligned} \tag{1.28}$$

It is interesting to compare this estimate with the corresponding determination for the Υ (9.4) by the PLUTO group at DESY which was presented at this School [48]. Inserting their measurements, $\Gamma^{\Upsilon}_{ee} = 1.10 \pm 0.05$ keV, $R(\Upsilon)_{\text{continuum}} = 3.70 \pm 0.12$, $B_l = 3.5 \pm 1.4\%$ (where $\Gamma_{\text{tot}} = 35^{+25}_{-10}$ keV), they obtain

$$\alpha_s(\Upsilon) = 0.16^{+0.04}_{-0.02}. \tag{1.29}$$

Taken together, these two estimates (1.27) and (1.29) by exactly the same method and for values of Q^2 differing by a factor of 9.2 begin to constitute an interesting test of the predicted variation of $\alpha_s(Q^2)$. Comparing with Fig. 1.3 one sees that the observed variation of α_s is somewhat less than that suggested by *any* of the curves. (Note, however, that if we were to switch from the $N_f = 4$ curve at the J/ψ to the $N_f = 5$ curve at the Υ (9.4), the variation would be slightly *less* than that observed).

There exist analogous formulae involving the η_c (2.98) [49]. For example

$$\Gamma_{\eta_c} \approx \frac{20}{\alpha_s} \Gamma_{J/\psi}, \tag{1.30}$$

which suggests that Γ_{η_c} should lie in the range 2 to 5 MeV whilst new measurements by the "Crystal Ball Group" at SPEAR indicate a larger value $\Gamma^{\text{EXP}}_{\eta_c} = 20^{+16}_{-11}$ MeV [50] corresponding to a rather small α_s.

The simple perturbative formula for the $J/\psi - \eta_c$ mass-splitting [51] provides another relation

$$\Delta^{\psi\eta}_{\text{m}} = \frac{1}{2\alpha^2} \alpha_s \left(\frac{M_\psi}{m_c}\right)^2 \Gamma^{\psi}_{ee}, \tag{1.31}$$

which on inserting the observed mass splitting, $\Delta_m = 115$ MeV yields

$$\text{“}\alpha_s(\psi)\text{”} \approx 0.28 m_c^2 \approx 0.6 \quad (\text{for } m_c = 1.5 \text{ GeV}). \tag{1.32}$$

Finally, there is the analogue of (1.26) [49]

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(\eta_c \to gg)} = \tfrac{8}{9}\left(\frac{\alpha}{\alpha_s}\right)^2, \tag{1.33}$$

which will be very interesting once the experimental γγ branching ratio for η_c (2.98) is known.

A further interesting prediction for which there already exists some relevant data [52] concerns the decay $J/\psi \to \gamma$ + hadrons. In QCD terms, this is interpreted as $J/\psi \to \gamma gg$ so that the branching ratio compared to the all-hadronic decay reflects the simple substitution $g \to \gamma$. The theoretical prediction [49] is

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to 3g)} \equiv B_\psi^\gamma = \frac{72e_Q^2\alpha}{10\alpha_s} = \frac{0.023}{\alpha_s(\psi)}. \tag{1.34}$$

The observed branching ratio (after allowance for "hadronic" γ's from η's and π^0's) comes out roughly of the expected magnitude for $\alpha_s \approx 0.2$ but the spectrum shape is not in accord with simple theory [53].

The above results stem [49] from a combination of a simple zero range (non-relativistic) approximation for the initiating annihilation (probability of the $c\bar{c}$ pair "presenting" themselves for annihilation proportional to $|\psi(0)|^2$) and lowest order perturbative QCD for the subsequent reaction which transforms the constituents. Formula (1.33) is known to receive large corrections from this latter source [53a]. The hope is that wave functions corrections may be to a large extent eliminated by expressing predictions in the form of ratios as above. There are certainly liable to be large corrections if this is not done and there may be remnant effects even if it is done [54].

(*ii*) Analysis of Jet Broadening in e^+e^- collisions

In QCD, this well documented phenomenon (cf. talk by P. Söding at this school [55]) is attributed to gluon bremsstrahlung leading to predictions [56] of the type

$$\langle p_\perp^2 \rangle \approx (\text{known const}) \times \alpha_s(Q^2) \times Q^2 + \text{"primordial" contributions arising from confinement.} \tag{1.35}$$

In order to extract α_s from data on jets, one has to model the non-perturbative effects [56]. The outcome from analyzing the TASSO group's data is [55]

$$\alpha_s(30\ \text{GeV}) = 0.17 \pm \underset{(\text{statistical})}{0.02} \pm \underset{(\text{systematic})}{0.03} \tag{1.36}$$

with similar findings from other groups at DESY. In relation to (1.29) and the expected variation with Q^2 (Fig. 1.3), 0.17 would be an unexpectedly large value for α_s at $Q^2 = 10\,M_\Upsilon^2$.

(*iii*) Exclusive EM Form-Factors of Hadrons

As discussed at this School by J. Gunion [57], there exist predictions [58] from perturbative QCD for the asymptotic Q^2 dependence of EM form factors, for example

$$Q^4 G_M^p(Q^2) \underset{\text{large } Q^2}{\sim} \text{const}\,[\alpha_s(Q^2)]^\varepsilon, \tag{1.37}$$

where ε is a known positive power. Since there is little variation of $Q^4 G_M^p$ over the considerable range of large Q^2 covered by experiment, this leads to small estimates [58] for α_s or equivalently for the QCD scaling parameter, $\Lambda : \Lambda \lesssim 0.1$ GeV.

(*iv*) $\Delta R_{\text{plateau}}$ in e+e− annihilation

In principle, the departure of $R \equiv \sigma(e^+e^- \to \text{hadrons})/(\sigma(e^+e^- \to \mu^+\nu^-)$ from its parton model plateau value, $R_{\text{part}} = 3 \times \sum_{\text{active flavours}} e_Q^2$ affords a measure of $\alpha_s(Q^2)$ (cf. §2) and there exist dispersion relation methods [59] which accomplish the necessary smoothing in Q^2. What is lacking is precise comprehensive data.

(*v*) Light flavour spectroscopy

Notional averaged values of $\langle\alpha_s\rangle$ at low Q^2 may be inferred from the phenomenology of light quark hadrons, especially from spin-orbit splittings. It is interesting to note the spread of resulting values [60].

(*vi*) Deep inelastic lepton scattering

By far the best researched and most discussed phenomenon for calibrating QCD is inclusive inelastic lepton scattering [61] (Fig. 1.4(a))

$$l + N \to l' + X. \tag{1.38}$$

The cross-section at given incoming lepton beam energy E is a function of two variables $Q^2 = -q^2$ and $\nu = p \cdot q$ (Fig. 1.4(a)) (corresponding to measurement of the angle and momentum of the outgoing lepton) which are often re-expressed in terms of the dimensionless variables $x \equiv Q^2/\nu$ (Bjorken x) and $y \equiv \nu/E$. Factorizing out the appropriate (known) point-like cross-section, one can express the observed intensity in terms of structure functions

$$\frac{d^2\sigma}{dxdy}\Bigg/\left(\frac{d^2\sigma}{dxdy}\right)_{\text{point-like}} = y^2 x F_1 + \left(1 - y - \frac{m_N x y}{2E}\right) F_2 + \begin{pmatrix} 0 \\ \mp \end{pmatrix} xy(1 - y/2) F_3 \tag{1.39}$$

with

$$F_i = F_i(x, Q^2). \tag{1.40}$$

The last term in (1.39) is absent for charged lepton scattering, has the coefficient (−) for ν scattering and (+) for ν̄. For practical reasons, F_1 is not well-measured.

