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# LORENTZ-CONTRACTED HADRON WAVE FUNCTIONS AND PION-NUCLEON ELASTIC SCATTERING AT HIGH ENERGIES

By Z. Wąs

Institute of Physics, Jagellonian University, Cracow*

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$\pi p$ elastic scattering at 200 GeV is calculated using the Glauber model with Lorentz-contracted oscillator type wave functions of hadrons. The model predicts a large increase of the slope of differential cross-section at small $t$ which is not observed in the data.

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## 1. *Introduction*

The purpose of this paper is to study the elastic $\pi p$ scattering at high energies using the model proposed by Goloskokov et al. [1]. In the model of Ref. [1] hadrons are treated as the systems of a finite number of dressed valence quarks, which interact independently of each other, according to the Glauber prescription [2]. The model differs from the standard applications of the Glauber model by the form of the hadronic wave functions. The authors of Ref. [1] proposed so called "relativistically invariant" wave functions [3, 4] which take into account the Lorentz contraction. They applied the model to pp scattering and obtained a reasonable description of the data. In particular, it was possible to explain the complicated structure of proton-proton scattering amplitude without assuming a similar structure for quark-quark scattering amplitudes.

Therefore, it seems interesting to apply the same model to $\pi p$ scattering, particularly in view of the known problems which arise in description of the pion form-factor by "relativistically invariant" wave function [5]. Our calculations show that the model fails for $\pi p$ scattering; it predicts a very large value for the slope of $\pi p$ elastic cross-section at small $t$ which is not observed in the data. In the next Section we describe the model and its application to $\pi p$ scattering. The results of numerical calculations and conclusions are presented in Section 3.

* Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

## 2. *Description of the model*

Let $p_A$, $p_C$ denote proton 4-momentum before and after scattering, and $p_B$, $p_D$ — pion 4-momentum before and after scattering.

Let us choose the coordinate system so that the 4-momentum transfer $\Delta = p_A - p_C = p_B - p_D$ is perpendicular to the $z$ axis: $\Delta_\mu = (0, \vec{\Delta}, 0)$ then $t = -\Delta^2$. Let $\vec{b}$ be the distance

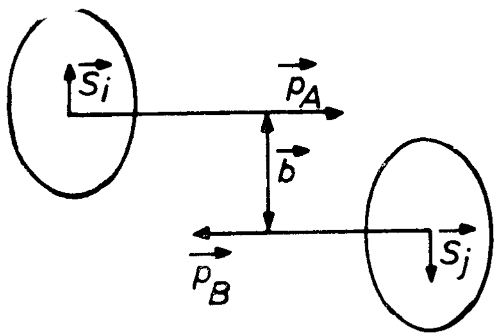


Fig. 1. Impact parameter representation of hadron-hadron scattering

between the centers of mass of colliding particles in the impact parameter plane and $\vec{S}_i$ are relative coordinates of an $i$-th ($j$-th) quark in the same plane (see Fig. 1).

According to the Glauber model [2] we have:

$$T_{fi} = \frac{i}{2\pi}\int d^2\vec{b}\langle f|\Gamma_{tot}(\vec{b}, \ldots)|i\rangle e^{i\vec{\Delta}\vec{b}}, \tag{1}$$

where

$$\Gamma_{tot} = 1 - \prod_{i=1}^{3}\prod_{j=1}^{2}(1-\Gamma_{ij}(\vec{b}+\vec{S}_i-\vec{S}_j)),$$

$$\Gamma_{ij} = \frac{1}{i(2\pi)^2}\int d^2\vec{q}e^{i\vec{q}\vec{b}}T_{q_iq_j}$$

and $T_{q_iq_j}$ is the quark-quark scattering amplitude. Then

$$\frac{d\sigma}{dt} = \pi|T_{fi}|^2, \tag{2}$$

$$\sigma_{tot} = 4\pi \,\mathrm{Im}\, T_{fi}|_{t=0}. \tag{3}$$

Following Ref. [1] we have used

$$T_{q_iq_j} = i(\alpha+i\beta)e^{-Bq^2}. \tag{4}$$

The proton wave function is of the form [1]:

$$\psi(\xi, \zeta, p) = \frac{1}{(2\pi a)^2}e^{\frac{1}{4a}}\{\xi^2-2(\xi u_p)^2+\zeta^2-2(\zeta u_p)^2\}; \quad u_p = \frac{p}{m_p}, \tag{5}$$

