Vol. B12 (1981) ACTA PHYSICA POLONICA No 11

FURTHER APPLICATION OF ANGULAR MOMENTUM DEPENDENT POTENTIALS TO PROTON ELASTIC SCATTERING*

BY A. M. KOBOS

Institute of Nuclear Physics, Cracow**

AND R. S. MACKINTOSH

Physics Discipline, The Open University, Milton Keynes***

(Received April 24, 1981)

In this paper we extend our application of the l-dependent model to a wider range of cases than in a previous paper. We include more non-closed shell nuclei and some heavy nuclei as targets, getting better fits than previously found, with no substantial exceptions to the systematic properties of the l-dependent potential. For one mass sequence we find shell effects, but note that the results would be more certain if more analysing powers data were available. This paper makes the case that a simple pattern of l-dependence is a universal feature of proton scattering between 20 and 60 MeV. Since also, as we show, the effect on (p, p′) is large, the effect of l-dependence upon direct reactions should not be ignored.

PACS numbers: 24.10.Ht, 25.40.Cm

1. Introduction

We have recently shown that proton elastic scattering data which could not be fitted by the usual models could be very well fitted when angular momentum dependent real and imaginary terms were included in the optical potential, [1–3]. Moreover, we have presented evidence that the proton optical potential really should be considered to be l-dependent. This evidence is of two kinds: firstly, we have demonstrated, [4], a close relationship between the effect of pickup coupling and l-dependence, and, secondly, we have shown the relationship of the oscillatory components of model independent l-independent fits to l-dependent terms in the potential, [5]. The relationships we have established

---

* Part of this work has been done when at Wheatstone Laboratory, King's College, London.

** Address: Instytut Fizyki Jądrowej, Radzikowskiego 152, 31-342 Kraków, Poland.

*** Address: Physics Discipline, The Open University, Milton Keynes, England.

are summarized in Fig. 1. Nevertheless, it has always been clear that to establish the physical meaningfulness of a 19 parameter potential, the potential must have regular properties over a range of targets and energies. So far, we have presented remarkable energy systematics for a few light targets and isolated cases for heavier ($A \leqslant 58$) targets. We are now

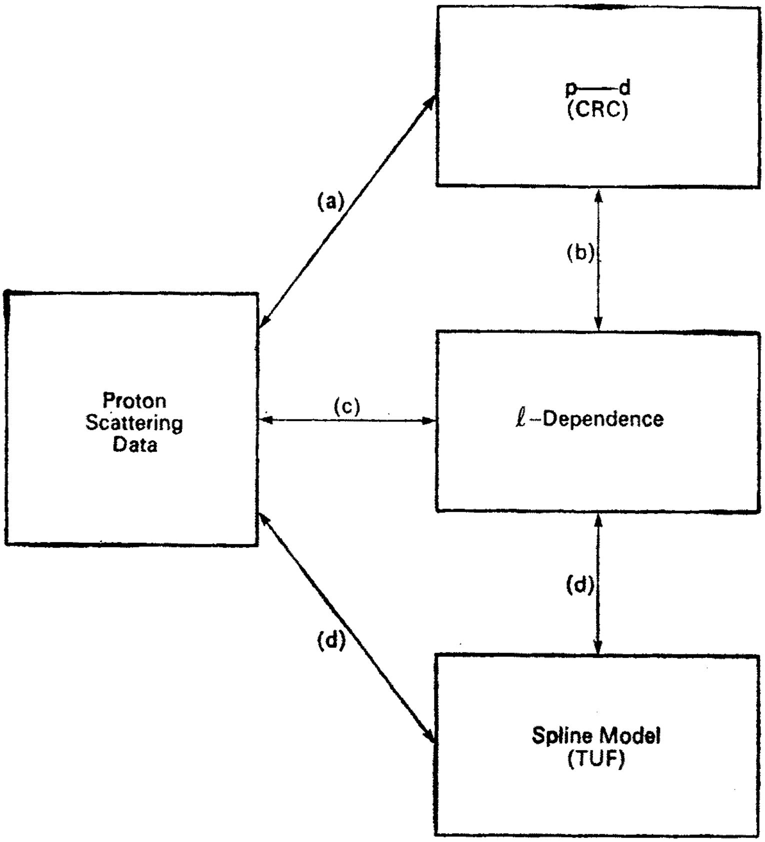

Fig. 1. Showing the interrelationships that have been established in the following references between different models for fitting proton scattering: (a) Mackintosh [8], Kobos and Mackintosh [7]; (b) Mackintosh and Kobos [4]; (c) Mackintosh and Cordero [1], and Kobos and Mackintosh [2], [3]; (d) Kobos and Mackintosh [5]

in a position to extend the systematics, and this paper, which is essentially a continuation of Ref. [3], contains fits to various data which have not previously been fitted to a high standard of accuracy. As well as extending the energy systematics found previously for ^{16}O, we have extended the model both to heavier nuclei and to a range of non-closed shell nuclei. The major limiting factor in our attempt to find systematics is the lack of "suitable" data; that is to say data which include precise asymmetry data and which cover a wide angular range. Without these features, the data are simply too easy to fit. In particular there is an interplay between the spin-orbit and other components. One purpose of this paper is to exhibit the shell effects to which we have briefly alluded previously, [6].

Indeed, the way in which these results are rendered less than definitive by the lack of analysing power data will be made plain and will lead to one theme of this paper, i.e. a plea for more high precision data including analysing powers and an account of the interesting phenomena which might be revealed by such data.

Other topics discussed in this paper are alternative parametrizations of the l-dependency and the application of l-dependent potentials to DWBA calculations of (p, p′) and (p, d) reactions.

2. *The l-dependent potential and fitting procedure*

For most of the calculations to be described, the form of the potential is that used previously. We define by f_i the Woods-Saxon form factor depending on r_i and a_i, i.e.:

$$f_i(r) = f(r, r_i A^{1/3}, a_i) = (1+\exp{(r-r_i A^{1/3})/a_i})^{-1} \tag{1}$$

and similarly define the derivative (surface peaked) form $g_i = -4a_i f_i'$. The l-dependent potential we shall generally use consists of a standard l-independent term

$$U(r) = -Vf_1(r) - iWf_2(r) - iW_d g_2(r) \tag{2}$$

to which we add a term depending upon the partial wave angular momentum, l:

$$U_l(r) = -f(l^2, L^2, \Delta^2)\{V_l g_3(r) + iW_l g_4(r)\} \tag{3}$$

(except, of course, we do not include an $A^{1/3}$ with the L^2).

Eqs. (1), (2) and (3) define the parameters employed when the potentials are presented in tabular form.

The overall potential $U+U_l = U$ alone for $l > L$, but contains extra surface peaked real and imaginary terms for $l \leqslant L$. This behaviour has always been found for actual values of L and Δ necessary to fit the data. For actual values of L and Δ, very much the same effects could certainly be reproduced with suitable L' and Δ' parameters bearing different powers in the l-dependent form factor.

The Coulomb and spin-orbit terms are of the usual form except that we have sometimes searched upon the imaginary spin-orbit term, [2]. All of the calculations reported in this paper involve a further development of our program which had been basically Perey's JIB-3 code. We have now incorporated the CERN comprehensive minimizing package MINUIT [9] which can handle a very efficient search for many parameters simultaneously.

