PHENOMENOLOGICAL APPROXIMATION OF INELASTIC π-N INTERACTIONS AT HIGH ENERGIES

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The phenomenological expressions for differential single-particle inclusive cross sections of nucleon and π-meson production in inelastic π-N collisions at energies from 5 GeV up to some thousands GeV are obtained. Using these expressions the analysis of experimental data is performed.

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In papers [1, 2] the phenomenological expressions for inclusive nucleon and meson distributions in inelastic N-N collisions have been obtained which are in good agreement with experimental data at energies exceeding 5 GeV. The aim of the present paper is to derive an analogical expression for inelastic π-N interactions.

To describe the distributions of "conserved" particles (meson at $x > 0$ and a nucleon emitted to the opposite direction with $x < 0$) we use the expression coinciding up to numerical values of coefficients with the one suggested in paper [1] for the spectrum of the leading particle in N-N collisions. The new values of coefficients are given in Tables I and II[1].

TABLE I

Coefficients for the calculation of the conserved π-meson and nucleon spectra in the region $|x| < 0.7$

Reaction \ Coeff.	a	b	c	d	f	g	μ	u	v
$\pi^{\pm}p \to \pi^{\pm} + \ldots$	85	15	−0.6	6.4	0.3	−3.2	1.26	5	0.04
$\pi^{\pm}p \to p + \ldots$	2.1	−0.16	0.04	6.2	0.1	5.5	1.26	1.3	−0.25

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[1] We use the same standard notation as in the previous papers [1, 2]. In what follows T is the kinetic energy of a projectile in the laboratory coordinate system.

TABLE II

Coefficients for the calculation of the conserved π-meson and nucleon spectra in the region $|x| > 0.7$

Equal for all reactions				$\pi^{\pm}p \rightarrow \pi^{\pm} + \ldots$		$\pi^{\pm}p \rightarrow p + \ldots$	
A				—		15	
R				—		0.02	
α				—		0.8	
i	α_i	β_i	γ_i	G_i	R_i	G_i	R_i
1	0	0	1.5	3.7	1.7	32	1.1
2	0.5	−0.5	1.5	6.5	4.6	36	5.3
3	0	−1.0	0.2	0.1	1.4	—	—

In the case of π-N collisions all particles with the changed charge sign as well as the particles retaining the charge sign but emitted to the hemispheres opposite to the direction of corresponding primaries must be considered as "unconserved" (for example, in π^--p interaction it is proton at $x > 0$ and π^--meson at $x < 0$). For the inclusive spectra of con-

TABLE III

Coefficients for the unconserved particles spectrum

Coeff.	$\pi^{\mp}p \rightarrow \pi^{\pm} + \ldots$	$\pi^{\pm}p \rightarrow \pi^{0} + \ldots$	$\pi^{\pm}p \rightarrow p + \ldots$	$\pi^{\pm}p \rightarrow \pi^{\pm} + \ldots$
a	20	22	1.5	20
b	1.8 (2.6)*	1.7 (2)*	4	2.6
c	0.02	0.01	−0.02	0.02
d	2.2	1.4	1.5	1.4
f	0.01	0.04	−0.1	0.01
g	1.6	0.55	1.8	1.6
h	11	11	6	6

* For the region $x < 0$.

served particles, mesons and nucleons, $Ed^3\sigma_c/dp^3$ we use the same expression as in the case of N-N collisions (see formula (1) in Ref. [2], where $x \equiv |x|$). The corresponding coefficients are presented in Table III. These coefficients are applicable for all values of x: $x > 0$ and $x < 0$. The exception are "b" values only, which are given in parenthesis for $x < 0$.

Good agreement of the selected expressions with experiment is illustrated by Figs. 1 and 2.