where $p$ is the 4-momentum of the nucleon, $\xi_\mu = \frac{(x_2-x_3)_\mu}{2\sqrt{3}}$; $\zeta_\mu = \frac{(x_2-x_3-2x_1)_\mu}{6}$ and $x_1$, $x_2$, $x_3$ are the four-dimensional coordinates of the dressed quarks in the proton. By analogy and following the results of Ref. [4] we take the pion wave function as

$$\psi = \frac{1}{2\pi A} e^{\frac{1}{4A}(\eta^2 - 2(\eta u_\pi)^2)}, \qquad u_\pi = \frac{p}{m_\pi}, \tag{6}$$

where $p_\mu$ is the 4-momentum of the meson and $\eta_\mu = \frac{(x_1-x_2)_\mu}{2}$. The functions are normalized by the conditions

$$\int d^4\xi d^4\zeta \psi^2(\xi, \zeta, p) = 1; \qquad \int d^4\eta \psi^2(\eta, p) = 1.$$

Using the wave function given by Eq. (5) it was shown in Ref. [3] that the nucleon form factor is given by

$$F_p = \Gamma_p^2(t) e^{2a\Gamma_p(t)t}, \qquad \Gamma_p(t) = \frac{1}{1 - \frac{t}{2m_p^2}};$$

this formula agrees reasonably well with the experimentally measured form-factor. Using the same argument as in Ref. [3] one obtains for pion form-factor

$$F_\pi = \Gamma_\pi(t) e^{\frac{A}{2}\Gamma_\pi(t)t}; \qquad \Gamma_\pi(t) = \frac{1}{1 - \frac{t}{2m_\pi^2}}.$$

This formula fails badly in comparison with data.

$$F_\pi(t) \approx 1 + t\left(\frac{1}{2m_\pi^2} + \tfrac{1}{2}A\right).$$

And thus the electromagnetic radius of the pion is predicted to be

$$r_\pi = \sqrt{6\left(\frac{1}{2m_\pi^2} + \tfrac{1}{2}A\right)} > \frac{\sqrt{3}}{m_\pi} = 2.45 \text{ fm}$$

whereas the experimentally measured radius is 0.678 fm [6]. This is a serious problem for the model [5]. We shall see that this problem is also reflected in the description of elastic scattering.

Expanding Eq. (1) in powers in $\Gamma_{ij}$ one obtains

$$T_{fi} = T^{(1)} + T^{(2)} + \ldots + T^{(6)}.$$

We quote here only the first two terms

$$T^{(1)} = \frac{6i(\alpha+i\beta)2B}{4\pi B}\,\Gamma_p^2\Gamma_\pi e^{[B+2a\Gamma_p+\frac{1}{2}A\Gamma_\pi]t},$$

$$T^{(2)} = \frac{-i(\alpha+i\beta)^2}{(4\pi B)^2}\,B\Gamma_p^2\Gamma_\pi e^{\frac{B}{2}t}\left\{\frac{6\exp\left(\frac{1}{2}a\Gamma_p t\right)}{\sqrt{(1+3a\Gamma_p/B+A\Gamma_\pi/B)\left(1+\frac{3a}{B}+\frac{A}{B}\right)}}\right.$$

$$\left.+\frac{3\exp(2a\Gamma_p t)}{\sqrt{(1+A/B)(1+A\Gamma_\pi/B)}}+6\frac{\exp\left(\frac{1}{2}a\Gamma_p t+\frac{1}{2}A\Gamma_\pi t\right)}{\sqrt{(1+3a\Gamma_p/B)(1+3a/B)}}\right\}.$$

In numerical calculations described in the next Section we have used the full formula for $T_{fi}$.

### *3. Numerical estimates and comparison with data*

In the numerical estimates of the elastic πp cross section we have used the q-q amplitudes as determined in Ref. [1]

$$\alpha = 5.45\ \text{GeV}^{-2}, \quad \beta = -0.751\ \text{GeV}^{-2}, \quad B = 0.378\ \text{GeV}^{-2}$$

also the proton wave function parameter $a$ was taken from Ref. [1], $a = 0.754\ \text{GeV}^{-2}$. The pion wave function parameter $A$ was considered as a free parameter $A \geqslant 0$.

The total cross section $\sigma_{tot}$ obtained from Eqs. (1) and (3) is plotted in Fig. 2, $\sigma_{tot}$ increases with $A$ and changes from $\sigma_{tot} = 15$ mb at $A = 0.01\ \text{GeV}^{-2}$ to $\sigma_{tot} = 23$ mb at $A = 10\ \text{GeV}^{-2}$.