We have never claimed that the parametrization given above is unique, only that it does work rather well. Establishing the uniqueness or otherwise of optical potentials is evidently a long process. We have occasionally tried alternative forms and we shall make some remarks about them below. However, it is certainly the case that other forms should be tried.

One of the possibilities is to substitute for the l-dependency factor:

$$f(l^2, L^2, \Delta^2) - 1/2 \tag{4}$$

so that a surface term changes its sign around $l = L$, rather than disappears for high l.

A parametrization with fewer parameters was suggested in [5] and that is to multiply standard form real and imaginary terms by, respectively, $X(l)$ and $Y(l)$ given by:

$$X(l) = (1-f(l^2, L^2, A^2), \tag{5a}$$

$$Y(l) = (1+f(l^2, L^2, \Delta^2). \tag{5b}$$

The main purpose of this paper is to give a systematic account of the application of the form given in Eqs. (1), (2) and (3).

3. Proton scattering from ^{15}N between 18 and 44 MeV

We have fitted data from the Milan Group, [10], for proton scattering from ^{15}N at 13 energies between 18 and 44 MeV. These data extend to reasonably backward angles but there are *no* accompanying analysing power data.

The conventional optical model analysis of these cross section data essentially failed to fit them [11]. Initially, in our search, we allowed the parameters of the spin orbit potential to vary along with the remaining parameters of the central potential. We were able to fit the data precisely but such properties of the potentials as the volume integrals and r.m.s. radii behaved much less regularly than they had for ^{16}O where analysing power data were also fitted. We then adopted the alternative approach of either fixing the spin--orbit potential at that for ^{16}O at a nearby energy, or interpolating between ^{16}O parameters at different energies. Without such a constraint there was evidently too much freedom in the potential to get fully meaningful results. The volume integrals and r.m.s. radii of the new potentials behaved in quite a different manner from those found with the first method. With both ways of getting the spin-orbit term, it is nevertheless the case that the qualitative features of the l-dependent terms are just those we have always found. It is true that reasonably good fits to these cross section data with l-independent potentials exist, [11], but the potentials found there have unphysical spin-orbit terms which would be immediately eliminated had analysing power data been fitted.

Subsequently, for three energies, 22.4, 24.5 and 26.0 MeV, the analysing power data became available to us (M. Pignanelli, private communication). It turned out that the potentials we had determined before gave poor fits to the analysing power data, but we easily found new parameters which fitted these and the respective cross sections. We also refitted the cross section data for 20 and 28 MeV with the spin-orbit parameters fixed at the values found at 22 and 26 MeV, respectively. The fits to cross-sections and analysing powers for these 5 energies are shown in Figs. 2a and 2b. For obvious reasons we did not extend this procedure to more distant energies. It is remarkable that now the volume integrals exhibit a resonance behaviour at about 24 MeV, strikingly similar to that observed for ^{16}O (see Fig. 5). Fig. 3 shows the behaviour of the volume integrals and r.m.s. radii for the three approaches to the spin-orbit potential as described above. From Fig. 3 it is clear that no reliable quantitative (and moreover qualitative) features of the potential can be extracted without fitting analysing power data. For this reason we present in Table I the parameters of the potential defined by Eqs. (1), (2) and (3) only for 20, 22.4, 24.5, 26 and

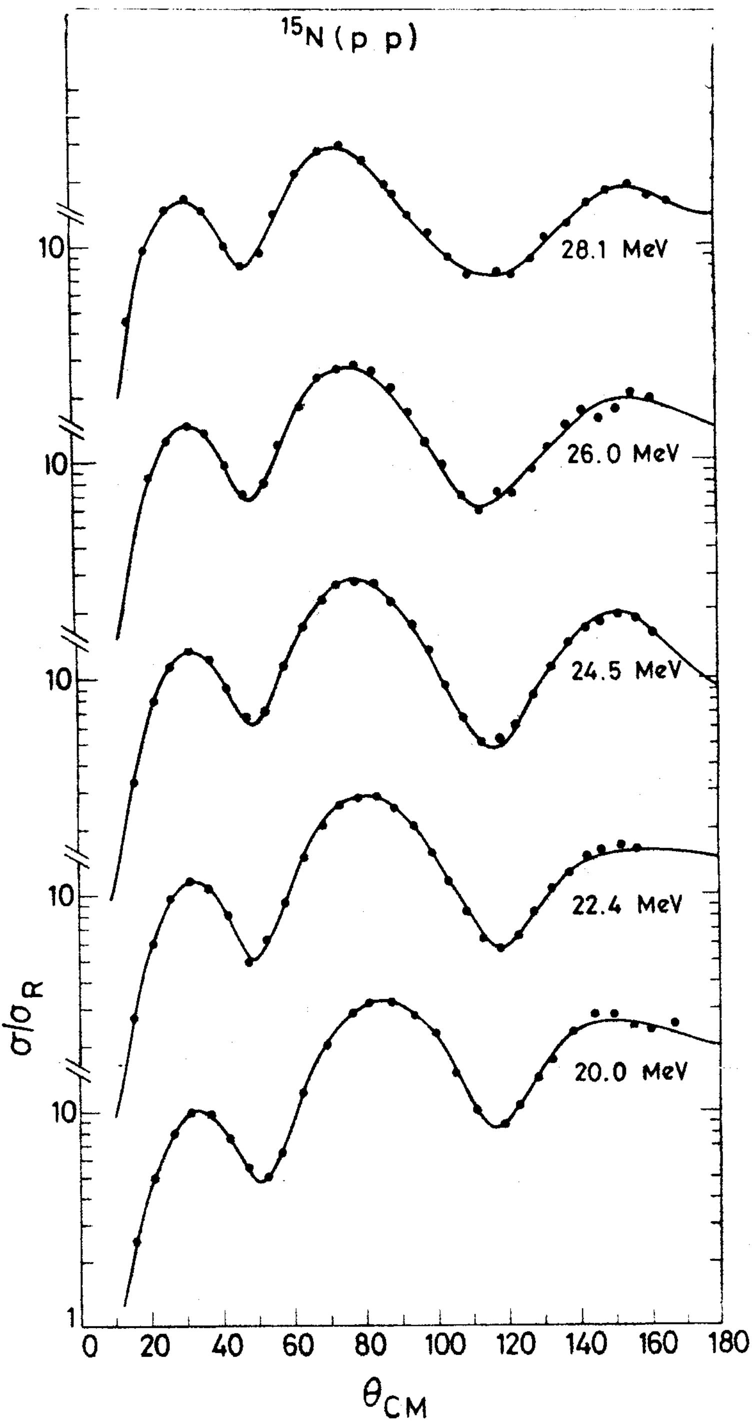

Fig. 2a. Fit to proton elastic scattering from ^{15}N at 20.0, 22.4, 24.5, 26.0 and 28.1 MeV. Except at 20.0 and 28.1 MeV the analysing power was fitted simultaneously. The spin-orbit potential for 20.0 was fixed at that for 22.4 MeV; that for 28.1 was fixed at that for 26.0.