In the parton model [62], absorption of the incoming gauge quantum ("γ" or "W±") is pictured as proceeding via a simple point-like coupling to a freely moving constitutent of the struck hadron (Fig. 1.4(b)). In the absence of further interactions, the F_i would then be independent of Q^2 (i.e. would "scale") with their x-dependence reflecting the distribution of parton momenta. In QCD, this simple picture gets modified by coupling of the struck quark to the gluon field (Fig. 1.4(c)) which issues in a specific Q^2 dependence of the F_i (violation of scaling). Since the F_i are also functions of the kinematic variable x, and there are several of them available to experiment, they provide an exceptionally versatile laboratory for calibrating QCD.

Predictions are most conveniently expressed in terms of moments [63] ($M_{ni}(Q^2) \equiv \int_0^1 dx\, x^{n-2} F_i(x, Q^2)$ or simple variants thereof [64]) of the structure functions. For example

$$M_{n2}^{\mathrm{NS}}(Q^2) = A_n(\alpha_s(Q^2))^{\varepsilon_n}\,[1+C_n/\alpha_s^{\mathrm{LO}}(Q^2)+\ \ldots]$$
$$(\equiv [M_{n2}^{\mathrm{NS}}(Q^2)]_{\text{leading log}}) \qquad (1.41)$$

with ε_n a known power, C_n a known coefficient depending like α_s on the renormalization scheme [63] (recall the discussion above Eq. (1.23) and see Eq. (1.21) for definition of α_s^{LO}) and A_n an overall constant to be taken from experiment. This is the dominant evolution pattern [65] for the "non-singlet" (NS) combination of nucleon structure functions ($F_2^{\mathrm{NS}} \equiv F_2^{\mathrm{p}}-F_2^{\mathrm{n}}$) at large Q^2 according to perturbative QCD; the corresponding singlet moments ($M_i^{\mathrm{S}} \equiv \frac{1}{2}(M_i^{\mathrm{p+n}})$) obey a slightly more complicated equation [66] since there are contributions from both quark and gluon constituents and they "evolve" differently. The non-singlet moments have therefore received most attention.

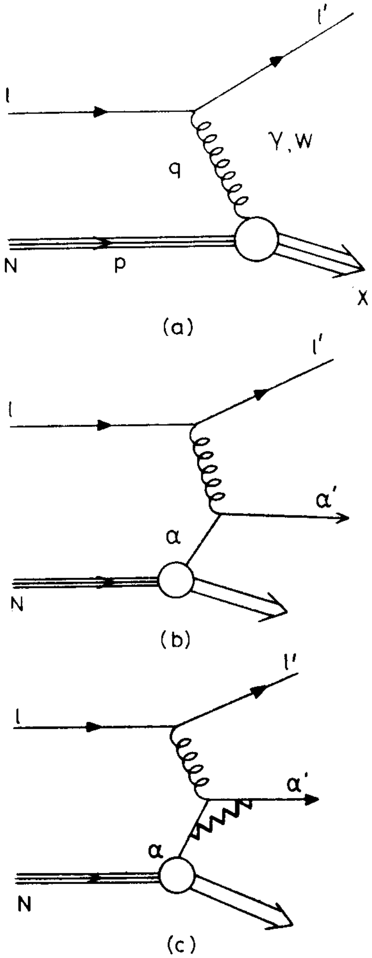

Fig. 1.4. Deep Inelastic Lepton Scattering: (a) the basic physical process (b) the parton picture (c) example of a QCD radiative correction to (b)

If Eq. (1.41) were guaranteed to be the whole story, it would be straightforward to extract α_s from the charged lepton and neutrino data. However, various mechanisms are known to exist which could produce appreciable power corrections (so called "higher twist" contributions [27])

$$M_{n2}^{\mathrm{NS}}(Q^2) = [M_{n2}^{\mathrm{NS}}(Q^2)]_{\text{leading log}} \times \left[1 + \frac{\mu_{n,4}}{Q^2} - \frac{\mu_{n,6}}{Q^4} + \dots\right] \tag{1.42}$$

to the leading logarithmic dependence via $\alpha_s(Q^2)$ which is explicitly shown in (1.41). In order to distinguish powers from logarithms, one needs a long lever arm in Q^2; this is achieved [67] by combining the very precise ep data from SLAC [68] with high energy neutrino data [69] (using and verifying the famous relation [70] $F_2^{eN} = \frac{5}{18} F_2^{\nu N}$). Two recent analyses along these lines [67, 71] find a substantial α_s dependence (i.e. a relatively large Λ) and little scope for power corrections; if the high energy data is omitted, there is almost indefinite scope for trading α_s dependence for power corrections [72]. Since the new analyses essentially uphold the neglect of power corrections, they find values of Λ similar to those originally determined from the high energy neutrino data [73]. Duke and Roberts' best fit yields [67]

$$\begin{aligned} \Lambda_{\overline{\mathrm{MS}}} &= 0.41 \pm .05 \text{ GeV (proton)}, \\ \Lambda_{\overline{\mathrm{MS}}} &= 0.52 \pm .10 \text{ GeV (NS n}-\text{p)}. \end{aligned} \tag{1.43}$$

The value $\Lambda_{\overline{\mathrm{MS}}} = 0.41$ corresponds (Fig. 1.3) to $\alpha_s^{\overline{\mathrm{MS}}}(\psi) \approx 0.3$; the associated $\Lambda_{\mathrm{MOM}} \approx 1$ GeV (Eq. (1.23)) and yields $\alpha_s^{\mathrm{MOM}}(\psi) \approx 0.45$.

This whole area of phenomenology is still controversial; for example, Donnachie and Landshoff [74] have pointed to the discrepant power behaviour of proton and neutron structure functions at large x and Q^2 as evidence of persisting higher twist effects [74]. Time plus improved and extended data will tell. Meanwhile, for the following discussion, we shall take the view that scaling violation in deep inelastic scattering *is* largely controlled by leading perturbative corrections and consequently that Λ is similar to the estimate (1.43) given above. It is interesting that comparable values are inferred from lattice calculations [75].

2. *QCD and duality*

An obvious challenge to QCD is to describe the systematics of hadron spectroscopy. This entails somehow accommodating the fact of quark confinement and adapting calculational methods to comprehend bound states. Some alternative methods were mentioned in §1.3 [2, 3].

