Vol. B12 (1981) ACTA PHYSICA POLONICA No 11

ON CALCULATION OF HADRON-NEUTRON TOTAL CROSS SECTIONS FROM THE DEUTERON DATA

BY M. JEŻABEK AND K. ZALEWSKI

Institute of Nuclear Physics, Cracow*

(Received April 16, 1981)

We develop a simple method for calculation of the total cross sections on neutron from the data taken on deuteron targets. The systematic error due to this procedure is smaller than 0.3 mb for beam momenta above 50 GeV/c. Using Glauber's formula without inelastic screening, as often done in experimental papers, may introduce a much bigger error.

PACS numbers: 13.85.-t

1. Introduction

The measurement of the hadron-deuteron total cross section $\sigma_{\rm hd}$ is the most efficient practical way to obtain $\sigma_{\rm hn}$, the corresponding cross section for the neutron target. Direct measurements, though possible, are difficult, because neutron beams are not monochromatic [1]. Moreover, the direct method can be used only for neutron-proton collisions. Consequently, the question how to extract $\sigma_{\rm hn}$ from the data on hadron-deuteron and hadron-proton interactions is of great practical importance.

In principle the answer is well-known:

$$\sigma_{\rm hn} = \sigma_{\rm hd} - \sigma_{\rm hp} + \delta\sigma_1 + \delta\sigma_2, \tag{1.1}$$

where $\delta\sigma_1$ and $\delta\sigma_2$ are the corrections due to elastic [2,3] (Fig. 1a) and inelastic [4, 5, 6] (Fig. 1b) screening, respectively. Compact formulae exist for both $\delta\sigma_1$ [2, 6] and $\delta\sigma_2$ [5, 6]:

$$\delta\sigma_1 = 2\,{\rm Re} \int\limits_{t_{\rm max}}^{0} dt[S(t)+S_\varepsilon(t)]F_{\rm el}^{\rm hp}(t)F_{\rm el}^{\rm hn}(t), \tag{1.2}$$

$$\delta\sigma_2 = 2\,{\rm Re} \sum_{{\rm x}\neq{\rm h}} \int\limits_{t_{\rm max}}^{t_{\rm min}} dt[S(t)+S_\varepsilon(t)]F^{\rm hp\to xp}(t)F^{\rm xn\to hn}(t). \tag{1.3}$$

---

* Address: Instytut Fizyki Jądrowej, Kawiory 26a, 30-055 Kraków, Poland.

In (1.3) the summation extends over all $x \neq h$ as seen in Fig. 1b. The form-factors S and S_ε are given by [6]:

$$S(\vec{Q}) = \int d^3\vec{r}\,|\psi(\vec{r})|^2 e^{i\vec{Q}\cdot\vec{r}}, \tag{1.4}$$

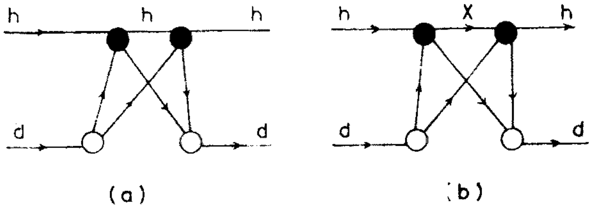

Fig. 1. Contributions to the defect of the total cross section on deuteron: a) elastic, b) inelastic screening

and

$$S_\varepsilon(\vec{Q}) = \int d^3\vec{r}\,|\psi(\vec{r})|^2 e^{i\vec{Q}\cdot\vec{r}}\frac{z}{|z|}, \tag{1.5}$$

where $\psi(\vec{r})$ denotes the wave function of the deuteron, and the z-axis is parallel to the momentum of the incident hadron.