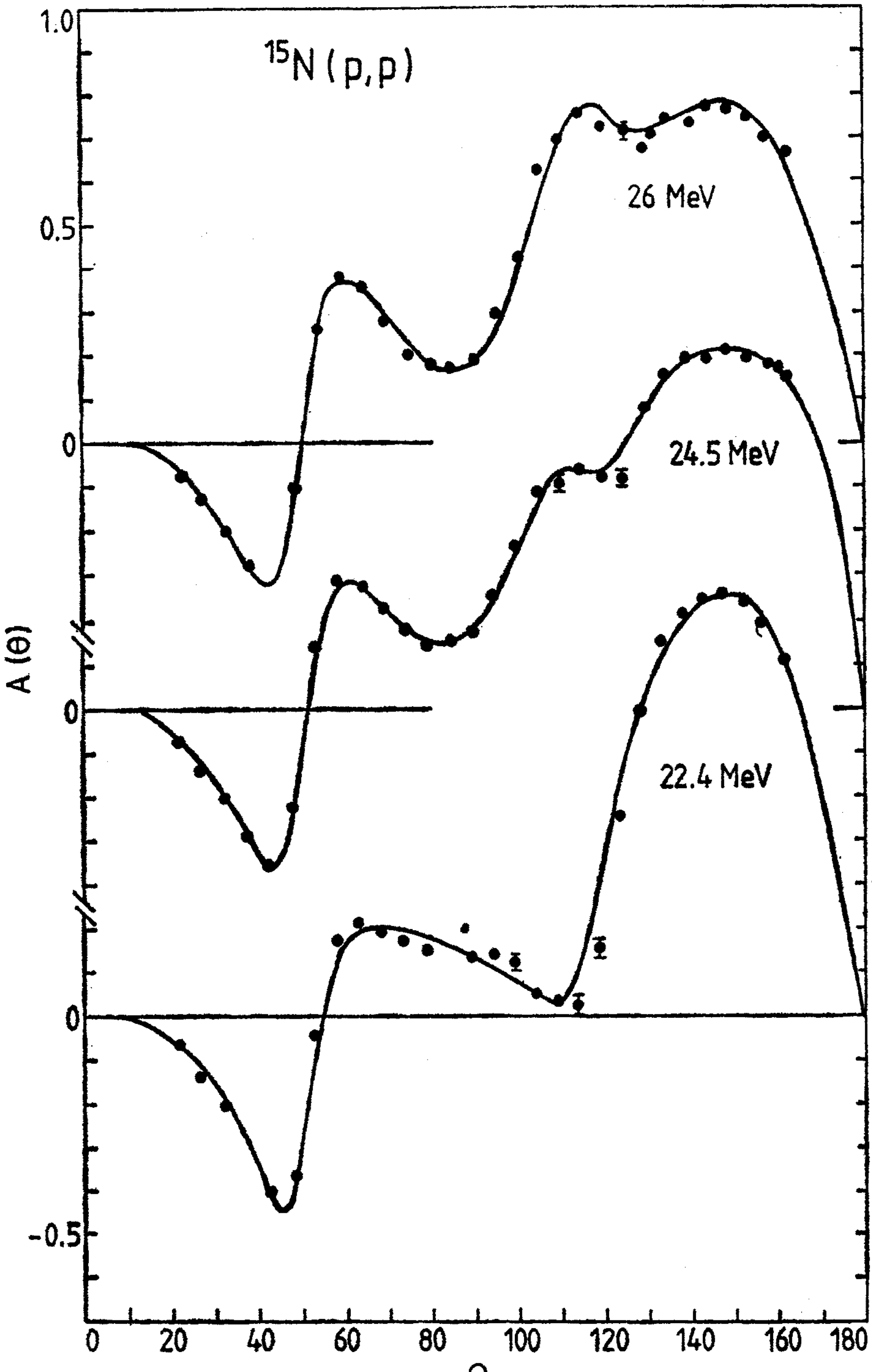

Fig. 2b. For the elastic scattering of protons from ^{15}N we show analysing power fits. The energies were 22.4, 24.5 and 26.0 MeV and the parameters were as for corresponding fits in Fig. 2a

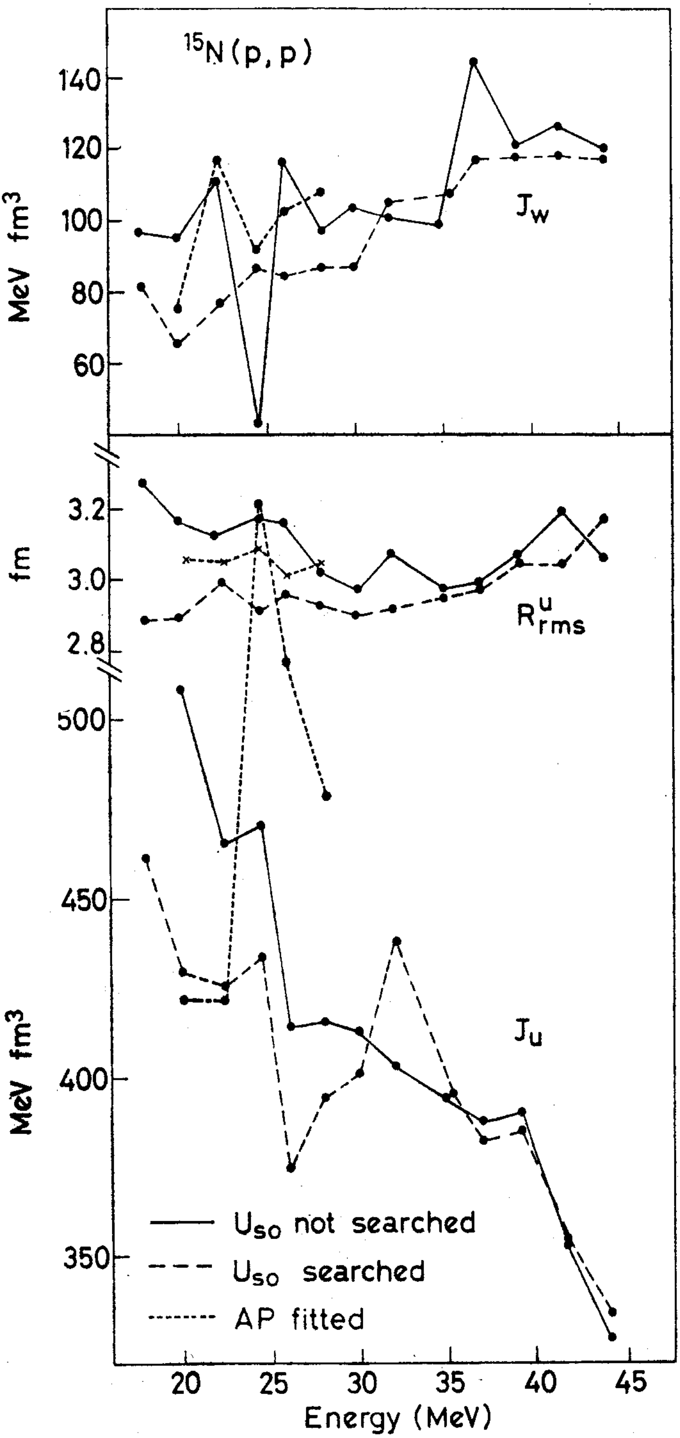

Fig. 3. Characteristics of the l-independent part of the ^{15}N l-dependent potential obtained in three ways: the imaginary volume integral J_w, the real r.m.s. radius R^u_{rms} and the real volume integral J_u. The three procedures are described in the text, but note that there were necessarily fewer energies for the AP fitted (analysing power fitted) procedure