The next picture and Table IV present the data on average multiplicity of secondaries

$$\langle n_x \rangle = \sigma_{\text{in}}^{-1} \int \left(E \frac{d^3\sigma_c}{dp^3} \right) \frac{d^3p}{E} + \sigma_{\text{in}}^{-1} \int \left(E \frac{d^3\sigma_{nc}}{dp^3} \right) \frac{d^3p}{E}, \tag{1}$$

where the spectra of conserved and unconserved particles are integrated over the corresponding x values (for fore- and backward directions of particle emission, respectively). Calculated curves are close to experimental points. At $T \gtrsim 100$ GeV the experimental points $\langle n_{ch} \rangle$ are systematically somewhat above the curve; this is because the experimentally measured summary multiplicity of all charged particles, contrast to the calculated value $\langle n^*_{ch} \rangle = \langle n_p \rangle + \langle n_{\pi^\pm} \rangle$, includes additionally the multiplicity of strange particles and antiprotons $\langle n_{S,\bar{p}} \rangle$.

As in the case of p-p interactions (see Ref. [2]) the multiplicity of the particles with conserved charge sign is the biggest one. Meson multiplicity becomes charge independent at very high energies $T \gtrsim 10^3$ GeV only. Meanwhile, the share of created π^0-mesons remains constant in the whole energy region $T \gg 1$ GeV: $\langle n_{\pi^0} \rangle / \langle n_{\pi^\pm} \rangle \simeq 40$–$50\%$.

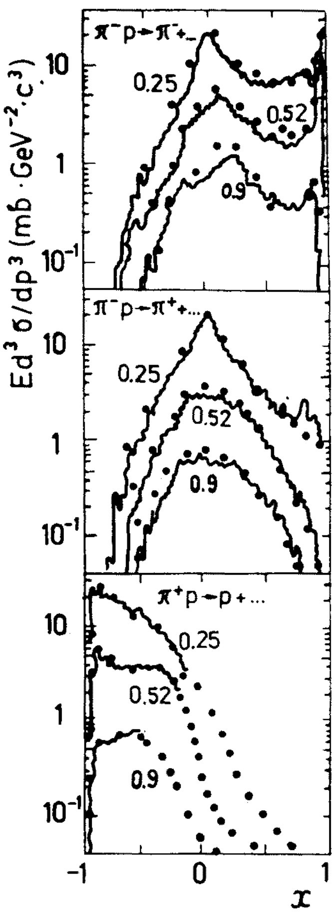

Fig. 1. Differential distributions of charged particles in inelastic π-p collisions. Dots correspond to calculations for $T = 16$ GeV. Experimental curves are from Refs. [3, 4]. Near the curves the corresponding $p_\perp$(GeV/c) values are shown. Theoretical spectra for protons at $x < 0$ and π^--mesons at $x > 0$ are calculated by the formula for conserved particles. The remaining parts of these particles and π^+-mesons spectra are calculated by the formula for unconserved particles

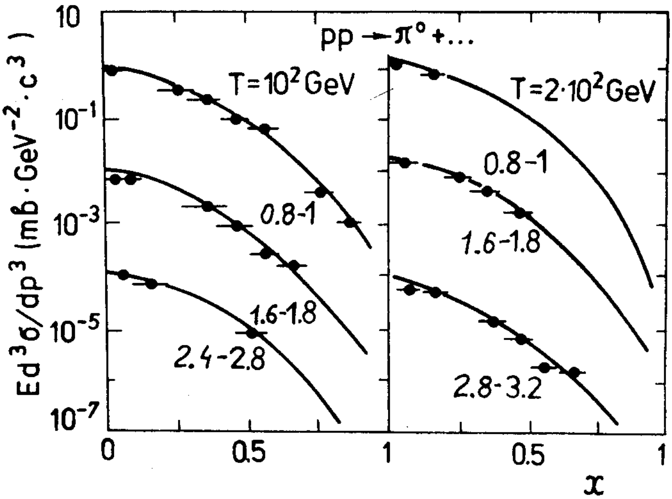

Fig. 2. Differential distributions of π^0-mesons created in π^--p collisions at energy T. Curves correspond to calculations. The experimental points are taken from Ref. [5]. Near the curves $p_\perp$(GeV/c) values are shown