Though simple in principle, the formulae (1.2) and (1.3) are not easy to use in practical calculations. The standard practical approach [7, 8] is to use Glauber's method without corrections for inelastic screening. The authors of the experimental papers warn the readers that this approach is controversial [7], but use it, because it is simple. Our main goal is to find a reliable approximation to the formulae (1.2)–(1.3) which is sufficiently simple to be used by experimentalists analysing data. We limit our discussion to process at incident momenta $p \gtrsim 50$ GeV/c and do not expect a precision better than about 0.3 mb for the σ_{hn} cross section. For lower energies and/or better precision a much more difficult analysis would be required.

The paper is organized as follows. In Section 2 we derive the simplified formula for σ_{hn}, and in Section 3 we calculate the neutron total cross sections from the available data. Our results are summarized in Section 4.

2. *Derivation of the formula for* σ_{hn}

A careful analysis of the different contributions to σ_{hn} leads to the following results [6]:

— the contribution of S_ε to $\delta\sigma_2$ is negative and small (~ 0.2 mb); its reliable calculation would require an additional highly nontrivial theoretical analysis of the form-factor S_ε. The contribution of S_ε to $\delta\sigma_1$ is negligible.

— $\delta\sigma_2$ can be expressed as a sum over Regge terms:

$$\delta\sigma_2 = -2\sum_{i,j,k}\int_{\xi_0}^{1} d\xi \int_{t_{max}}^{t_{min}} dt S(\sqrt{-t})\eta_j \cos[\pi\alpha_j(t)]$$
$$\times\, G_{ijk}(t,t,0)\xi^{\alpha_k(0)-\alpha_i(t)-\alpha_j(t)} s^{\alpha_k(0)-1}, \tag{2.1}$$

where latin indices stand for reggeons, $\alpha_j(t)$ is the trajectory of the reggeon j, η_j is its signature, and G_{ijk} denotes the three-reggeon vertex function. The summation over intermediate inelastic states x (cf. Eq. (1.3) and Fig. 1b) is replaced by integration over $\xi = \frac{M^2}{s}$, where M is the mass of x and $\xi_0 = \frac{M_0^2}{s}$ a cut off parameter. At energies above 50 GeV the dominant contribution to $\delta\sigma_2$ comes from the triple Pomeron (PPP) term, and it grows logarithmically with energy. The contributions from PPR terms stay constant above 50 GeV and are of order 0.2 mb[1]. The other contributions decrease with increasing energy, and at high energies are negligible.

We assume now that elastic amplitudes at high energies are purely imaginary and can be parametrized as:

$$F_{\rm el}^{\rm hN}(t) = i\,\frac{\sigma_{\rm tot}^{\rm hN}}{4\sqrt{\pi}}\,e^{1/2bt}, \tag{2.2}$$

where N stands for the nucleon. Eq. (1.2) can be rewritten in the form:

$$\delta\sigma_1 \approx \frac{f(b)}{4\pi}\,\sigma_{\rm hp}\sigma_{\rm hn}, \tag{2.3}$$

where

$$f(b) = \tfrac{1}{2}\int\limits_{-\infty}^{0} dt S(\sqrt{-t})e^{bt}. \tag{2.4}$$

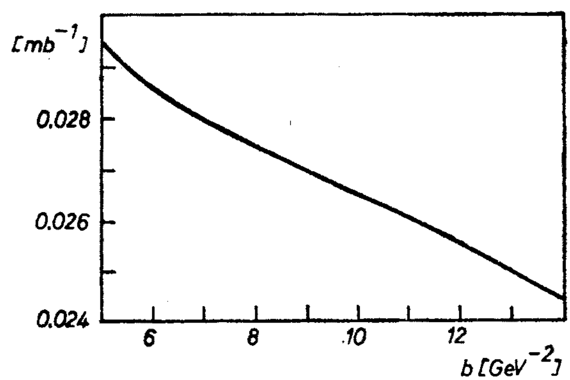

Fig. 2. Effective inverse radius squared $f(b)$ for deuteron (see Eq. (2.4)) as a function of the elastic slope b

The function $f(b)$ depends little on b for realistic values of elastic slopes. In Fig. 2 we have plotted this function for Mc Gee's [9] parametrization fo the deuteron wave function. When calculating $\delta\sigma_2$, we keep in Eq. (2.1) only the triple pomeron term and get:

$$\delta\sigma_2 = 2\int\limits_{\xi_0}^{1}\frac{d\xi}{\xi}\int\limits_{t_{\max}}^{m^2\xi^2} dt\,\xi^{2\alpha_P' t}G_{\rm PPP}^{\rm hNN}(t)S(\sqrt{-t}). \tag{2.5}$$

---

[1] These statements are not true at asymptotia. However, the approximation we use should be good enough for the energies accessible in this decade.