TABLE I

The l-dependent optical potential for proton elastic scattering from ^{15}N

E	20.0	22.4	24.5	26.0	28.1
V	60.622	53.881	66.514	68.830	63.689
r_1	1.0002	1.0550	1.1644	1.0985	1.1020
a_1	0.6422	0.6429	0.4890	0.5237	0.5150
W	0.0	0.0	1.2179	1.0367	0.0144
W_d	1.2549	1.9313	2.3554	2.4695	3.0902
r_2	1.9886	1.9998	1.8290	1.7318	1.7049
a_2	0.6968	0.7004	0.3790	0.4895	0.5634
V_{so}	6.4440	6.4440	7.1113	7.1773	7.1487
W_{so}	1.2440	1.2440	0.6476	0.2757	0.2560
r_{so}	0.6007	0.6007	0.6636	0.6687	0.6697
a_{so}	0.5951	0.5951	0.6173	0.6272	0.6224
V_l	−8.6581	−14.761	−16.191	−14.072	−15.857
r_3	0.7001	0.83854	0.8764	0.7914	0.7055
a_3	0.2574	0.2156	0.7239	0.7998	0.6406
W_l	5.4435	3.1265	4.7615	4.2407	2.4988
r_4	2.0809	1.8953	1.0249	1.0084	1.1300
a_4	0.1882	0.2356	0.1502	0.1518	0.1701
L	3.5094	3.0656	3.1043	3.0296	3.0670
Δ	0.7010	0.7151	1.0475	0.7549	0.6163
J_v	421.52	421.98	561.60	519.36	480.04
$\langle r_v^2\rangle^{\frac{1}{2}}$	3.057	3.125	2.873	2.862	2.845
J_w	74.58	116.69	94.98	101.81	108.76
J_{vl}	−23.69	−46.87	−247.13	−217.17	−146.10
J_{wl}	90.14	53.98	15.36	13.41	11.12
χ_σ^2/N_σ	1.02	2.83	1.52	3.39	1.51
N_σ	29	30	30	29	29
χ_P^2/N_P	—	5.11	1.61	1.66	—
N_P	—	29	29	29	—

28.1 MeV. Although we cannot attribute too much meaning to the actual values of the parameters found at the other energies, there is nevertheless a positive conclusion to be drawn: It is clear that the proton scattering from a non-closed shell nucleus, ^{15}N, over a wide range of energies, can be fitted exceedingly well with much the same l-dependent potentials as those required for proton scattering from ^{16}O nucleus. We fitted the 22.4 MeV data with an l-independent Woods-Saxon potential for which $\chi_\sigma^2/N_\sigma = 16.3$ and $\chi_P^2/N_P = 19.3$, i.e. about five times and three times higher, respectively, than for the l-dependent potential.

4. *Proton scattering from the mass sequence $A = 12$ to $A = 24$ at 35.2 MeV*

The Milan Group have measured the elastic scattering of 35.2 MeV protons from many nuclei, [10]. We have fitted their data for a sequence of 12 nuclei between ^{12}C and ^{24}Mg. There are again no analysing power data and once more the problem was that of fixing the spin-orbit term. This was allowed to adjust over restricted ranges of its parameters. For nuclei from ^{12}C to ^{19}F the spin-orbit potential appeared to be consistent with known spin-orbit potentials at this energy. However, for ^{20}Ne, ^{22}Ne, ^{23}Na and ^{24}Mg the resulting depths of the spin-orbit potential were unusually small (2–3 MeV). For these last nuclei we then fixed the spin-orbit parameters at the values found for ^{19}F and refitted the central potential. The new potentials had rather different volume integrals. As before we achieved great improvements to the quality of fits by including l-dependent terms. We have been able to obtain fits which go through virtually every experimental point for all cases. In particular the "anomalous" large angle enhancement of the cross section for nuclei near to closed shells (^{16}O) is accurately fitted. Because of uncertainties in the actual values of the parameters engendered by the lack of analysing power data, we have not presented a table of parameters. However, over the entire mass range the qualitative features of the l-dependent terms are exactly those as previously found for closed shell nuclei: repulsive real term peaked within the nuclear surface and absorptive imaginary term peaked outside the nucleus. We note here that these data could not be fitted well with *any* l-independent Woods-Saxon potential, even raised to some powers, [12]. We explicitly established that the required l-dependence could not be confined to one or two partial waves only, contrary to various suggestions.

In Fig. 4 we show the mass dependence of the l-independent and l-dependent volume integrals. Although the detailed behaviour is dependent on the chosen spin-orbit potential, it is clear that there are discontinuities which may be associated with the shell closure of ^{16}O and the onset of deformation at around $A = 19$. This is what we referred to in [6] as "shell effects" for the l-dependent potentials. We reiterate that any precise estimation of the effect depends strongly on the spin-orbit potential. However, given analysing power data we could then establish shell effects through discontinuities in the parameters corresponding to precise fits. The shape of the backward angle angular distribution changes considerably over this mass range. This is not really anomalous and it can be fitted with an l-dependent potential with a regular variation that should ultimately be related to nuclear structure. Of course, some nuclei in the range are deformed and this is certainly reflected in the r.m.s. radii and the 2^+ inelastic coupling. This latter certainly gives a somewhat l-dependent contribution to the optical potential but apparently weaker than pickup effects. Fabrici et al. [12] attribute the difficulties in explaining the experimental cross section to correlations of the backward angle enhancement in cross section with the collective properties of nuclei, although it was impossible to reproduce the effect with coupled channel calculations. With the l-dependent potential, reflecting probably some shell closure effects, we have been able to precisely fit the effect and that is why we claim that the coupling to reaction channels [7] could be responsible for the large angle enhancement of the elastic scattering cross section for nuclei near to the closed 1p shell. However, a more precise estimate of the effect would require analysing power data. We believe our results show that it may not be useful to consider some features of the cross section as "phenomena" as they may correspond to more or less simple changes in the scattering potential which precisely fits the data (cf. ALAS for alpha particles). The most obvious correlate in the present case is the large change in (p, d) Q-value at the closed shell.

