A LOW-ENERGY DIRECT CHANNEL REGGE-POLE APPROACH TO α-^{12}C ELASTIC SCATTERING PROCESS

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Differential scattering cross-sections for the elastic scattering of α particles by ^{12}C at low bombarding energies have been evaluated in the direct channel Regge-pole formalism, taking into account the contributions from a few nearby dominant excited levels of the compound nucleus ^{16}O and incorporating the background effect. The relevant pole parameters for the scattering process are also evaluated from the "least-squares-fit" with the experimental data. The overall agreement with the experiment is found to be satisfactory.

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1. Introduction

The success of Regge-pole theory in high energy scattering processes has given rise to considerable interest in studying its application to nuclear scattering and reaction processes at low energies. It is to be noted here that of the methodologies advocated so far for nuclear scattering processes the most important ones are the optical model [1], the diffraction model or the sharp cut-off model which has been subsequently developed by Ackhiezer, Pomeranchuk, Blair and McIntyre and it is now known as the APBM model :2], the compound nucleus analysis :3] and also the Regge-pole formalism [4], However, the most remarkable triumph of the pole representation is that one does not need refer to the potential at all.

Though the analytic S-matrix theory in the complex momentum plane (complex K-plane) has achieved remarkable success in high-energy physics, it is yet to have such an appeal to nuclear scattering phenomena due to its certain limitations [5]. On the other hand the representation in the complex angular momentum plane (λ-plane) is more realistic,

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because it is free from most of these limitations. The complex angular momentum approach to particle scattering phenomena in terms of the poles in the λ-plane was first introduced by Regge et al. [6]. In this Regge prescription the Sommerfeld-Watson representation of the scattering amplitude in the form of a contour integral in the λ-plane enclosing the real λ-axis in the right half plane is modified by distorting the contour in such a way as to include the poles (known as Regge poles) of the partial-wave amplitude in the first quardrant. In its simplest form the "Background Integral" parallel to the imaginary axis is neglected with respect to the pole terms. However, this Regge simple pole formula has shortcomings too. As for example, the S-matrix satisfies neither the correct asymptotic behaviour in the λ-plane nor the proper threshold behaviour in K in addition to its incapability of satisfying the elastic unitarity condition. Again, the Regge simple pole formula for low energy resonant scattering leads only to a power law decrease of the non-resonant scattering phases with angular momentum instead of an exponential fall which is characteristic for forces with a finite effective radius. This has been accounted for by an incorrect analytic behaviour of the Regge amplitude relative to the crossed-channel variable "t". So, different modified Regge-pole models have been suggested by different authors [7]. As the background integral represents the contributions from the singularities in the left-half λ-plane, so the main object of the different modified models was of taking some contribution from the background integral and lumping it into the pole terms, as was done in Khuri's representation [7]. Khuri's model though fails to give the correct threshold behaviour for the imaginary part of the partial-wave amplitude in K, yet it gives correct asymptotic behaviour in λ. However, one of the main drawbacks of this model and many other modified models [7] is that they fail to satisfy the elastic unitarity condition explicitly. The pole model which overcomes all these shortcomings is an elegant and very simple model, the "Modified Pole Model" (MPM) of Grushin and Nikitin [8] who have taken into account the effect of the background integral by adding an integral to the simple Regge-pole formula and have successfully analysed the low energy n-^{12}C elastic scattering phenomena within the framework of this model.

The application of the Regge-pole phenomenology to nuclear scattering process was first made by the group of Italian workers [9] who have made a parametrized fit to the experimental scattering data using the simple Regge-pole formula and hence their results may be treated as unrealistic due to a complete neglect of the background effect which do not need to be very small for nuclear scattering processes.

We have, therefore, taken recourse to the Modified Pole Model (MPM) of Grushin and Nikitin [8] for the analysis of low energy resonant scattering of a spinless projectile (α-particle) by a spinless target (^{12}C).