The basic idea is to by-pass detailed aspects of confinement and determine a simplified set of universal dynamical coefficients by appeal to the duality principle [76]: analytic approximations to analytic functions continue to approximate on the average. In equations, if

$$\Pi(s) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\operatorname{Im} \Pi(s')}{s' - s}\, ds' \underset{\text{for large } s}{\approx} \hat{\Pi}(s) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{\operatorname{Im} \hat{\Pi}(s')}{s' - s}\, ds', \tag{2.1}$$

then

$$\int_{s_0}^{\infty} \frac{\operatorname{Im} \Pi(s')}{s'-s}\, ds' \xrightarrow[s\to\infty]{} \int_{s_0}^{\infty} \frac{\operatorname{Im} \tilde{\Pi}(s')}{s'-s}\, ds' \tag{2.2}$$

and for (2.2) to be true, $\operatorname{Im}\tilde{\Pi}(s')$ has to continue to resemble $\operatorname{Im}\Pi(s')$ *on the average* as s' is continued below the asymptotic region. Pictorially this notion, of which a familiar example is Bloom–Gilman duality [77] in electro-production, is illustrated in Fig. 2.1. An immediate consequence is the possibility of deriving a whole variety of sum rules

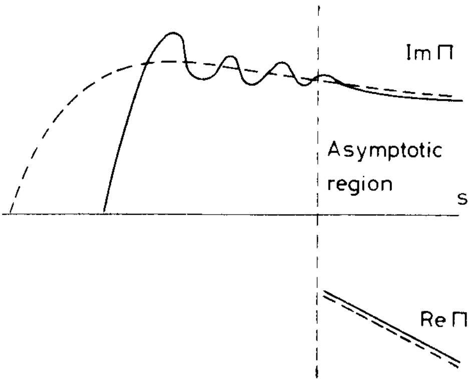

Fig. 2.1. Qualitative illustration of duality

$$\int_{s_0}^{\infty} W(s') \operatorname{Im} \Pi^{\mathrm{EXP}}(s') ds' \cong \int_{s_0}^{\infty} W(s') \operatorname{Im} \Pi^{\mathrm{THY}}(s') ds' \tag{2.3}$$

relating weighted sums (in the applications to be described, the weight $W(s')$ is either a negative power $(s')^{-n-1}$ or negative exponential e^{-s'/M^2}) of experimentally accessible cross-sections, $\operatorname{Im}\Pi^{\mathrm{EXP}}$, to the corresponding relatively simple theoretical expressions, $\operatorname{Im}\Pi^{\mathrm{THY}}$, which are valid at high and intermediate energies. Such relations will be useful providing there is a significant overlap between the energy regions covered by theory and experiment.

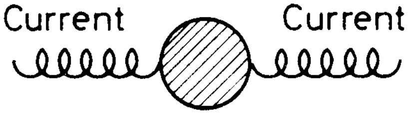

Fig. 2.2. The generic vacuum polarization graph

Applications [78] concern various species of vacuum polarizations (Fig. 2.2), the example which will feature most in the sequel being that associated with the charm current,

$\Pi^{(c)}$, defined by:

$$\Pi^{(c)}_{\mu\nu} \equiv i\langle 0|\int dxe^{iqx}T\{J^{(c)}_{\mu}(x), J^{(c)}_{\nu}(0)\}\,|0\rangle \equiv (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(c)}(q^2),$$
$$(J^{(c)}_{\mu} \equiv \bar{c}\gamma_\mu c). \qquad (2.4)$$

Alternative Π's come from changing either the flavour or tensor character of the currents entering (2.4). $\Pi^{(c)}$ is particularly suitable for phenomenology because its absorptive part is directly proportional to R_c, the contribution to $R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ from charm [79]

$$\Pi^{(c)}(s) = \frac{s^2}{12\pi^2 Q_c^2}\int \frac{R_c(s')}{(s'-s)\,(s')^2}\,ds', \quad (Q_c = \tfrac{2}{3}). \qquad (2.5)$$

Before entering into details, it is worth remarking that for this case, we know a priori that we have a good chance of deriving sensible duality relations [80]. This observation comes from considering alternative expressions for elementary contributions to R_c: on the one hand, the dominant low-energy contribution to the left hand side of Eq. (2.3) comes from the J/ψ

$$R_\psi \approx \frac{9\pi^2}{\alpha^2}\,\Gamma_{\psi\to ee}M_\psi\delta(s-M_\psi^2); \qquad (2.6)$$

on the other hand

$$R^{\text{parton model}} = 3\times\tfrac{4}{9} = \tfrac{4}{3} \equiv R^{\text{pm}}. \qquad (2.7)$$

Defining a "duality interval", ΔM^2 by the requirement, $\langle R_\psi\rangle = \langle R^{\text{pm}}\rangle$, one obtains a "sensible" number in relation to the charmonium level spacing

$$\Delta M^2 = \frac{27\pi}{4\alpha^2}\,\Gamma_{ee}M_\mu = 5.8\ \text{GeV}^2$$
$$\approx M_{\psi'}^2 - M_\psi^2 = 4\ \text{GeV}^2. \qquad (2.8)$$

One is thus encouraged to pursue duality relations of the type (2.3) with more detailed and realistic insertions for $\text{Im}\Pi^{\text{EXP, THY}}$.

Two main routes have been pursued. The first [59] aims to devise precise determinations of specific parameters e.g. $\alpha_s(Q^2)$; for this, the best strategy is to evaluate sum rules at high s where perturbative QCD supplies a simple prediction for the dominant correction to R^{pm}. The alternative [24] is to aim at a more global picture, evaluating the sum rules at lower s and thereby, hopefully, achieving some overall parameterization of confinement. Provided this is successful, one should be able to apply the same description in a variety of situations — $(\bar{c}c)$, $(\bar{b}b)$, $(\bar{c}q)$, $(\bar{q}q)$ … .

This latter has been the programme of the ITEP group, Shifman, Vainshtein and Zakharov (SVZ) [24] and co-workers. Their idea is that as one reduces s from asymptotic, values, departures from perturbative QCD at first enter in a simple and controlled way via specific *power corrections* (higher twist contributions) (Fig. 2.3); such power corrections arise from spontaneous symmetry breaking (§1.3(c)) expressed through non-vanishing vacuum expectation values such as

$$\langle \alpha_s G^2 \rangle_0 \equiv \langle \alpha_s G^a_{\mu\nu} G^{a\mu\nu} \rangle_0 \neq 0. \tag{2.9}$$

This circumstance enables SVZ to push the theoretical interpolation down into the resonance region; and thence, via duality relations, to build connections between resonance parameters and QCD. In this way, they achieve a complete outline parameterization akin to the bag model but much more flexible. Parameters can be determined in one situation then applied in another.

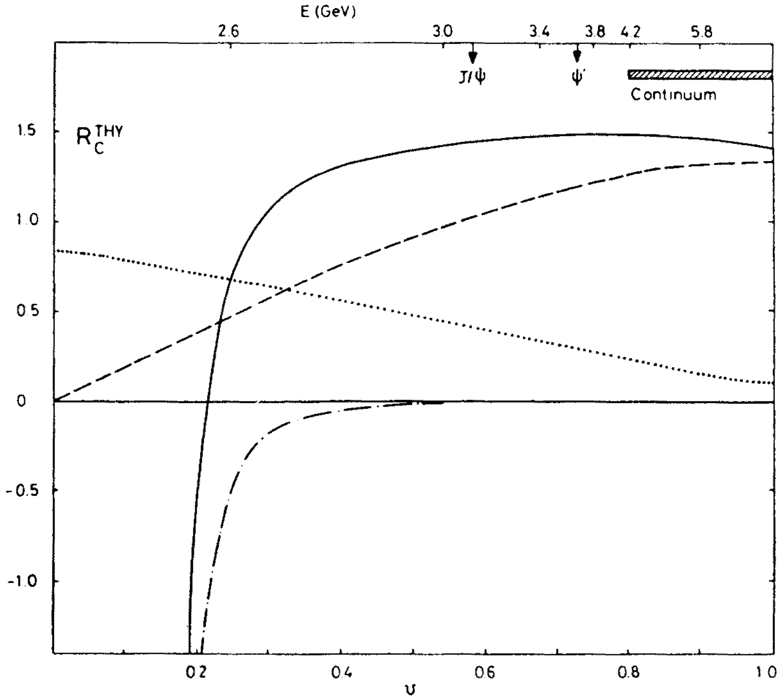

Fig. 2.3. Contributions to R_c^{THY} (2.15) as a function of $v = [1-4m_c^2/s]^{1/2}$: (———) total; (– – –) parton contribution (2.16); (· · · · ·) perturbative correction based on (2.17); (—·—·) power correction based on (2.18)