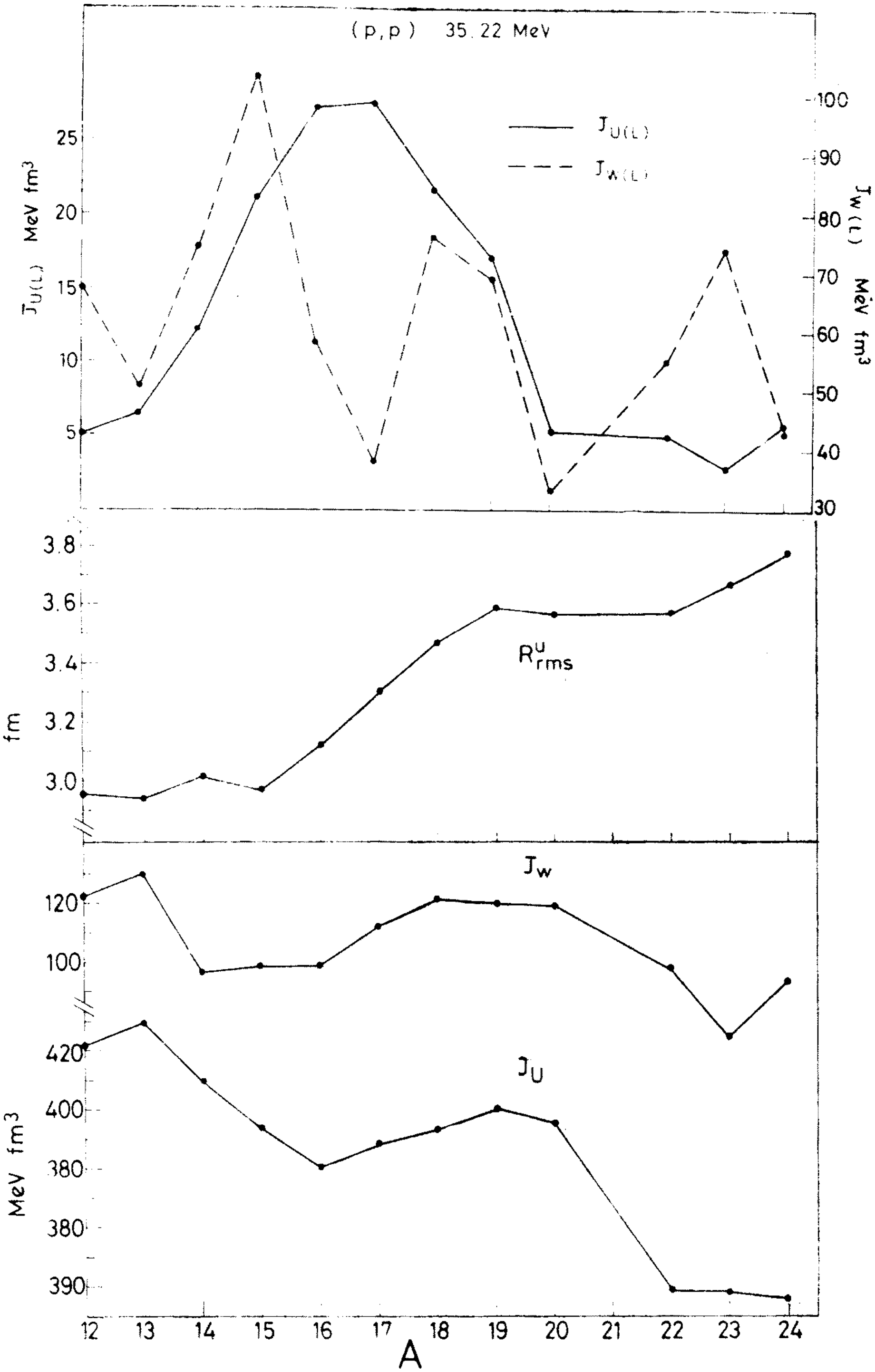

Fig. 4. Dependence upon mass number of l-dependent optical potential for 35.22 MeV protons. R^u_{rms} is rms radius of real l-dependent term and J_u is the corresponding volume integral. J_w is the volume integral of the l-independent imaginary term. $J_{u(L)}$ and $J_{w(L)}$ are the volume integrals for the l-dependent term. Note that $J_{u(L)}$ is plotted positive, but is of opposite sign to all the other terms, a universal feature of this

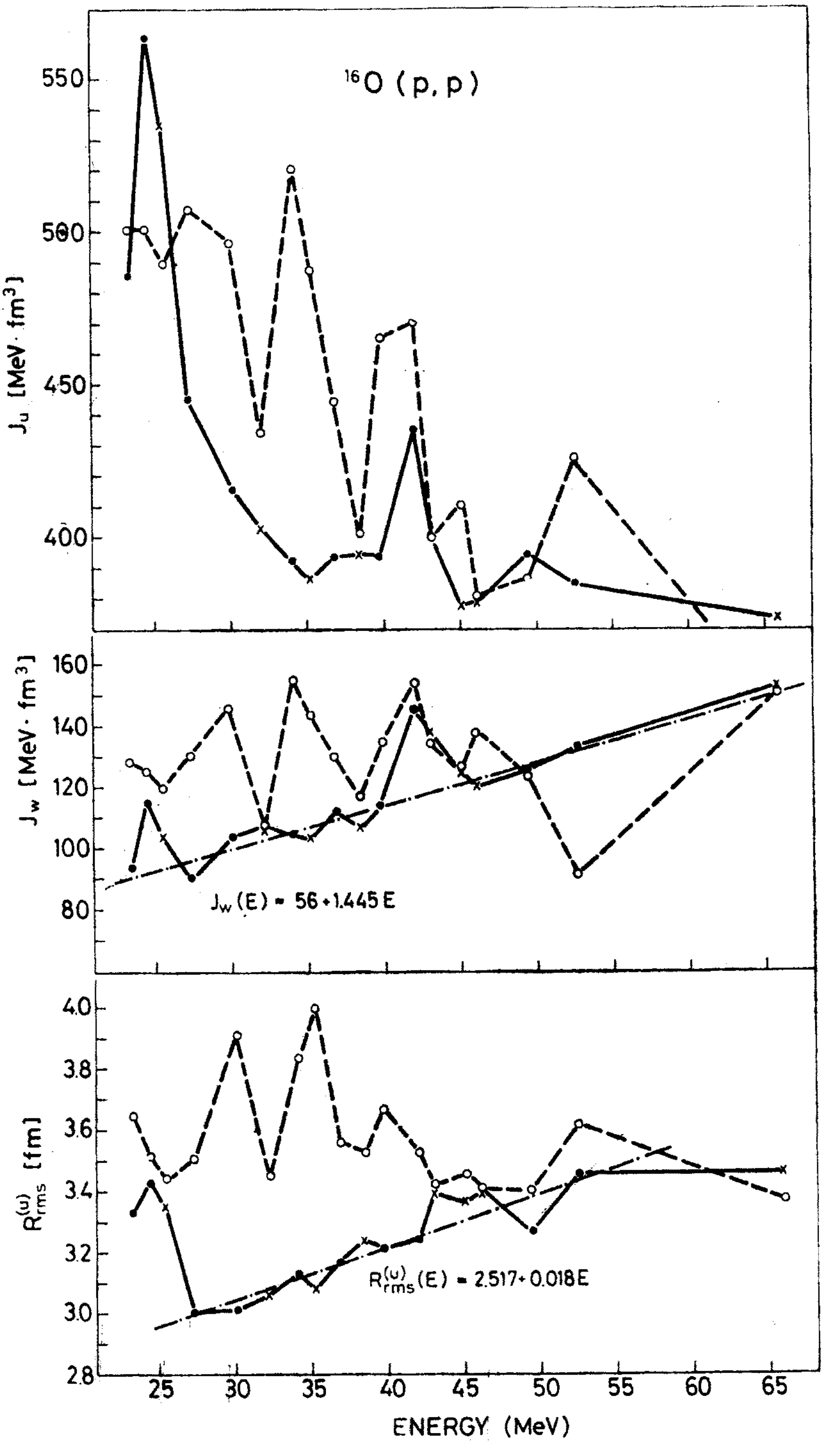

Fig. 5. Energy dependence of the ^{16}O l-dependent potential. We show how the behaviour of the l-independent parts becomes much more regular for the l-dependent potential (solid lines) than for standard l-independent potentials (dashed lines). The solid points represent cases where analysing power data was fitted, the crosses where it was not