It is worth mentioning here that as the information about the crossed channel processes for these types of nuclear reactions is not available, we Reggeize in the direct channel. The energy of the incoming α-particles considered varied from 11.00 MeV (Lab.) to 16.00 MeV (Lab.) at 4 different randomly chosen energy values.

In the next section we discuss our method of calculation in the framework of MPM starting from the simple Regge formulation for the sake of completeness and in the last section we present the results of our calculations and discussion.

2. Method of calculations

The elastic scattering amplitude for a spinless target and a spinless projectile is given by

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)(S_l - 1) P_l(\cos\theta), \tag{1}$$

where $S_l(E)$ is the scattering matrix.

Applying the Sommerfeld-Watson transform to (1) we have

$$f(\theta) = \frac{1}{2ik} \int_C \frac{(2\lambda+1) f_\lambda(E) P_\lambda(-z)}{\sin \pi\lambda} d\lambda, \tag{2}$$

where $z = \cos\theta$ and the contour C encloses the real λ-axis in the right-half plane.

Now, Reggeizing (2), we get

$$f(\theta) = \frac{1}{2ik} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{(2\lambda+1) f_\lambda(E)}{\sin \pi\lambda} P_\lambda(-z) d\lambda + \frac{\pi}{k} \sum_{j=1}^{N} \frac{(2\lambda_j+1) R_j(E) P_{\lambda_j}(-z)}{\sin \pi\lambda_j}. \tag{3}$$

In the simple Regge-pole formulation the background integral is neglected with respect to the "pole term" and hence the scattering amplitude reduces to the following form:

$$f(\theta) = \frac{\pi}{k} \sum_{j=1}^{N} \frac{(2\lambda_j+1) R_j(E) P_{\lambda_j}(-z)}{\sin \pi\lambda_j}. \tag{4}$$

Here λ_j is the j-th Regge-pole given by $\lambda_j(E) = \alpha_j(E) + i\beta_j(E)$; $f_\lambda(E)$ is the analytic continuation of the partial wave amplitude with the integral value l to complex λ and $R_j(E)$ is the residue of the partial wave amplitude at λ_j and is complex in general, and N represents the number of poles considered at a given energy, and $P_\lambda(z)$ is a Legendre polynomial of complex order λ. The equation can also be written down as

$$f(\theta) = \frac{1}{k} \sum_{j=1}^{N} (2\lambda_j+1) R_j(E) P(\lambda_j, z), \tag{5}$$

where

$$P(\lambda, z) = \frac{\pi}{\sin \pi\lambda} P_\lambda(-z). \tag{6}$$

The scattering amplitude given by Eqs. (5) and (6) has the shortcomings referred to above.

In the Modified Pole Model (MPM) a different structure is taken for the function $P(\lambda, z)$. In this Model [8] it has been assumed that in the complex z-plane, the function $P(\lambda, z)$ has a square-root branch-point at $z = z_0 = \cosh \xi$, where $z_0 = 1 + t_0/2k^2$, t_0 being the "branch-point" in the crossed t-plane. ξ may also be defined as $\xi = \cosh^{-1}\left(1 + \frac{\mu_0^2}{2k^2}\right)$

where μ_0 is the mass of the lowest-mass exchanged system. This assumption leads to accounting for the potential scattering with the amplitude depending on ξ, the phenomenological choice of which denotes the introduction of an effective short-range potential which has an effective radius defined as $R \sim [2k^2(\cosh \xi - 1)]^{1/2}$. The modified form of the function $P(\lambda, z)$ is chosen as

$$P(\lambda, z) = \left[\frac{\pi}{\sin \pi\lambda} P_\lambda(-z) + \int_0^{\exp(\xi)} \frac{h^\lambda dh}{(h^2 - 2hz + 1)^{\frac{1}{2}}}\right] \quad \text{for Re}\,\lambda > -1. \tag{7}$$

The scattering amplitude given by (5) with $P(\lambda, z)$ as shown in (7) is the required amplitude free from all the shortcomings referred to above.