In (2.5) α'_P denotes the slope of the pomeron trajectory and $G_{PPP}^{hNN}(t)$ is the triple pomeron coupling. If factorization holds

$$G_{PPP}^{hNN}(t) = \frac{\beta_{hh}^{P}}{\beta_{NN}^{P}} G_{PPP}(t), \tag{2.6}$$

where the function $G_{PPP}(t)$ can be obtained from inclusive single-particle spectra in the reaction

$$pp \to p+X,$$

and β's are the pomeron-hadron-hadron and pomeron-nucleon-nucleon vertex functions evaluated at $t = 0$. Finally, assuming factorization for 2-body processes, one gets

$$\delta\sigma_2 = g(\xi_0)\frac{\sigma_{hN}}{\sigma_{NN}}, \tag{2.7}$$

where

$$g(\xi_0) = 2\int_{\xi_0}^{1}\frac{d\xi}{\xi}\int_{t_{max}}^{m^2\xi^2} dt\xi^{2\alpha_P' t}G_{PPP}(t)S(\sqrt{-t}). \tag{2.8}$$

According to a recent analysis of Ganguli and Roy [10]:

$$G_{PPP}(t) = 4.74e^{14.8t}+2.23e^{3.1t} \tag{2.9}$$

for

$$\alpha'_P = 0.25 \text{ GeV}^{-2}. \tag{2.10}$$

For this parametrization one obtains the following approximation of g:

$$g(\xi_0) = -0.78-0.35 \ln \xi_0 \tag{2.11}$$

The accuracy of the last formula is better than 5% for $3. \leq \xi_0 \leq 12.$, and it will become clear that for our purposes the approximation is good enough. Finally, we arrive at the following expression for σ_{hn}:

$$\sigma_{hn} = \sigma_{hd}-\sigma_{hp}+\frac{f(b)}{4\pi}\sigma_{hp}\sigma_{hn}+(-0.78-0.35 \ln \xi_0)\frac{\sigma_{hN}}{\sigma_{NN}}, \tag{2.12}$$

which is our main result.

The r.h.s. of Eq. (2.12) depends on: measurable cross sections, the slope of the $\frac{d\sigma}{dt}$ distribution for hadron-nucleon elastic scattering, and the cut-off $\xi_0 = \frac{M_0^2}{s}$, which is a free parameter. However, it should be stressed that M_0, the mass at which the integration over triple pomeron parametrization of inelastic diffraction is cut off, must be equal, within a factor ~ 1.5, to the mass of the first diffractive excitation of hadron h. Thus the uncertainty due to ξ_0 is of order of 0.2 mb.

3. *Analysis of the data*

We use the results obtained by Carrol et al. [7] for $\pi^\pm$, $K^\pm$ and $p^\pm$ total cross sections on protons and deuterons at Fermilab energies.

To test our parametrization, we start from the pion induced reactions. From isospin invariance

$$\sigma_{\pi^-p} = \sigma_{\pi^+n} \quad \text{and} \quad \sigma_{\pi^+p} = \sigma_{\pi^-n},$$

thus, all the cross sections in (2.12) are known. We have taken $b = 8.5\ \text{GeV}^{-2}$ and $M_0 = 1.5$ GeV, which is close to the position of A_1 resonance. Then, we have found that the fit (2.7) and (2.11) to

$$\delta\sigma_2 = \sigma_{\pi^+p} + \sigma_{\pi^-p} - \sigma_{\pi d} - \frac{f(b)}{4\pi}\sigma_{\pi^+p}\sigma_{\pi^-p} \tag{2.13}$$

is very good ($\chi^2 = 6.8/11$) for the energies in the range 50–340 GeV (cf. Fig. 3a). The masses of A_1 and Q_1 resonances are not very different, so, we have used the same fit to