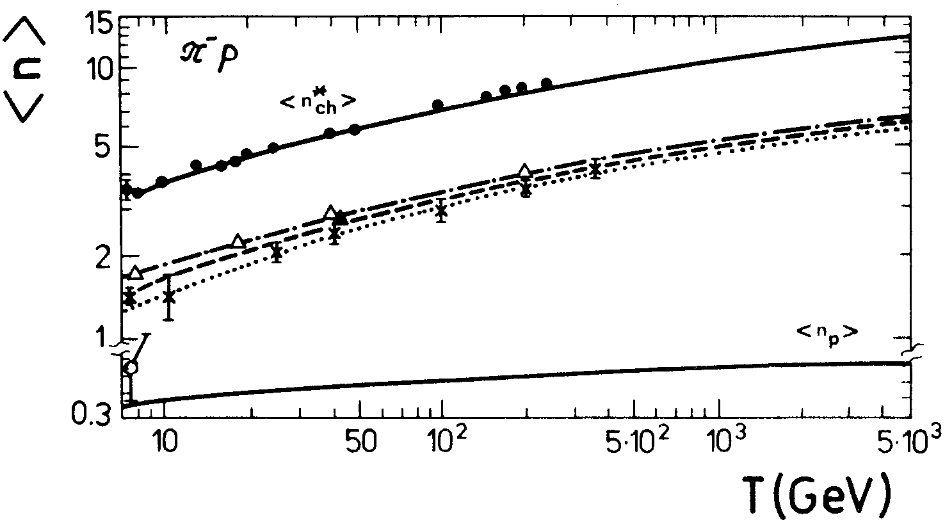

Fig. 3. Average multiplicity of particle production in inelastic π^--p interactions. Solid curves correspond to calculated values of $\langle n_p \rangle$ and $\langle n^*_{ch} \rangle$. Dashed-and-dotted, dotted and dashed lines correspond to calculations of $\langle n_{\pi^-} \rangle$, $\langle n_{\pi^0} \rangle$ and $\langle n_{\pi^+} \rangle$, respectively. Experimental multiplicities $\langle n_p \rangle$, $\langle n^*_{ch} \rangle$, $\langle n_{\pi^-} \rangle$, $\langle n_{\pi^+} \rangle$, $\langle n_{\pi^0} \rangle$ are shown by the marks ○, ●, △, ▲, × [6, 21]

Average transverse momenta of conserved particles, nucleons and mesons, are appreciably larger than for the other particles (see Table V, where π^--p interaction is considered as an example). The energy dependence of $\langle p_\perp \rangle$ is very weak.

The distinguishing nature of the conserved particles becomes apparent in their energy also, which considerably, especially for nucleon, exceeds the energy of the other particles. For example, in π^--p interaction the average proton energy in C.M.S. $\langle T_p \rangle \gtrsim T^{1/2}$, as in the case of p-p collisions. At the same time the meson energy $\sim T^\alpha$, where $\alpha = 0.3$; in particular $\langle T_{\pi^+} \rangle \simeq \langle T_{\pi^0} \rangle$ and the energy of π^--meson, which includes rapid leading particles along with conserved particles, is about 1.5 times higher (see Fig. 4).

TABLE IV

Average multiplicity ratio for neutral and charged π-mesons in inelastic π^--p interactions

T, GeV	$\langle n_{\pi^0} \rangle / \langle n_{\pi^\pm} \rangle$	
	Calculation	Experiment
7	0.41	0.40±0.12 [5–7]
10	0.44	—
25	0.46	0.47±0.05 [8]
40	0.46	0.45±0.01 [9]
100	0.47	0.46±0.06 [18]*
205	0.48	0.56±0.09 [19]*
10^3	0.49	—

* Using $\langle n_{\pi^\pm} \rangle = \langle n_{ch} \rangle_{exp} - \langle n_p \rangle_{theor}$.