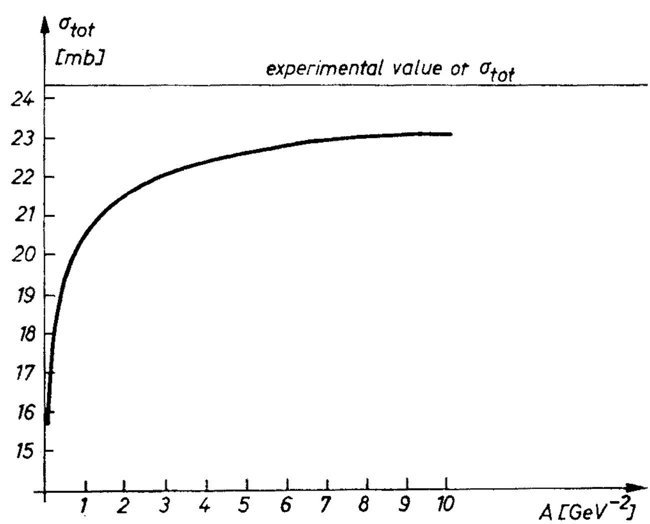


Fig. 2. Total cross-section calculated from the model

For $\pi^-p$ scattering the experimental value of $\sigma_{tot}$ is 24.33 mb [8]. Theoretical values are a bit too small, but cannot be considered as a serious discrepancy with data.

We have also calculated the differential cross-section using the formulae (1) and (2). This cross-section is plotted versus $t$ for $A = 0.01\ \text{GeV}^{-2}$, $A = 1\ \text{GeV}^{-2}$ and $A = 10\ \text{GeV}^{-2}$ in Fig. 3. One sees a diffractive minimum at about 3 $\text{GeV}^2$. The model predicts also the structure at small $t$, where the slope increases dramatically reflecting the very large pion electromagnetic radius. Even for $A = 0.01\ \text{GeV}^{-2}$ this structure is quite visible and already inconsistent with data [7] which are also plotted in Fig. 4. The origin of this disagreement can be traced back to the effect of the Lorentz contraction of the pion wave function which implies much too fast change of the pion form-factor with the momentum transfer. The effect is so dramatic because the pion mass is very small. We feel, therefore, that the Lorentz--contracted oscillator type wave functions are not a good choice for the description of hadrons.

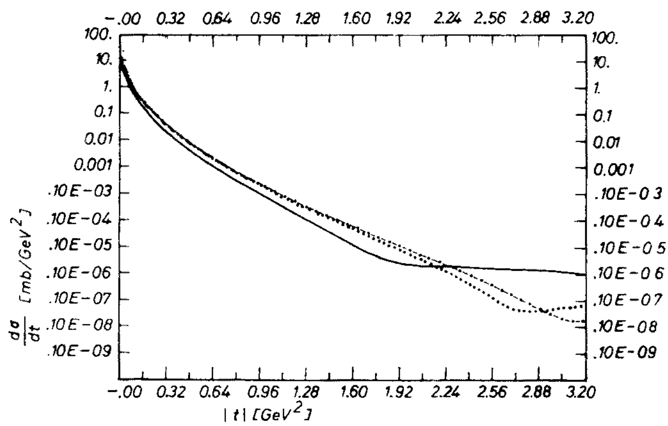


Fig. 3. Differential elastic cross-section: ——— $A = 0.01\ \text{GeV}^{-2}$; ······ $A = 1\ \text{GeV}^{-2}$; ·——· $A = 10\ \text{GeV}^{-2}$

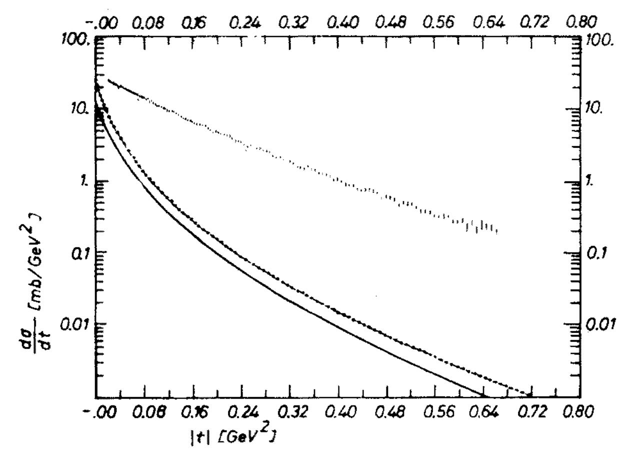


Fig. 4. Differential elastic cross-section. Data taken from Ref. [8]

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