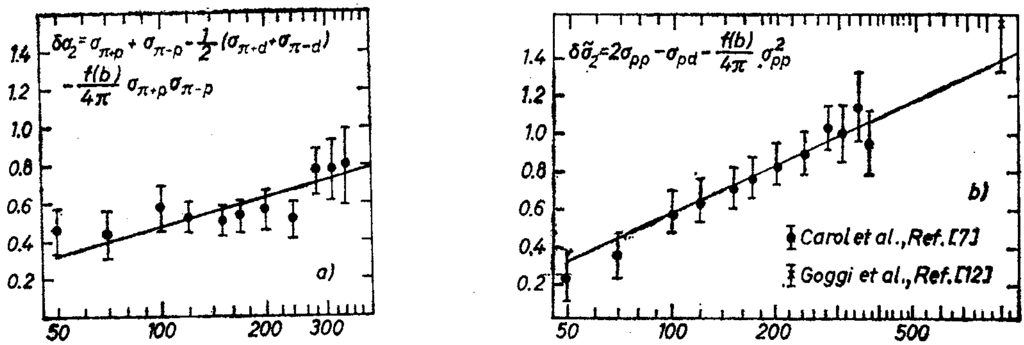

Fig. 3. Comparison between the inelastic screening contributions to a) nd, b) pd total cross section defects and the parametrization (2.12)

TABLE I

p_{lab}(GeV)	K⁺p [a]	K⁺n [b]	K⁻p [a]	K⁻n [b]	p̄p [a]	p̄n [b]
50	18.06±.08	18.54±.15	20.30±.10	19.71±.17	43.93±.10	42.84±.22
70	18.52±.08	18.56±.15	20.38±.05	19.76±.10	43.05±.06	42.28±.17
100	18.88±.07	19.05±.13	20.45±.06	19.95±.13	42.12±.08	41.54±.19
120	19.14±.07	19.30±.14	20.59±.06	20.21±.12	41.70±.15	41.85±.27
150	19.36±.07	19.69±.13	20.60±.07	20.18±.14	41.79±.17	40.94±.32
170	19.64±.06	19.82±.12	20.65±.06	20.33±.12	41.69±.15	41.21±.30
200	19.91±.11	19.85±.15	20.79±.05	20.62±.11	41.51±.15	41.19±.32
240	20.22±.06	20.50±.12	21.30±.07	20.69±.12	41.90±.20	41.07±.35
280	20.45±.07	20.76±.12	21.32±.08	21.05±.13	41.91±.21	41.05±.37
310	20.67±.15	20.95±.26	21.45±.12	21.23±.19	—	—

[a] Carrol et al., Ref. [9]; [b] only statistical errors included

$\delta\sigma_2$ for kaon induced reactions. The $K^{\pm}n$ total cross sections calculated in this way are given in Table I.

It is noteworthy that within our accuracy $\sigma_{K^+p} = \sigma_{K^+n}$, as duality suggests. We use this observation to fix the parametrization of $\delta\sigma_2$ for baryons. The elastic slopes for proton above 50 GeV can be given in the form [11]:

$$b = 0.56 \ln p_{lab} + 8.43. \tag{2.14}$$

Fixing $\sigma_{np} = \sigma_{pp}$, and comparing

$$\delta\tilde{\sigma}_2 = 2\sigma_{pp} - \sigma_{pd} - \frac{f(b)}{4\pi}\sigma_{pp}^2 \tag{2.15}$$

with formula (2.11) one gets the best agreement for $M_0 = 2$ GeV a value close to the mass of the first diffractive excitation of the nucleon. This fit is again very good ($\chi^2 = 4.8/12$)