5. Further systematics for ^{16}O and ^{40}Ca

In our previous paper [3], we published fits for proton scattering from ^{16}O at 11 energies. There are certain regularities in the potential which provided some of the best evidence for l-dependence as a general phenomenon. At the same time there were "resonance-like" features in the energy dependence of certain volume integrals. Now, we have extended those systematics by fitting further data. For most of these new cases there are no analysing power data and we have fixed the spin orbit potentials to values found for scattering from the same nucleus at nearby energies. The new data are those of Snelgrove and Kashy, [13], at 25.46, 32.07, 38.43 and 45.13 MeV, of the Milan Group (De Leo et al. [10] at 35.2 MeV, Clarke et al. [14] at 49.4 MeV, and Lerner et al. (K. Kwiatkowski, private communica-

The l-dependent optical potentials for proton elastic scattering for various nuclei

Nucleus	Fe^{56}	^{56}Fe	^{58}Ni	^{58}Ni
E	30.3	35.2	30.3	40.0
V	49.872	47.902	52.068	49.883
r_1	1.1524	1.1824	1.1589	1.0975
a_1	0.6456	0.6362	0.5839	0.7304
W	3.4943	4.8250	2.9910	4.0640
W_d	2.2004	2.7083	2.1507	3.5153
r_2	1.4944	1.2761	1.4679	1.5802
a_2	0.6982	0.6978	0.7678	0.7136
V_{so}	7.6053	7.7377	6.9745	6.1172
W_{so}	0.2301	0.3107	0.0	0.0
r_{so}	0.8468	0.8283	0.8165	0.9408
a_{so}	0.7028	0.7376	0.6956	0.6295
V_l	−5.8345	−4.6551	−10.385	−5.1329
r_3	0.9956	1.0168	1.0195	0.9620
a_3	0.3440	0.5356	0.3844	0.2596
W_l	0.6256	0.8172	0.9115	1.8305
r_4	2.3988	2.5732	2.4084	2.3343
a_4	0.1501	0.5444	0.1501	0.5483
L	5.5589	6.7138	5.5003	6.9702
Δ	0.900	1.9124	1.7364	0.9015
J_v	384.51	393.53	393.38	354.21
$\langle r_v^2\rangle^{\frac{1}{2}}$	4.174	4.227	4.097	4.266
J_w	102.57	92.94	95.03	84.08
J_{v_l}	−26.65	−35.71	−55.16	−16.15
J_{w_l}	9.198	38.81	10.23	71.35
χ_σ^2/N_σ	2.46 (6.71)	0.80	2.40 (6.93)	1.77 (3.26)
N_σ	80	30	80	60
χ_p^2/N_p	4.32 (13.32)	—	1.21 (7.99)	2.68 (8.81)
N_p	32	—	45	29

tion) at 65.8 MeV. At the lower energies χ^2/N was generally improved by more than an order of magnitude and there was a smaller but significant improvement for higher energies. What we have found is that the behaviour of resulting parameters, volume integrals and r.m.s. radii falls very well into the pattern found for the 11 cases fitted previously [3]. To illustrate this we show in Fig. 5 the energy dependence of volume integrals of real and imaginary parts, and of real r.m.s. radii of the l-dependent components of the potential. These figures are amended versions of those published in [3] but the extended systematics constitute a very important confirmation of our model. We comment that for ^{40}Ca we have also now analysed new data for ^{40}Ca at 24.5, 27.4 MeV, (B. W. Ridley, unpublished) at 35.2 MeV (De Leo et al. [10]) and at 61.4 MeV (Fulmer et al. [15]), the last three without analysing power data. The results follow closely the energy dependence of the cases described in [3].

TABLE II

from ^{56}Fe to ^{208}Pb. Figures in parentheses refer to best l-independent fits

^{58}Ni	^{90}Zr	^{120}Sn	^{208}Pb	^{208}Pb
61.4	40.0	30.3	30.3	40.4
45.512	44.601	52.494	53.262	58.369
1.1342	1.2282	1.1577	1.2042	1.1634
0.6198	0.5330	0.7457	0.7261	0.6172
4.5986	3.4811	0.013	2.5466	3.3363
2.9351	3.6789	8.7540	10.1100	2.8021
1.4003	1.3072	1.3093	1.2840	1.4951
0.5681	0.7590	0.6476	0.5471	0.8826
7.5808	7.1845	6.0927	6.4462	4.2724
0.0	0.0	0.0	0.0	0.0
0.9660	1.0148	1.0310	1.2281	0.8978
0.7223	0.8262	0.7151	0.4983	0.4783
−7.0556	−4.8957	−4.7990	−7.9636	−24.230
0.9570	1.0642	0.8418	0.9130	1.0444
0.2794	0.3984	0.1501	0.3406	0.4637
1.3100	1.2388	6.4973	0.8166	3.5496
2.3981	1.7898	1.1076	1.7463	1.5001
0.1730	0.1500	0.4440	0.1600	0.6678
7.7011	7.8771	5.3879	6.1745	8.9992
0.9813	0.9637	3.1150	2.5843	4.9010
330.46	375.39	395.72	426.22	412.49
4.107	4.701	5.219	6.149	5.812
101.87	93.74	101.55	101.66	98.41
−23.71	−25.16	−5.18	−19.29	−105.13
16.33	6.64	36.58	3.36	45.76
0.36	6.23	2.20	0.99	12.47
	(9.39)	(2.99)	(3.04)	(22.18)
47	65	80	72	65
—	5.98	2.58	2.67	3.48
	(9.14)	(5.61)	(3.83)	(5.41)
—	24	36	40	29

6. *l*-dependence of the proton potential for heavier nuclei

Elsewhere [16] we described results and fits at energies below 25 MeV for the targets ^{54}Fe, ^{56}Fe, ^{58}Ni and ^{120}Sn (Van Hall et al. [17]). Here we record fits to published scattering data for the following energies and targets: 30.3 MeV protons on ^{58}Ni, ^{120}Sn and ^{208}Pb (Ridley and Turner [18], and Greenlees et al. [19]), 35.2 MeV protons on ^{56}Fe (De Leo et al. [10]; no analysing power data), 40 MeV protons on ^{58}Ni, ^{90}Zr and ^{208}Pb (Blumberg et al. [20]) and 61.4 MeV protons on ^{58}Ni (Fulmer et al. [15]; no analysing power and limited angular range).

The parameters and characteristics of the potentials fitting these data are given in Table II. As a rough average, χ^2/N is reduced by about a factor of two which is less improvement than was found for lighter nuclei. Nevertheless, the persistent combination of $V_l < 0$ and $W_l > 0$ is found in every case, as for light nuclei, and in fact all the general properties found for light nuclei hold here with a few minor exceptions which we note. For ^{120}Sn at 30.3 MeV, we find $r_4 < r_2$. Curiously, this also occurs for ^{120}Sn at 20.4 MeV [16]. We note in this connection extremely large values of r_4 for all ^{58}Ni cases which also occur for ^{58}Ni at the lowest energies, [16]. For ^{208}Pb at 30.3 MeV we found an unusually large