Now, if the leading poles in the complex λ-plane for the process considered are situated near some integral values (l) of the angular momentum close to the real λ-axis (as in the case of resonant scattering), then the imaginary part of the pole is very small and in that case the modified amplitude is reduced to the form:

$$f(\theta) = \frac{1}{k} \sum_{j=1}^{N} (2\lambda_j + 1) R_j(E) \left(\frac{P_{l_j}(z)}{i\beta_j} + T_{l_j}(z)\right) \tag{8}$$

where

$$T_0(z) = \ln\left(\frac{c + \exp(\xi) - z}{2}\right); \qquad T_1(z) = c + z(1 + T_0),$$

$$lT_l(z) = (2l-1)zT_{l-1}(z) - (l-1)T_{l-2}(z) + \frac{1}{(2l-1)}\{P_l(z) - P_{l-2}(z)\} + c\exp(\xi(l-1)), \tag{9}$$

where $c = [\exp(2\xi) - 2z\exp(\xi) + 1]^{\frac{1}{2}}$. Then the differential scattering cross-section is given by

$$\sigma(\theta) = |f(\theta|^2. \tag{10}$$

The centre of mass differential scattering cross-sections at incident laboratory energies of 11.00, 13.00, 14.00 and 16.00 MeV of α-particles have been computed using equation (10) along with the equations (8) and (9) over the whole of the angular region by means of a least-squares fit to the experimental data [10]. At each energy value, the contributions from 2 or 3 dominant resonances corresponding to the excited states [11] of ^{16}O have been considered.

3. *Results and discussion*

The angular distributions at laboratory energies of 11.00, 13.00, 14.00 and 16.00 MeV calculated from the modified pole representation are shown together with the experimental ones in Fig. 1 to Fig. 4. The theoretical angular distributions, like the experimental ones,

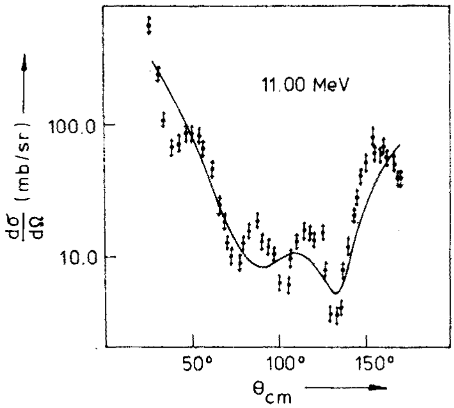

Fig. 1

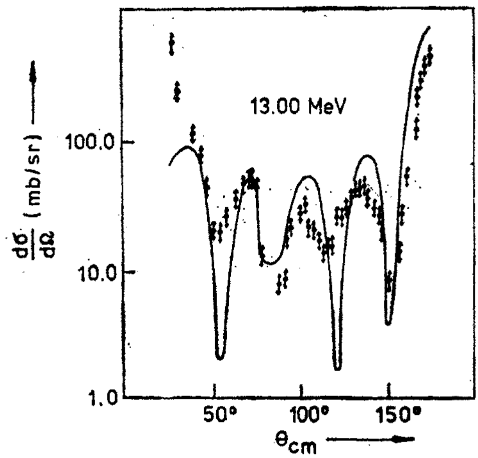

Fig. 2

Fig. 1. Angular distributions (c.m.) at a laboratory energy of 11.00 MeV. Points indicate experimental values while the line represents the theoretical results

Fig. 2. Same as for figure 1 at 13.00 MeV (Lab.)