TABLE II

p_{lab}(GeV)	Σ^-p [a]	Σ^-n [b]	Ξ^-p [a]	Ξ^-n [b]
74.5	33.08 ± .31	32.07 ± .43	—	—
86.1	33.21 ± .27	32.65 ± .40	—	—
101.5	—	—	29.19 ± .29	28.39 ± .42
119.8	33.29 ± .33	32.99 ± .47	—	—
133.8	—	—	29.35 ± .31	29.21 ± .46
136.9	34.14 ± .30	32.81 ± .42	—	—

[a] Biagi et al., Ref. [12]; [b] only statistical errors included.

above 50 GeV [2] (cf. Fig. 3b), and when applied to $\bar{p}$ total cross sections gives the results collected in Table I. In Table II we present also the results for Σ^-n and Ξ^-n cross sections. In the calculation we have used the cross sections obtained by Biagi et al. [8].

4. *Summary*

At presently accessible energies above about 50 GeV the inelastic screening corrections to the scattering on deuterium are comparable with those from elastic screening. Consequently, the Glauber theory must be extended to include this effect. Neglecting it one gets nonsense results for the form factor of deuterium. For example the Glauber formula, when applied to hd, yields $f \approx 0.039$ mb^{-1} (cf. (2.3)), and this value should be compared with 0.027 mb^{-1} which follows from the known deuteron wave function and, consequently, from e.g. electron-deuteron scattering experiments. Moreover, the contribution to the cross section defect from the inelastic screening grows faster with energy than the contribution from the elastic scattering, thus, a naive application of the Glauber

[2] We have included also ISR point [12].

formalism gives f which grows with energy—obviously an unphysical result. At high energies the largest contribution to inelastic screening comes from inelastic diffraction. This contribution grows whereas the others stay constant or decrease with increasing energy. Thus, in this paper we have proposed the simple formula (2.12) which is a generalization of Glauber's formula and includes the term which describes inelastic diffraction screening on deuterium. This formula, when applied to pion induced reactions, gives a very good description of the data at Fermilab energies. The systematic error of our approximation is about 0.2 mb, and we have argued that a better approximation requires an additional theoretical analysis of the corrections from the ordering of nucleons in deuteron, i.e. of the S_ε form-factor, not to mention the need for complete triple-Regge analysis. On the other hand, the accuracy obtained is comparable with the statistical errors of the present day experiments, so, we find the approximation (2.12) quite satisfactory. We have used it in the calculation of $K^{\pm}n$, $\bar{p}n$, $\Sigma^{-}n$ and $\Xi^{-}n$ total cross sections from the existing data. The results of this calculation are given in Table I and II.

REFERENCES

[1] M. J. Longo et al., *Phys. Rev. Lett.* **33**, 727 (1974).
[2] R. J. Glauber, *Phys. Rev.* **100**, 242 (1955).
[3] R. J. Glauber, V. Franco, *Phys. Rev.* **156**, 365 (1965).
[4] E. Albers et al., *Nuovo Cimento* **42A**, 365 (1965).
[5] V. N. Gribov, *Zh. Eksp. Teor. Fiz.* **56**, 892 (1969).
[6] J. Kwieciński, L. Leśniak, K. Zalewski, *Nucl. Phys.* **B78**, 251 (1974).
[7] A. S. Carrol et al., *Phys. Lett.* **61B**, 303 (1976); *Phys. Lett.* **80B**, 423 (1979).
[8] S. F. Biagi et al., *Nucl. Phys.* **B186**, 1 (1981).
[9] J. I. Mc Gee, *Phys. Rev.* **151**, 772 (1966).
[10] S. N. Ganguli, D. P. Roy, *Regge Phenomenology of Inclusive Reactions*, Tata Institute of Fundamental Research preprint TIFR-TH-80-13 (1980).
[11] V. A. Nikitin, in *Proceedings of International Conference on High Energy Physics*, vol. 2, p. 547, Geneva 1979.
[12] G. Goggi et al., *Nucl. Phys.* **B149**, 381 (1979).