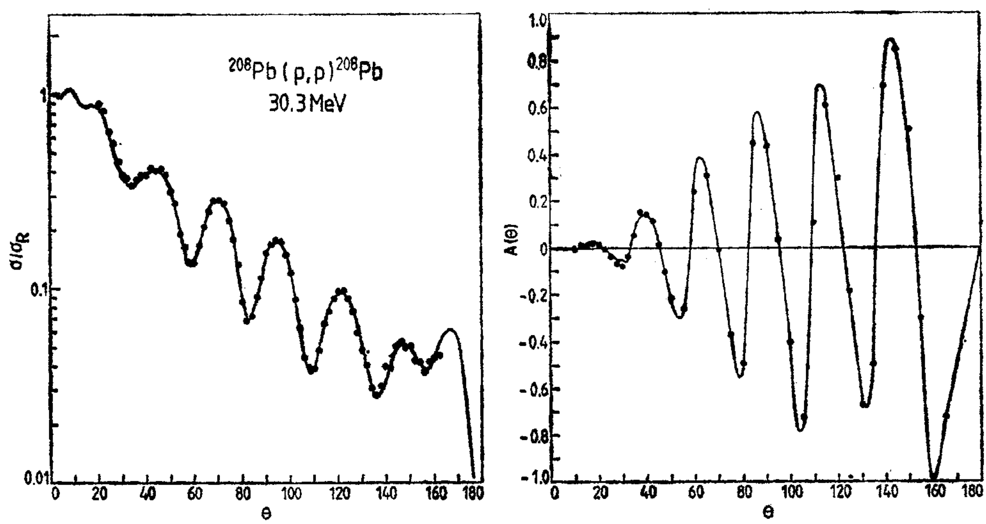

Fig. 6. Fit to 30.3 MeV proton scattering from ^{208}Pb using *l*-dependent potential

spin-orbit radius in contradiction to our results, elsewhere in this paper and in [2], for lighter nuclei. We failed to find alternative solutions with a smaller r_{so}. The fit in this case shown in Fig. 6 is good ($\chi^2/N_\sigma = 0.99$, $\chi_P/N_P = 2.67$ c.f. 3.04 and 3.83, respectively, for Woods-Saxon fits) for precision data over wide angular range ($N_\sigma = 72$, $N_P = 40$).

The volume integrals of the *l*-dependent terms are somewhat less than those for light nuclei, but this should be expected for a surface term which would tend to go like $A^{-1/3}$. The searching procedure in all cases found L within one unit of l from the value for which $|\eta(l)|$ has its deep minimum, (or its outermost minimum if there is more than one). It cannot be said that the values of volume integrals of the l-dependent terms, J_{V_l} and J_{W_l}, follow very regular patterns, but in every case one or the other is large, in the sense that setting U_l or W_l zero would make very drastic effect on the cross section. On the evidence of the existing data, there is no reason to believe that the l-dependence disappears for heavy nuclei; nor does it disappear rapidly above 35–40 MeV and it is probably substantial to twice that energy.

The cases considered here practically exhaust the data we consider suitable but they are not really sufficient to relate the potentials reliably to the many interesting physical effects that may be occurring. The combined nucleus/energy systematics should reveal effects due to the energy dependence of Pauli breakup effects on the deuteron and the effects of different momentum matching conditions of the (p, d) reaction as well as varying influence of deuteron breakup corresponding to the varying of the surface density gradient from nucleus to nucleus.

7. *Alternative parametrizations of the l-dependence*

In order to study different parametrizations of the l-dependent potential we performed some trial calculations for ^{16}O at 34.1 MeV, ^{40}Ca at 30.3 MeV and ^{56}Fe at 30.3 MeV. With the l-dependence as given by Eq. (4) we have been able to get almost equivalent fits to those reported earlier [3], providing that we allow for the l-dependent potential to be considerably deeper, about twice, with the remaining l-dependent parameters very similar to those previously found. It seems that the additional attractive effect for higher partial waves can be balanced by a stronger repulsive effect in the low partial waves. We tried to fit the above data with an alternative l-dependent potential as defined by Eqs. (5a) and (5b). We were not able to obtain any fit at all. We take this to imply that, as expected, the l-dependence of the potential must be localized in the surface region of the potential, and that the real and imaginary l-dependent terms must have independent geometries. Obviously, simply adding parameters does not ensure better fits unless they have some correspondence to the physics.

Finally, we note the l-dependence introduced by Chatwin et al. [21], for heavy ions. The imaginary part of the form factor is multiplied by $(1+\exp((l-L)/\Delta))^{-1}$. With light ions we have nearly always found that the real l-dependent terms were at least as great as the imaginary terms, although often less as a fraction of the respective l-independent component. In fact, dispersion theory makes it unlikely that l-dependence could exist only in one part of the potential. Although the stated physical justification of the potential of Chatwin et al. [21] is quite different from the physical picture which we believe underlies ours, we now believe that it may be possible to find a deeper link between our ideas and those of Chatwin et al. [21].

8. *l-dependent potentials and inelastic scattering*

We have applied l-dependent optical potentials to DWBA calculations of ^{16}O (p, p′) (3^-, 6.13 MeV level) for 30.1 MeV incident protons using optical potentials from [3], for 30.1 and 24.5 MeV. Real and complex macroscopic form factors of a derivative

Woods-Saxon form were used, and in each case the l-dependent optical potentials gave inelastic scattering angular distributions which were very different from those found with the best l-independent potentials. The difference was significant for forward angles and for the whole backward hemisphere. As can be seen in Fig. 7 there is even a backward peaking effect. More details of these calculations can be found in [22]. We wish to draw attention to the following points. First, these potentials can make large differences to inelastic scattering cross sections, and this fact should be faced when various other physical inferences