It is instructive to follow through SVZ's analysis [81] of $\Pi^{(c)}$ in some detail, since it provides their estimate of $\langle \alpha_s G^2 \rangle_0$. For this case, they employ simple power moments

$$M_n \equiv \frac{1}{12\pi^2 Q_c^2} \int \frac{R_c(s')}{(s')^{n+1}}\, ds' = \frac{1}{n!}\left(-\frac{d}{ds}\right)^n \Pi^{(c)}(s)\bigg|_{s=0}. \tag{2.10}$$

An alternative to taking s^{-n} moments, which SVZ adopt for analyzing light quark systems (see Eq. (2.28) below), is to use "Borel-ization"

$$\mathop{L}_{\substack{s,n\to\infty\\ s/n=M^2\ \text{fixed}}} \frac{1}{(n-1)!}\, s^n \left(-\frac{d}{ds}\right)^n \underset{(s)}{\overset{(f)}{\prod}} = \frac{1}{12\pi^2 Q_f^2}\int ds e^{-s/M^2 R^{(f)}(s)} \tag{2.11}$$

(f) = {flavour, tensor character}. For heavy quark systems, such as that described by $\Pi^{(c)}$, it is not necessary to introduce this elaboration, since $s = 0$ is already far removed from the threshold, $s = 4m_c^2$; one can also generalize (2.10) to refer to an arbitrary negative (space-like) squared energy $Q_0^2 = -4m_c^2\xi$ [82]. SVZ just use (2.10) unmodified and their sum-rules take the simple form

$$M_n^{\text{THY}} = M_n^{\text{EXP}}. \tag{2.12}$$

It is straightforward to compute the right hand side of (2.12) from (2.10) using an R_c^{EXP} comprising a sum of resonance terms like (2.6) and a contribution which models the continuum. The distinctive feature of SVZ's analysis is their treatment of R_c^{THY}.

Their starting point is an operator product expansion (OPE) [26] for the quantity

$$\Omega_{\mu\nu} \equiv T\{J_\mu(x), J_\nu(0)\} \tag{2.13}$$

appearing in (2.4). The outcome, which immediately extends to the analogous operators appearing in other $\Pi^{(f)}$'s, is exemplified by the Feynman graphs shown in Fig. 2.4. Up to this point, it is natural that two classes of operator should appear: those like (a) and (b) which communicate with the perturbative vacuum of QCD; and those like (c) and (d) containing extra external lines which do not. (It is *assumed* [83] that the coefficient functions of either class may be computed perturbatively.) The effects of spontaneous symmetry breaking [84] manifest themselves in the following step in which the vacuum expectation value of $\Omega_{\mu\nu}$ is formed to yield $\Pi_{\mu\nu}^{(c)}$

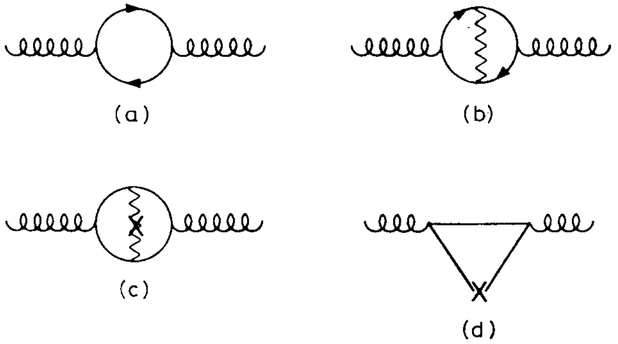

Fig. 2.4. Feynman graphs contributing to $\Omega_{\mu\nu}$ (2.13)

$$\Pi_{\mu\nu}^{(c)} = \langle\Omega_{\mu\nu}\rangle \tag{2.14}$$

and consequently R_c^{THY} (using (2.4), 2.5)).

For heavy quark systems, only (a), (b) and (c) of Fig. 2.4 contribute to leading order in α_s [85], and SVZ thus derive the estimate [86]

$$R_c^{\mathrm{THY}} = R_c^{\mathrm{pm}}\left\{1+\frac{\alpha_s}{\pi}K\right\} - \frac{\langle\alpha_s G^2\rangle_0}{Q^4}L+O(\alpha_s^2), \tag{2.15}$$

where

$$R_c^{\mathrm{pm}} = \tfrac{4}{3}\left[\frac{v(3-v^2)}{2}\theta(s-4m_c^2)\right], \tag{2.16}$$

$$K = \frac{4\pi}{3}\left[\frac{\pi}{2v}-\frac{(v+3)}{4}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right], \tag{2.17}$$

$$L = \frac{\pi}{6v^5}(1-v^2)^2(1+v^2), \tag{2.18}$$

and the charmed quark mass, m_c, enters the formulae via the quantity

$$v = [1-4m_c^2/s]^{1/2}. \tag{2.19}$$

Given (2.15), it is simple to derive the moments M_n^{THY} and thence to compute the ratios, r_n, of successive moments [87]

$$r_n^{\mathrm{THY}} \equiv \frac{M_n^{\mathrm{THY}}}{M_{n-1}^{\mathrm{THY}}} = \left(\frac{n^2-1}{n^2+\frac{3}{2}n}\right)\frac{1}{4m_c^2}\{1+K_n\alpha_s-L_n\phi\}. \tag{2.20}$$

Here, ϕ is a dimensionless measure of the gluon condensate

$$\phi \equiv \frac{4\pi^2}{9}\left\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^aG^{a\mu\nu}\right\rangle_0(4m_c^2)^{-2} \tag{2.21}$$

and K_n and L_n are known coefficients. The corresponding experimental ratios are easily calculated from data on the ψ's (for large n, r_n^{EXP} is dominated by the J/ψ: $r_n^{\mathrm{EXP}} \to (M_{J/\psi})^{-2} \sim 0.1\ \mathrm{GeV}^{-2}$) with allowance for an extra contribution to R_c from the continuum [87]. All is now set for a comparison of theory and experiment, hopefully vindicating the approximation (2.20) and determining its parameters. The result [87] which is shown in Fig. 2.5 from reference [24] constitutes the primary evidence for and calibration of ϕ. If ϕ were zero, r_n^{THY} would follow the open circles on the figure (SVZ compute for $\alpha_s = 0.2$ as given by (1.27) but this is unimportant since the perturbative corrections to r_n are small e.g. 7% for r_5). Inclusion of a suitable non-zero ϕ contribution yields the inverted solid triangles in good agreement with "experiment" for $n \leqslant 8$. The lowest moments are used to fix m_c, the effective "duality" charmed quark mass, which should probably be viewed as the current mass of the c quark (cf. discussion in §1.2 vi(c)). The values which SVZ determine for these parameters are