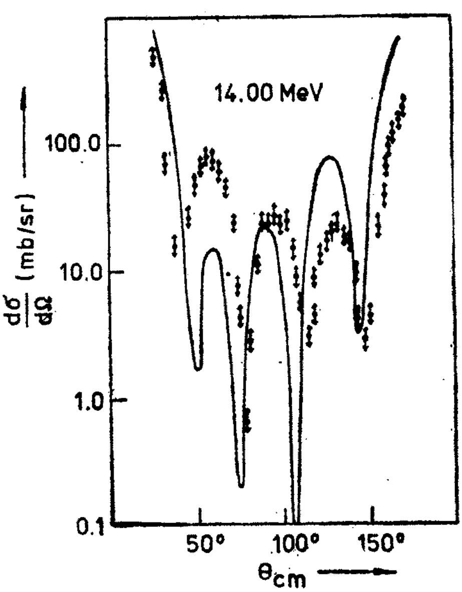

Fig. 3

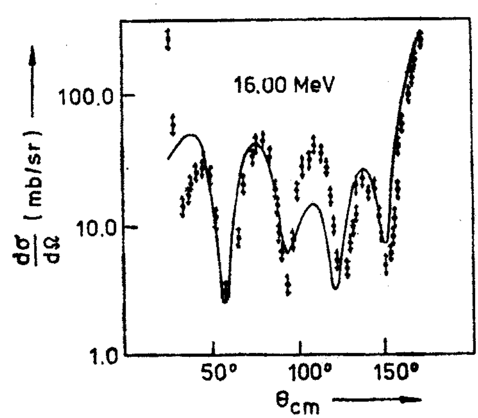

Fig. 4

Fig. 3. Same as for figure 1 at 14.00 MeV (Lab.)

Fig. 4. Same as for figure 1 at 16.00 MeV (Lab.)

exhibit a distinct diffraction pattern. At all energies the theoretical angular distributions reproduce all the experimental maxima and minima except at 11.00 MeV. At this energy though fewer maxima and minima are obtained, nevertheless, overall agreement is quite satisfactory. At 14.00 MeV the third theoretical minimum is too deep while, except first minimum and maximum other theoretical maxima and minima are slightly displaced towards lower angles.

However, over the entire energy region considered, it has been found that 2 or 3 nearby leading poles are sufficient to account for the scattering phenomena at least qualitatively. The imaginary parts (β) of the poles and the corresponding residues (both real and imaginary parts) evaluated by a least-squares fit to differential cross-sections data are tabulated together with their real parts and excitation energies (E_x) in Table I.

TABLE I

Pole parameters and the residues at different incident energies

E (lab.) (MeV)	J^π (J = real part of the pole π = parity)	E_x (MeV) (Excitation) energy)	β	Re R	Im R
11.00	1^-	15.42	0.0199	0.4777	0.5659
	3^-	15.42	0.2270	2.5654	0.7750
	5^-	16.9	0.0016	1.3875	2.8408
13.00	2^+	16.94	0.0049	−1.7744	−3.1657
	4^+	16.8	0.0395	−44.4480	−13.0420
	0^+	17.7	0.0251	−1.4959	−0.0347
14.00	2^+	17.7	0.0018	0.0208	0.0059
	4^+	17.81	0.1562	2.3132	0.2214
	2^+	19.12	0.0037	0.4734	0.3099
16.00	4^+	19.12	0.0450	−0.4124	7.6517
	5^-	19.25	0.0021	−0.6075	0.0264

At 11.00 MeV the tentatively assigned levels (1^-) and (3^-) with an excitation energy of 15.42 MeV have been confirmed by the present theoretical work. Again at 16.00 MeV the existence of three resonance states 2^+ (E_x = 19.12 MeV), 4^+ (E_x = 19.12 MeV) and 5^- (E_x = 19.25 MeV) of the compound nucleus ^{16}O which were tentatively assigned by experiment [11] are supported by this work. We conclude that if the effect of the background is taken into account then two or three nearby dominant Regge-poles may hopefully reproduce the essential features of the low energy nuclear scattering of a spinless projectile by a spinless target.

All the required computational work has been performed on a Burroughs-6700 computer at the Regional Computer Centre, Calcutta.

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