TABLE V

Average transverse momentum of secondary particles in inelastic π^--p interaction

T, GeV	$\langle p_{\perp p} \rangle$, GeV/$c$	$\langle p_{\perp \pi^-} \rangle$, GeV/$c$	$\langle p_{\perp \pi^+, \pi^0} \rangle$, GeV/$c$
10	0.42	0.36	0.30
	(0.42±0.03) [23, 24][1]		(0.30±0.02) [23, 24]
20	0.43	0.37	0.32
	(0.42±0.04) [22]	(0.37±0.01) [25, 26]	
40	0.43	0.38	0.33
		(0.356±0.004) [27]	(0.369±0.004) [27]
10^2	0.44	0.39	0.34
10^3	0.45	0.40	0.35
$5 \cdot 10^3$	0.46	0.42	0.36

[1] Experimental data are shown in parenthesis.

At the same primary energy T the kinetic energy of secondary mesons in π-N collisions is, on the average, 10–20% higher than in N-N interactions (in C.M.S.).

In experimental papers the inelasticity coefficient $\langle K \rangle$ characterizing the share of energy consumed for the formation of secondaries is often considered. For π-N interactions this coefficient is less definitely determined than for N-N interactions. Taking into account its physical nature the coefficient $\langle K \rangle$ could be defined as

$$\langle K \rangle = 1 - \{\langle T_N \rangle + \langle T_{\pi_L} \rangle + M_N + M_\pi\}/\sqrt{s}\,, \tag{2}$$

where $\langle T_N \rangle$ and $\langle T_{\pi L} \rangle$ are the average kinetic energies of the secondary nucleon and the leading π-meson, M_N and M_π are their masses. However, at present there is no generally accepted criterion for the selection of the leading particles among the other conserved particles, so the equation (2) is often replaced by

$$\langle K \rangle = \langle \Delta E_\pi \rangle / \sqrt{s}\,, \tag{3}$$

where $\langle \Delta E_\pi \rangle$ is the summarized average energy of all secondary mesons in C.M.S. with the exception of the average energy of one secondary meson $\langle E_{\pi^\pm} \rangle = \langle T_{\pi^\pm} \rangle + M_{\pi^\pm}$. The

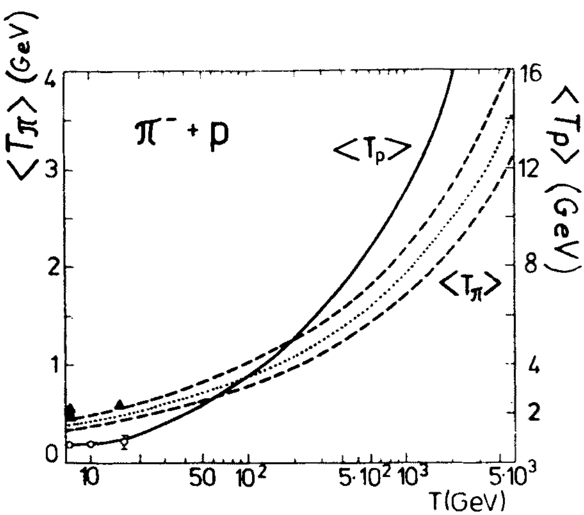

Fig. 4. Average kinetic energy of secondaries in inelastic π^--p interactions (C.M.S.). Solid curve shows the energy of protons. The upper and lower dashed lines correspond to $\langle T_{\pi^-} \rangle$ and $\langle T_{\pi^+} \rangle$, respectively. Dotted curve illustrates the average energy of charged mesons. Averaged experimental data for the charged mesons and protons are shown by the marks ▲ and ○, respectively [25, 28–31]

values of $\langle K \rangle$ calculated by this method along with the known experimental data for the case of π^--p interactions are shown in Table IV.