$$m_c \cong 1.26\ \mathrm{GeV}, \tag{2.22}$$

$$\phi = 1.35\times10^{-3}, \tag{2.23}$$

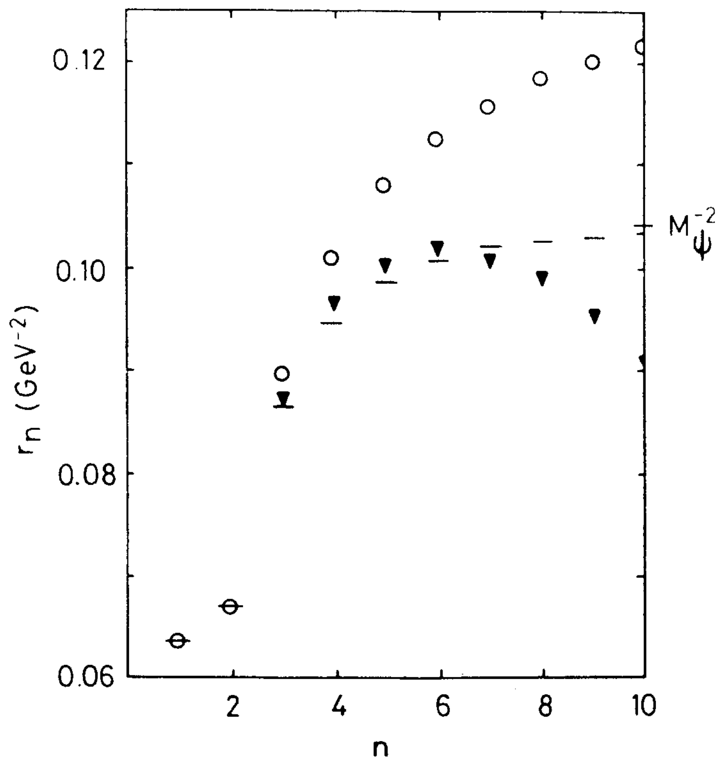

Fig. 2.5. Theory and experiment compared for ratios of power moments of R_c (Eqs. (2.10), (2.20), $r_n = M_n/M_{n-1}$: – experiment, ○ theory omitting power corrections; ▼ theory including power corrections — parameters as in Eqs. (2.22)–(2.24)) (based on Ref. [81])

whence

$$\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0 \approx 0.012\,\text{GeV}^4. \tag{2.24}$$

The latter corresponds [25] to a negative vacuum energy density $\varepsilon_0 = -0.0035\,\text{GeV}^4$ akin to the MIT bag pressure [88].

SVZ have extended their analysis to various other systems; examples concerning light quark compounds [89] are mentioned below. By the nature of things, applications of duality to light quark systems tend to be more speculative. Great importance therefore attaches to extending the scope of application within the heavy quark sector. Such has been the work of Reinders, Rubinstein and Yazaki (RRY) [82, 90] who have extended the above analysis to embrace all the currents coupling to S and P wave $c\bar{c}$ compounds. Determining the theoretical formulae analogous to (2.20) entails laborious Feynman graph evaluations [82]; once these are done, it is straightforward to confront the various $[r_n^J]^{\text{THY}}$ with the corresponding experimental ratios [90]

$$[r_n^J]^{\text{EXP}} = \frac{1}{(M_R^J)^2 + Q_0^2}(1+\delta_n^J) \equiv [r_n^J]_{\substack{\text{Lowest}\\ \text{resonance}}} \times (\text{correction for higher states}). \tag{2.25}$$

Here, M_R^J is the mass of the lowest resonance contributing (e.g. M_{η_c} for the 0^- current) and Q_0^2 designates the space-like point at which the moments (2.10) are evaluated. The factor $(1+\delta_n^J)$ which represents the effect of higher states is unconstrained by experiment (except for the vector current), however its effects may be minimized either by avoiding small values of n or by forming ratios $r_n^J/r_n^{J'}$. Arguing in this way, RRY determine preferred values for the theoretical parameters m_c, α_s and ϕ (as defined in (2.21)) and predict masses for the η_c and intermediate χ states [90]:

$$\begin{aligned} m(\mathrm{J}/\psi)-m(\eta_c) &= 80\pm 30\ \mathrm{MeV}\ \ [115], \\ m(^3P_2) &= 3.54\pm.01\ \mathrm{GeV}\ \ [3.55], \\ m(^3P_1) &= 3.50\pm.01\ \mathrm{GeV}\ \ [3.51], \\ m(^3P_0) &= 3.40\pm.02\ \mathrm{GeV}\ \ [3.41], \end{aligned} \tag{2.26}$$

in good agreement with experiment [91]. They also predict a mass for the as yet unobserved $C = -1$ axial; $m(^1P_1) \simeq 3.51$ GeV. That $m_{\eta_c} \approx 3.0$ GeV (not 2.85) was predicted in several previous calculations [92] including SVZ's. The most impressive success of the new calculations is thus the emergence of the observed splittings of the P states [93]. Theoretical parameters for the $S(P)$ wave calculations come out similar to SVZ's:

$$m_c = 1.26(1.23)\ \mathrm{GeV}; \quad \alpha_s = 0.27(0.23); \quad \phi = 0.0012\ \ [94] \tag{2.27}$$

except that α_s is larger. It will be extremely interesting to see how the scheme transfers to other $\overline{\mathrm{Q}}\mathrm{Q}$, $\overline{\mathrm{Q}}\mathrm{q}$ compounds.

A final example of the application of QCD sum rules is provided by SVZ's discussion [89] of the ϱ and $\pi-\mathrm{A}_1$ dominated spectral functions. This entails using the machinery of QCD duality for much lower Q^2 and is consequently much more speculative. Nonetheless, the results are interesting; furthermore, adapting the method to light quark systems brings out some new points. Firstly, because of the proximity of $s = 0$ to threshold some modification of the simple moment procedure (2.10) is necessary. SVZ's answer is to opt for "Borel-ization" (Eq. (2.11)) which affords a more flexible sampling of low energy contributions. As previously, the major task is to assemble the various theoretical contributions to $R^{(f)}$ analogous to (2.15). Because light quarks are involved contributions like Fig. 2.4(d) now appear and there are more complicated four-quark operators which SVZ estimate by saturating with the vacuum intermediate state. The upshot for the $I = 1$ vector spectral function, $R^{I=1}(s)$ is a sum rule

$$\begin{aligned} \int_{s_0}^{\infty} ds\, e^{-s/M^2} R^{I=1}(s) = \tfrac{3}{2} M^2 \Bigg\{ 1 + \frac{\alpha_s(M^2)}{\pi} + \frac{4\pi^2}{M^4} \langle 0 | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle \\ + \frac{\pi^2}{3M^4} \left\langle 0 \left| \frac{\alpha_s}{\pi}\, GG \right| 0 \right\rangle + \frac{4\ \text{quark terms}}{M^6} \Bigg\}. \end{aligned} \tag{2.28}$$

