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A COMMENT ON A CRITICISM OF THE NONSYMMETRIC UNIFIED FIELD THEORY

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An objection due to C. P. Johnson to the nonsymmetric unified field theory is critically examined.

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An objection due to C. P. Johnson [1] to the nonsymmetric unified field theory of Einstein and Straus [2] appears to have been largely ignored in the literature. Nevertheless, it is perhaps the most damaging of all that have been raised (and are now known to be unfounded). In view of the recent developments in the theory (see for example, a review article by Klotz and Gregory, to be published in GRG, or Klotz [3] and following articles) it is therefore necessary to see if the objection can be substantiated.

Using a similarity solution

$$x \to kx, \quad k \text{ constant}, \tag{1}$$

for a hypothetical arrangement of two charged and one uncharged, massive bodies, Johnson correctly concludes that similarity solution corresponds to the situation in which the distances are decreased in the ratio of $1/k$ and accelerations become k times those in the original solution. So far this is straightforward Riemannian geometry and Newtonian kinematics to which general relativistic relations must, of course, reduce in the 0th approximation. It seems to me, however, that he then makes an additional assumption in 'equating' gravitational field and acceleration, namely, that the Newtonian gravitational constant N remains unchanged. But N is not a dimensionless quantity and in prerelativistic physics a similarity transformation corresponds to a change of scale. If for mass and charge we have, under (1),

$$m' = k^{\alpha}m, \quad e' = k^{\beta}e \tag{2}$$

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respectively, then a balance between gravitational, electrostatic and dynamical forces (mass × acceleration) results if

$$\alpha = 2\beta + 1. \tag{3}$$

Then if N remains unchanged, $\alpha = \beta = -1$.

Johnson's contradiction arises, however, not so much from any assumption about N (I have only pointed it out to show that an additional assumption is being made) but from his insistence that the transformation of charge should be derived from Einstein's definition of charge density

$$e = \tfrac{1}{6}\,\varepsilon^{0ijk} g_{[ij],k}. \tag{4}$$

This definition was indeed attractive at the stage at which Einstein left the theory but recent investigations (Klotz loc. cit.) have shown that it is impossible. The similarity mapping implies that for the affine connection

$$\Gamma^{\lambda}_{\mu\nu} \rightarrow k\Gamma^{\lambda}_{\mu\nu}, \tag{5}$$

and the current reformulation of the theory requires the electromagnetic potential vector to be proportional to

$$\Gamma_{\mu} = \Gamma^{\sigma}_{[\mu\sigma]}. \tag{6}$$

(This is in fact an old suggestion of Einstein dating to mid-twenties; what was not known then was that it is perfectly adequate, allowing us in particular to derive the Lorentz force on a charged test particle from the field equations without having to modify the approximation techniques). But it then follows that the charge is invariant under similarity transformations, $\beta = 0$, $\alpha = 1$ and $N' = k^2 N$, which I have already shown to be consistent with the classical balance of forces. Hence, Johnson's objection is invalid.

In the original formulation of the theory, the symmetric and skew-symmetric parts of the nonsymmetric 'metric' tensor $g_{\mu\nu}$ were respectively taken as the gravitational and electromagnetic potentials. This is no longer the correct identification. The tensor $g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]}$ does represent the macroscopic fields but in interaction with each other and $g_{(\mu\nu)}$ reduces to the gravitational potentials only when symmetry is reimposed and General Relativity recovered. And the equation of motion of test particles tells us that for sufficiently weak fields, the pre-relativistic situation is reestablished as is required by laboratory evidence.

Incidentally, the transformation (5) is also consistent with my definition of the Riemannian metric $a_{\mu\nu}$

$$\tilde{\Gamma}^{\lambda}_{(\mu\nu)} = \left\{ {\lambda \atop \mu\nu} \right\}_a, \tag{7}$$

$\tilde{\Gamma}^{\lambda}_{(\mu\nu)}$ being the symmetric part of the Schrödinger connection for which

$$\tilde{\Gamma}_{\mu} = 0. \tag{8}$$

Let me conclude with a brief reference to Einstein's own comments on Johnson's objection [4]. Actually only one point in Einstein's reply is relevant and that is the observation that a similarity solution may correspond to the situation in a 'different world'. This can now be shown explicitly. Under the identification (7) of the metric, the local Riemannian geometry in the vicinity of a stationary charge e, is given by [5]

$$ds^2 = \left(1+e\sqrt{\frac{r_0^2}{r^2}-1}\right)dt^2 - \frac{dr^2}{\left(1-\frac{r^2}{r_0^2}\right)\left(1+e\sqrt{\frac{r_0^2}{r^2}-1}\right)} - r^2(d\theta^2+\sin^2\theta d\phi^2). \tag{9}$$

Because of the cut-off at $r = r_0$, and because the solution corresponds to a strictly Coulomb field, the constant r_0 is identified with the 'radius of the universe'. A similarity solution then corresponds to a universe whose radius (and hence mass, volume etc.) is different from the original one.

Most of the above work was done while I was on leave at the Institute of Astronomy, Cambridge, U.K.

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