The quantity $\langle K \rangle$ defined by formula (2) approximately twice exceeds the inelasticity coefficient for N-N interactions (see Ref. [1]) and increases with increasing primary energy T. It is caused by the large contribution of the leading meson.

The situation is simpler for the partial inelasticity coefficients of secondary nucleons and π-mesons with the charge differing from the charge of primary meson; they are defined as

$$\langle K_x \rangle = \langle n_x \rangle \{\langle T_x \rangle + M_x\} / \sqrt{s}\,. \tag{4}$$

The ambiguity of these coefficients is due to the fact of using C.M.S. or laboratory coordinate system only. As it is in the case of N-N interactions the proton inelasticity coefficient $\langle K_p \rangle$ is practically energy independent and meson inelasticity coefficients increase slowly with increasing T (see Tables VI and VII). At all energies $T \gtrsim 10$ GeV the ratio of inelasti-

TABLE VI

Inelasticity coefficients for π^--p interactions in C.M.S.

T, GeV	$\langle K_p \rangle$	$\langle K \rangle$	
		Calcul.	Exper. [28]
7	0.30	0.47	0.49 ± 0.08
10	0.30	0.48	0.57 ± 0.05
16	0.30	0.51	0.56 ± 0.06
10^2	0.29	0.59	—
10^3	0.26	0.67	—

TABLE VII

The relative share of energy consumed for π^0-meson formation in π^--p interactions in the laboratory coordinate system

T, GeV	Calculation	Experiment
10.5	0.19	0.22 ± 0.01 [32]
18.5	0.20	0.21 ± 0.01 [33, 34]
40	0.23	0.25 ± 0.01 [34]
10^2	0.24	—
10^3	0.26	—
$5 \cdot 10^3$	0.27	—

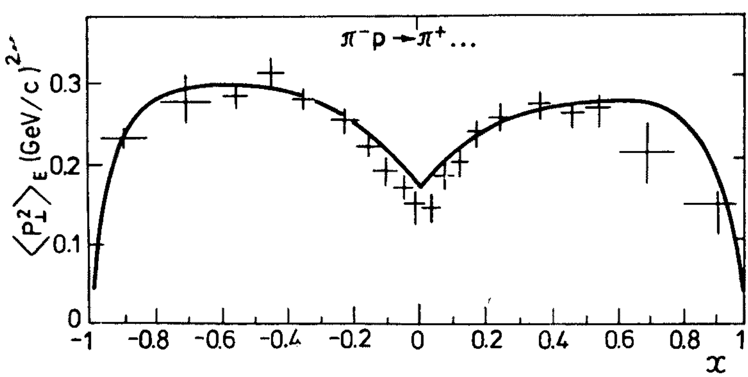

Fig. 5. Correlation of $\langle p_\perp^2 \rangle_E$ and x values for π^+-mesons in inclusive reaction $\pi^-\text{p} \to \pi^+ \ldots$ at energy $T = 15$ GeV. The curve is our result, the points are experimental data from Ref. [37]

city coefficients for unconserved mesons of different charge (for example, $\langle K_{\pi^0} \rangle / \langle K_{\pi^\pm} \rangle$ in the case of π^--p collisions) equals to unity with several percent accuracy.

Fig. 5 presents the comparison of theoretical and experimental data for pion weighted average transverse momentum

$$\langle p_\perp(x) \rangle_E = \int p_\perp E \frac{d^3\sigma}{dp^3} dp_\perp^2 \Big/ \int E \frac{d^3\sigma}{dp^3} dp_\perp^2, \tag{5}$$

where E is pion c.m.s. energy [35]. Calculated x-$p_\perp$ correlations reproduce the well-known "seagull effect" [36].

From above considerations we see that the approximating expressions for the differential distributions of conserved and unconserved particles $Ed^3\sigma/dp^3$ make it possible to "assemble" large amounts of experimental data. These expressions can be used to obtain sufficiently accurate estimations of various quantities in those regions of kinematic variables where measurements are still absent.

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