Finally, inserting a conventional estimate for the light quark spontaneous symmetry breaking term

$$\langle 0|m_u\bar{u}u+m_d\bar{d}d|0\rangle \approx -\tfrac{1}{2}f_\pi^2 m_\pi^2 \simeq -1.7\times 10^{-4}\ \mathrm{GeV}^4 \tag{2.29}$$

and their own estimate (2.24) for the gluon condensate, $\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_0$, SVZ obtain the prediction

$$I(M^2) \equiv \int e^{-s/M^2}R^{I=1}(s)ds \simeq \tfrac{3}{2}M^2\left[1+\frac{\alpha_s(M)}{\pi}+0.1\left(\frac{M_\varrho^2}{M^2}\right)^2-0.14\left(\frac{M_\varrho^2}{M^2}\right)^3\right]. \tag{2.30}$$

(Note, the M^{-6} term which arises from the extra approximation of saturating the four quark operator with the vacuum intermediate state comes out quite large). Saturating the left-side of (2.30) with the ϱ and evaluating for $M = m_\varrho$ gives $12\pi^2 e^{-1}m_\varrho^2/g_\varrho^2$ ($e = 2.7183\ldots$), corresponding to the conventional definition of g_ϱ which entails $R_\varrho^{I=1} = \frac{12\pi^2 m_\varrho^2}{g_\varrho^2}\delta(s-m_\varrho^2)$. If all corrections to the right hand side of (2.30) are suppressed, there results the remarkable prediction [89]

$$\frac{g_\varrho^2}{4\pi} \simeq \frac{2\pi}{e} \simeq 2.3. \tag{2.31}$$

SVZ go on to discuss predictions for m_ϱ using $\frac{d}{dM^2}$ (Eq. (2.30)) and claim that the power correction terms stabilize the prediction as a function of the parameter M.

They also extend the analysis to the corresponding axial current, J_A [89]. Because J_A is not conserved there are now two spectral functions $\Pi_{1,2}$ which may be chosen such that the physical contributions to $\mathrm{Im}\Pi_{1,2}$ take the form

$$\begin{aligned}\mathrm{Im}\,\Pi_2\text{``}&=\text{''}\ \pi f_\pi^2\delta(s)+\text{``}A_1\text{''}+\text{``continuum''},\\ \mathrm{Im}\,\Pi_1\text{``}&=\text{''}\qquad\qquad\ \text{``}A_1\text{''}+\text{``continuum''}.\end{aligned} \tag{2.32}$$

Again saturating at $M = m_\varrho$ for Π_2 and keeping only the π contribution, they obtain

$$f_\pi = \frac{m_\varrho}{2\pi} = 125\ \mathrm{MeV}, \tag{2.33}$$

the corresponding experimental value being 133 MeV (in their normalization). They go on to argue that the full sum rules are again "stabilized" by the power corrections but do not give full details.

SVZ have applied QCD sum rules to other aspects of the vector nonet [95] and to estimating the gluonic content of the $I = 0$ scalar meson [96] and of the η′ [97]. Mention of this last topic serves to introduce the following section and to conclude this short description of QCD sum rules.

3. *QCD and current algebra*

Much progress has recently been made in understanding how QCD impinges on current algebra. The key elements of the problem and of its resolution have been in place for some time. On the one hand, the octet of ground state pseudo-scalars $\{\pi, K, \eta_8\}$ are supposed to be the Goldstone bosons corresponding to spontaneously broken chiral symmetry [23]. This model requires that m_π and m_K should vanish as the corresponding quark masses tend to zero and justifies the use of PCAC for the π and K fields [23]. This leaves a problem (the U(1) problem(s) [98]) with the η and η' masses, with the mixing structure of the scalar nonet and with understanding how $\eta \to 3\pi$. On the other hand, in QCD the divergence of the SU(3) flavour singlet axial vector current has an anomaly [99] (see Fig. 3.1 and Eq. (3.4) below) akin to the originally discovered effect in spinor electro--dynamics [100] which successfully explains the decays $\pi^0(\eta, \eta') \to \gamma\gamma$ (provided $N_c = 3$). It was therefore immediately proposed [99] that this anomaly should dispose of the U(1) problem. Many subtleties arise in implementing this [101]. Fresh insight has come recently from looking at the problem within the large N_c approximation [102] to QCD which has generated confidence to re-examine the Ward identities of current algebra with anomaly effects included [103]. This yields phenomenological estimates of the gluon content of the η and η' (in a sense to be made precise below). Combined with extra assumptions and using the observed rates for $\pi^0, \eta, \eta' \to 2\gamma$, this leads to new predictions for decay processes involving η's and η''s: $J/\psi \to \eta(\eta')\gamma$ [104], and a relation between $\eta' \to \eta\pi^+\pi^-$ and $\eta \to \pi^+\pi^-\pi^0$ [105]. Besides being interesting in their own right, these discussions generate a whole new approach to gluon couplings and may even have bearing on the elusive bound configurations of gluons (gluonia or glueballs [106]) which theorists hypothesise [107].

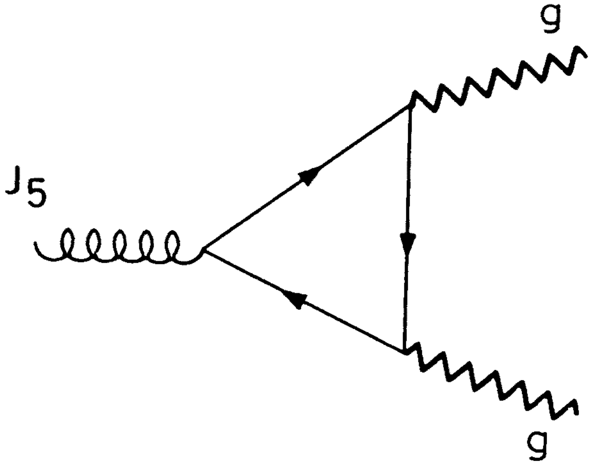

Fig. 3.1. Feynman graph yielding the axial anomaly in QCD

Ward Identities

The key parameters in the following discussion are the matrix elements of the two gluon operator [103] (cf. (1.12))

$$G\tilde{G} \equiv \sqrt{\tfrac{3}{2}}\,\frac{\alpha_s}{4\pi}\,G^{a\mu\nu}\tilde{G}^a_{\mu\nu} \tag{3.1}$$

between vacuum and η_a (η or η')

$$A_{\eta_a} \equiv (m_{\eta_a})^{-2}\langle 0|G\tilde{G}|\eta_a\rangle. \tag{3.2}$$