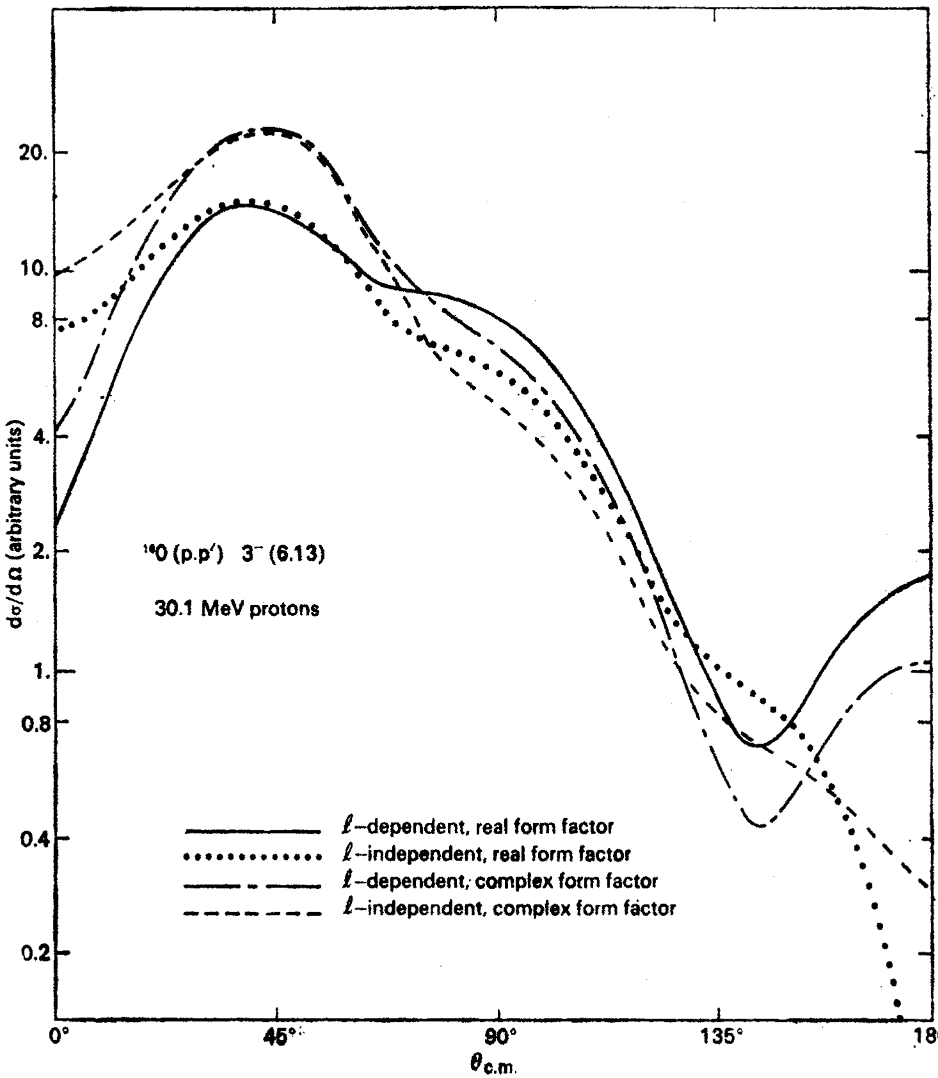

Fig. 7. Inelastic scattering of 30.1 MeV protons to the 6.13 MeV 3^- state of ^{16}O comparing results obtained with l-dependent and l-independent optical potentials. Results are shown for both real and complex excitation form factors

(concerning giant resonance contributions, for example) are made from inelastic scattering. Secondly, it should be possible to use inelastic scattering and other reactions to explore the optical potentials themselves since these are by no means fixed by 'scattering matrix' η_{lj} for elastic scattering. Unfortunately, considerations of the possible use of l-dependent optical potentials brings us to the question of l-dependent inelastic scattering form factors and of inelastic scattering in the presence of strong channel coupling effects. There is little that can be said for sure about these matters. Nevertheless, (p, p′) model calculations could be a useful monitor of the wavefunctions of phase equivalent l-dependent and spline-model l-independent fits. We note that we also attempted to use the l-dependent proton optical potentials for DWBA calculations of ^{16}O (p, d) ^{15}O reaction (data of Snelgrove and Kashy [13]). Unfortunately, the (p, d) cross section appeared to be determined mainly by the deuteron potential and no definite effect of the application of proton l-dependent potential could be established. On the other hand we have no information so far about a possible l-dependence of the deuteron optical potential.

9. *Conclusions*

The major conclusions are as follows:

(1) The l-dependent effects previously described are not confined to light, closed shell-nuclei, but are apparently a universal feature of the proton optical potential. Certain general features occur always, from ^{12}C to ^{208}Pb.

(2) When very good fits to scattering from a sequence of nuclei are obtained with an l-dependent potential, then shell effects reveal themselves as discontinuities in this potential. It is better to look upon such discontinuities as phenomena rather than upon particular features of the angular distribution (e.g. backward peaking) which might be contingencies of quite simple changes of geometry, etc.

(3) For a rigorous extraction of such effects, analysing power data are *essential*. Data which cannot be fitted with any simple (i.e. Woods-Saxon or Woods-Saxon raised to some powers) l-independent potential become all too easy to fit with an l-dependent potential unless the spin-orbit term is fixed by insisting on precise fits to analysing power data.

(4) Attempts to use other parametrizations of l-dependence do not seem to work, with one exception which is a simple modification of that which we have used generally. While we cannot claim that our parametrization is unique, it appears to have some truth. Simply adding parameters does not necessarily improve fits.

(5) The use of l-dependent potentials should be taken seriously when studying (p, p′), at least near ^{16}O.

A. M. Kobos is grateful to Professors S. Micheletti and M. Pignanelli for useful discussions, to Dr. R. J. Griffiths for the hospitality extended to him while at King's College, and to Dr. P. E. Hodgson for permitting the use of computing facilities of Nuclear Physics Laboratory, Oxford. The calculations of Section 8 were performed under the auspicies of the MSU Cyclotron Laboratory and the US NSF to whom R. S. Mackintosh is grateful. The (p, d) calculations mentioned also in Section 8 were done at Institute of Nuclear Physics, Kraków with the help of Dr. J. Albinski.

REFERENCES

[1] R. S. Mackintosh, L. A. Cordero, *Phys. Lett.* **68B**, 213 (1977).
[2] R. S. Mackintosh, A. M. Kobos, *J. Phys. G* **4L**, 135 (1978).
[3] A. M. Kobos, R. S. Mackintosh, *J. Phys. G* **5**, 97 (1979).
[4] R. S. Mackintosh, A. M. Kobos, *J. Phys. G* **5**, 359 (1979).
[5] A. M. Kobos, R. S. Mackintosh, *Ann. Phys. (N.Y.)* **123**, 296 (1979).
[6] R. S. Mackintosh, A. M. Kobos, Proc. Hamburg Workshop on Microscopic Optical Potentials, H. V. Geramb ed., in *Lectures in Phys.* **89**, 188 (1979), Springer Verlag, Berlin.
[7] R. S. Mackintosh, A. M. Kobos, *Phys. Lett.* **62B**, 127 (1976).
[8] R. S. Mackintosh, *Nucl. Phys.* **A230**, 195 (1974).
[9] F. James, M. Roos, *Comput. Phys. Commun.* **10**, 343 (1975).
[10] R. DeLeo, G. D'Erasmo, E. Fabrici, S. Micheletti, A. Pantaleo, M. Pignanelli, F. Resmini, Milan Report INFN/BE-78/8 (1978).
[11] E. Fabrici, S. Micheletti, M. Pignanelli, F. Resmini, R. DeLeo, G. D'Erasmo, A. Pantaleo, J. L. Escudie, A. Tarrats, *Phys. Rev.* **C21**, 830 (1980).
[12] E. Fabrici, S. Micheletti, M. Pignanelli, F. Resmini, R. DeLeo, G. D'Erasmo, A. Pantaleo, *Phys. Rev.* **C21**, 844 (1980).
[13] J. L. Snelgrove, E. Kashy, *Phys. Rev.* **187**, 1246 (1969).
[14] N. M. Clarke, E. J. Burge, D. A. Smith, J. C. Dore, *Nucl. Phys.* **A157**, 145 (1970).
[15] C. B. Fulmer, J. B. Ball, A. Scott, M. L. Whitten, *Phys. Rev.* **181**, 1565 (1969).
[16] P. J. VanHall, R. S. Mackintosh, A. M. Kobos, W. M. L. Moonen, Proc. Inter. Conf. on Polarisation Phenomena, Santa Fe, paper no. 2.16 (1980).
[17] P. J. VanHall et al., Cyclotron Lab., Eindhoven University of Technology Report (1979), unpublished.
[18] B. W. Ridley, J. F. Turner, *Nucl. Phys.* **58**, 497 (1964).
[19] G. W. Greenlees, V. Hnizdo, O. Karban, J. Lowe, W. Makofske, *Phys. Rev.* **C2**, 1063 (1970).
[20] L. N. Blumberg, E. E. Gross, A. Van der Woude, A. Zucker, R. H. Bassel, *Phys. Rev.* **147**, 812 (1966).
[21] R. A. Chatwin, J. S. Eck, D. Robson, A. Richter, *Phys. Rev.* **C1**, 795 (1970).
[22] R. S. Mackintosh, Michigan State Cyclotron Lab. Annual Report, 1978 p. 25.