(The symbol $\tilde{G}^a_{\mu\nu}$ in (3.1) denotes the "dual" (gluon) field [108] $\frac{1}{2}\,\varepsilon_{\mu\nu\sigma\varrho}G^{a\sigma\mu}$ and the factor $(m_{\eta_a})^{-2}$ in (3.2) is inserted for dimensional convenience (cf. Eqs. (3.5), (3.6) below).) The coefficients A_{η_a} thus introduced can be thought of as measuring the coupling of the gluon field to the $I = 0$ pseudo-scalars. They are important parameters because of the existence of the axial anomaly in QCD, whereby the divergence of the flavour singlet axial vector current possesses an extra component. Besides the standard contribution arising from non-vanishing quark masses

$$D^i = -[Q^i_5, L_{SB}] \equiv [Q^i_5, (m_u\bar{u}u + m_d\bar{d}d + m_s\bar{s}s)] \tag{3.3}$$

there is a term (whose source in perturbation theory is indicated in Fig. 3.1) proportional to $G\tilde{G}$

$$\partial^\mu A^i_\mu = D^i + \delta_{i0}G\tilde{G}. \tag{3.4}$$

(Here, as usual, $A^i_\mu \equiv \bar{q}\gamma_\mu\gamma_5(\lambda_i/2)q$ with λ_i the Gell-Mann matrices.) The pseudo-scalar decay constants, F_{ia}, defined conventionally as matrix elements of (3.4) by

$$\langle 0|\delta^\mu A^i_\mu|P_a\rangle \equiv m^2_a F_{ia}, \qquad i = 0, 1, \ldots, 8,$$

$$\{P_a\} \equiv \{\pi, \mathrm{K}, \eta, \eta'\}, \tag{3.5}$$

furnish a measure of the explicit symmetry breaking effects (3.3), provided the anomaly contribution (3.2) is subtracted off [109]

$$F_{ia} - \delta_{i0}A_{\eta_a} = (m_a)^{-2}\langle 0|D^i|P_a\rangle. \tag{3.6}$$

Eq. (3.6) asserts that the anomaly should control the departure of the pseudo-scalar nonet from the "ideal" [110]. To see if this works and to draw consequences one needs to know how big the A_η's are.

Several estimates have been given [111]. In all but one case [104], they stem from saturating the Ward identities of current algebra with the ground state pseudo-scalar nonet [112]. This approximation scheme [113] (which is sketched in Appendix I) yields four equations [114] for the six coupling parameters ($F_{8\eta}$, $F_{0\eta}$, $F_{8\eta'}$, $F_{0\eta'}$, A_η, $A_{\eta'}$) relating to η and η'. These comprise the three relations [115]

$$L_{88} \equiv m^2_\eta F^2_{8\eta} + m^2_{\eta'}F^2_{8\eta'} = \tfrac{1}{3}(4m^2_{\mathrm{K}}F^2_{\mathrm{K}} - m^2_\pi F^2_\pi), \tag{3.7a}$$

$$L_{08} \equiv m_\eta^2 F_{8\eta} F_{0\eta} + m_{\eta'}^2 F_{8\eta'} F_{0\eta'} = \frac{-2\sqrt{2}}{3}(m_K^2 F_K^2 - m_\pi^2 F_\pi^2), \tag{3.7b}$$

$$L_{00} \equiv m_\eta^2 F_{0\eta}(F_{0\eta} - A_\eta) + m_{\eta'}^2 F_{0\eta'}(F_{0\eta'} - A_{0\eta'}) = \tfrac{1}{3}(2m_K^2 F_K^2 + m_\pi^2 F_\pi^2), \tag{3.7c}$$

together with the constraint [116]

$$m_\eta^2 F_{8\eta} A_\eta + m_{\eta'}^2 F_{8\eta'} A_{\eta'} = 0. \tag{3.8}$$

Variants of (3.7a, b, c) have a long history in discussions of the pseudo-scalars [117]; for example, in the absence of the anomaly contributions, the combination $(2L_{00} + 2\sqrt{2}\,L_{08} + L_{88})$ yields Weinberg's famous inequality $(m_{\eta_a} \leqslant \sqrt{3}\,m_\pi)$ for the η and η′ masses [118]. The derivation requires ones making symmetry assumptions [119] on the F's, as does Veneziano's discussion of the η and η′ mass-matrix in the presence of the anomaly [120]. His formula

$$M_{88}^2 = \tfrac{4}{3}\,m_K^2 - \tfrac{1}{3}\,m_\pi^2, \qquad M_{08}^2 = -\frac{2\sqrt{2}}{3}(m_K^2 - m_\pi^2),$$

$$M_{00}^2 = \tfrac{2}{3}\,m_K^2 + \tfrac{1}{3}\,m_\pi^2 + 3\lambda/N_c \tag{3.9}$$

gives a good account of the pseudo-scalar masses with a mixing angle of +14°, similar to other estimates. The occurrence of the factor N_c^{-1} in the anomaly contribution to (3.9) witnesses to Veneziano's analysis being from the standpoint of the large N_c expansion [102] whose application in this context was pioneered by Witten [121]. An important achievement of this picture is to unify the status of the pseudo-scalars as Goldstone particles: whereas the octet pseudo-scalar masses vanish as $m_q \to 0$ that of the iso-singlet vanishes as $N_c \to \infty$ (again cf. (3.9)). This has led to the suggestion [122] of extending PCAC to the η′ using $G\tilde{G}$ as the interpolating field

$$\phi_{\eta'} = \phi_{\eta'}^{(g)} \equiv (m_{\eta'})^{-2}(A_{\eta'})^{-1} G\tilde{G}; \tag{3.10}$$

this idea has been used to relate the decays $\eta' \to \eta\pi^+\pi^-$ and $\eta \to \pi^+\pi^-\pi^0$ with striking numerical success [124] (see (3.18) below).

Goldberg [123] and subsequent authors [103] have adopted a somewhat different interpolation procedure when deriving two further constraints on the F_{ia}'s from the electro-magnetic decays $\pi^0, \eta, \eta' \to \gamma\gamma$:

$$\begin{pmatrix} \phi_{\pi^0}^{(r)} \\ \phi_\eta^{(r)} \\ \phi_{\eta'}^{(r)} \end{pmatrix} = \begin{pmatrix} F_\pi^2 m_\pi^2 & 0 & 0 \\ 0 & F_{8\eta}^2 m_\eta^2 & F_{8\eta'}^2 m_{\eta'}^2 \\ 0 & F_{0\eta}^2 m_\eta^2 & F_{0\eta'}^2 m_{\eta'}^2 \end{pmatrix}^{-1} \begin{pmatrix} \partial^\mu A_\mu^3 \\ \partial^\mu A_\mu^8 \\ \partial^\mu A_\mu^0 \end{pmatrix}. \tag{3.11}$$

This assures [123] that the extrapolation to zero pseudo-scalar 4-momentum yields the simple Adler formula [100] for ratios of the decay amplitudes

$$R_\eta \equiv \frac{M(\eta \to \gamma\gamma)}{M(\pi^0 \to \gamma\gamma)} \xrightarrow[p_\mu \to 0]{} \frac{(F_{0\eta'} - 2\sqrt{2}\,F_{8\eta'})F_\pi}{\sqrt{3}\,(F_{8\eta}F_{0\eta'} - F_{0\eta}F_{8\eta'})}, \tag{3.12a}$$

$$R_{\eta'} \equiv \frac{M(\eta' \to \gamma\gamma)}{M(\pi^0 \to \gamma\gamma)} \xrightarrow[p_\mu \to 0]{} -\frac{(F_{0\eta} - 2\sqrt{2}\,F_{8\eta})F_\pi}{\sqrt{3}\,(F_{8\eta}F_{0\eta'} - F_{0\eta}F_{8\eta'})}. \tag{3.12b}$$

Equating the experimentally observed ratios to the theoretical formulae for $p_\mu \to 0$ on the right of (3.12) (a very considerable extrapolation especially for η') supplies two final equations which can be used along with (3.7) and (3.8) to fix the decay constants.

A completely independent constraint on A_η and $A_{\eta'}$ has been derived by Novikov et al. (NSVZ) [104] from consideration of $J/\psi \to \eta(\eta')\gamma$ decays. In their model, the anomaly couplings control these decays yielding the relation

$$\frac{\Gamma(\psi \to \eta'\gamma)}{\Gamma(\psi \to \eta\gamma)} = \frac{(k_{\eta'\gamma})^3}{(k_{\eta\gamma})^3}\left(\frac{A_{\eta'}}{A_\eta}\right)^2. \tag{3.13}$$

This supplied the sixth condition for the analysis of Milton et al. [124] in which the constraint (3.8) was omitted; Goldberg [123] using an approximate version of (3.7c) [115] did not need a sixth condition and was thus able to use (3.13) for prediction — with numerically satisfactory results.

Van Herwijnen and Williams [114] used (3.7), (3.8) and (3.12) in the form given above with the following results (in each case followed in brackets successively by those of Goldberg [123] and of Milton et al. [124]):

$$\begin{aligned} F_{8\eta} &\approx 1.2F_\pi(1.1, 1.02) & F_{8\eta'} &\approx -0.25F_\pi(-0.17, -0.15)\\ F_{0\eta} &\approx -0.17F_\pi(0.17, 0.07) & F_{0\eta'} &\approx 1.2F_\pi(1.1, 1.12)\\ A_\eta &\approx 0.65F_\pi(0.82, 0.53) & A_{\eta'} &\approx 1.0F_\pi(0.81, 0.47) \end{aligned} \tag{3.14}$$

It is interesting to subjoin to the above catalogue the outcome of Veneziano's analysis [120] (cf. Eq. (3.9) above) in which a lot more flavour symmetry is pre-supposed:

$$\begin{aligned} F_{8\eta}/F_{\text{av}} &= F_{1\eta'}/F_{\text{av}} = \cos\phi = .97,\\ F_{1\eta}/F_{\text{av}} &= -F_{8\eta'}/F_{\text{av}} = \sin\phi = .24,\\ A_\eta/F_{\text{av}} &= .18, \quad A_{\eta'}/F_{\text{av}} = .70,\\ F_{\text{av}} &= F_\pi = F_{\text{K}}(\text{expt} \approx 1.2F_\pi), \end{aligned} \tag{3.15}$$

and, also, to cite the results of Novikov et al. [104] for A_η and $A_{\eta'}$, the former from a version of current algebra argumentation (with strong flavour symmetry assumptions) and with the estimate for $A_{\eta'}$ emanating from a species of QCD sum rule (cf. §2). NSVZ's result is

$$A_\eta \approx F_\pi; \quad A_{\eta'} \approx 0.7F_\pi. \tag{3.16}$$

All the above estimates concur in finding the anomaly contributions quite large; there is also reasonable agreement on the form of the F's. In view of the drastic approximations made in the various calculations, that is probably all that should be demanded at this point.

Goldberg [106] has recently extended the above techniques to embrace a genuine 0^- glueball such as might account for the observed γ spectrum in $J/\psi \to \gamma X$ at large x [52] (cf. the discussion adjoining Eq. (1.34)). The predominance of gluons in the conjectured resonance (to which a mass $m_G \approx 2$ GeV is tentatively assigned to reproduce the observed spectrum shape [52]) results in F_{0G} being essentially saturated by the anomaly contribution A_G. Various phenomenological consequences are drawn.

One other application of the anomaly couplings which deserves mention [105] is to the hadronic decays of η and η'. From its first discovery [99] it was hoped that the strong anomaly would bring a resolution [125] of the long standing puzzle [126] within current algebra as to the mechanism for $\eta \to \pi^+\pi^-\pi^0$. Milton et al. have recently translated this hope into a concrete calculation [105] in which they relate $M_\eta \equiv M(\eta \to \pi^+\pi^-\pi^0)$ to $M_{\eta'} \equiv M(\eta' \to \eta\pi^+\pi^-)$ assuming the standard form ($H_{SB} \equiv H'$ (Eq. (1.1)) $\equiv -L_{SB}$ (Eq. (3.3)) of the symmetry breaking Hamiltonian. Their evaluation proceeds via a series of standard current algebra manoeuvres whereby M_η is translated into a multiple of $(m_d - m_u)\langle \pi^+\pi^- | G\tilde{G} | \eta\rangle$ (zero in the absence of the anomaly [126]) which is then transformed into a matrix element of ϕ_η, using hypothesis (3.10), this in turn being manipulated into a multiple of $M_{\eta'}$. The final result is

$$M_\eta = \sqrt{\tfrac{2}{3}}\,\frac{m_d - m_u}{m_d + m_u}\,\frac{A_{\eta'}}{F_\eta}\,M_{\eta'} \tag{3.17}$$

which Milton et al. have evaluated by taking A_η from (3.16), $A_{\eta'}/A_\eta$ from (3.13) and adopting the estimate $(m_d - m_u)/(m_d + m_u) \approx .29$ for the ratio of quark masses [127]. The result

$$M_\eta / M_{\eta'} \approx 0.11 \tag{3.18}$$

appears to be in good agreement with experiment.

That concludes this short survey of topics in QCD and spectroscopy.

My thanks to the organisers of the XX Cracow School, to all the participants and to numerous colleagues and friends who have given helpful advice, especially to Colin Froggatt, Tim Jones, Hans Reinders, Gordon Ringland, Dick Roberts, Graham Ross, Hector Rubinstein, David Sutherland and Shigeo Yazaki.

APPENDIX I

Saturation of anomalous Ward identities with scalar nonet

The derivation [114] of Eqs. (3.7), (3.8) start with the Ward identities of current algebra which take the form [101]

$$\begin{aligned} L_{ij} &\equiv \int d^4x\, T\langle 0|\partial^\mu A^i_\mu(x)\,\{\partial^\nu A^j_\nu(0) - \delta_{j0} G\tilde{G}(0)\}\,|0\rangle \\ &= -\langle 0|\,[Q^i_5(0), D^j(0)]\,|0\rangle \equiv R_{ij}, \qquad (i, j = 0, 1, \dots 8). \end{aligned} \tag{A1}$$

Using Eq. (3.3), an equivalent from for R_{ij} is

$$R_{ij} = \langle 0|\,[Q^5_i, [Q^5_j, L_{SB}]]\,|0\rangle. \tag{A2}$$

The left side of (Al) is estimated by saturating with the ground state pseudo-scalars (assumed dominant) to yield

$$L_{ij} \approx \sum_{a} m_a^2 F_{ia}(F_{ja} - \delta_{j0} A_a). \tag{A3}$$

The right hand side follows from the assumed structure of L_{SB} (second part of Eq. (3.3) with $m_u = m_d$) and the U(3)×U(3) algebra of charges. This yields formulae for the five relevant R_{ij}'s in terms of two parameters. Eliminating these, yields the three equations (3.7). The additional condition (3.8) comes from requiring that (A3) be symmetrical in i and j, although it is in fact a more general property of (Al) [116].

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$$(g^2/4\pi) = \frac{g_0^2/4\pi}{1+(g_0^2/4\pi)\,(\beta_0/4\pi)\ln(Q^2/\mu^2)},$$

with g_0^2 the value of g^2 at $Q^2 = \mu^2$. The plus sign in the denominator is the crucial guarantor of *asymptotic freedom* ($g^2 \to 0$ as $Q^2 \to \infty$).

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