CLUSTER PROPERTIES OF THE GIANT MONOPOLE RESONANCES E0 IN THE LIGHT AND MEDIUM MASS NUCLEI

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(*Received February 10, 1981*)

The connection between the cluster spectroscopic factors of the giant monopole isoscalar resonances, E0, and the ground states of nuclei has been established by means of the formalism of the Sp (2, **R**) group. These relations allow one to analyze the dependence of the cluster properties of the isoscalar E0 giant resonances on the mass number A.

PACS numbers: 21.60.-n, 21.60.Fw

In the last years, much attention was paid to the experimental study of the hadron--nucleus inelastic scattering at the energy ~ 1 GeV. It was found that the atomic nuclei giant resonances of the different multipolarity E0, E1, E2 and E3 of both types (isoscalar and isovector in dependence on the bombarding particle type p, α, d, π etc.) are excited intensively in such processes. The important result of these experiments is that the inelastic scattering is accompanied by intensive emission of α-particles with moderate energies [1]. It has been demonstrated experimentally [2], that the emission of these α-particles is caused by decay of the multipole giant resonances and partly by decay of the excited daughter nuclei after output of the first α-particle. The intensive formation of the α-particles in the processes of the inelastic scattering was observed in many laboratories and for large number of nuclei. Therefore, this property may be considered today as a general nuclear property. This fact stimulated interest for the theoretical investigation of the cluster properties of the giant resonances.

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Such an investigation was begun by Hecht [3] where the α-particle spectroscopic factors, S_α, for the giant isoscalar resonances, E0 and E2, had been calculated for a number of the light nuclei. The overlap integrals between the cluster states (CS) and the states of these giant resonances (GR) also had been calculated. Values of the overlap integrals, $\langle CS|GR\rangle$, and values, S_α, were found to be rather high. This explains qualitatively the considerable contribution of the α-particle channel into the decay of the giant resonances.

Examples of specific nuclei with $A \leqslant 28$, which were considered in paper [3] showed that the values of S_α and $\langle CS|GR\rangle$ decrease on the whole with an increase in the nuclear mass number A. The cluster spectroscopic factors of the giant resonances E0 and E2 also have been obtained in [4, 5] for the ^{40}Ca nucleus.

However, it is interesting to estimate theoretically the cluster properties of the medium mass nuclei with $A \sim 50$–80, because for these nuclei sufficiently strong cluster effects were observed in the last few years. It is impossible to determine these properties using papers [3–5], in which calculations have been made only up to $A = 40$, because these results are exclusively of a numerical nature and do not contain analytical expressions.

Therefore, in this paper we shall obtain the analytical expressions, $\frac{S_\alpha(A^*)}{S_\alpha(A)}$, for the ratios of the α-particle spectroscopic factors for the gaint isoscalar resonances E0 and for the states of the lowest shell model configuration of nuclei as the function of the mass number, A. These expressions will allow us to estimate the order of the values $S_\alpha(A^*)$ for the medium mass nuclei and also to trace the evolution of the cluster properties of the gaint resonances E0 with increasing A.

It is convenient to use the Sp(2, R) group [6–9] in the consideration of the cluster properties of the giant resonances E0.

Let us introduce dimensionless Jacobi coordinates $x_{j\xi}$ ($j = 1, 2, \ldots, A-1$; $\xi = 1, 2, 3$) for the nucleus A. In these coordinates hyperradius, ϱ, is written as follows: $\varrho^2 = \sum_{j=1}^{A-1} \sum_{\xi=1}^{3} x_{j\xi}^2 r_0^2$, $r_0^2 = \frac{\hbar}{m\omega}$, where m is the nucleon mass. ϱ^2 is the operator of the isoscalar monopole excitation of nucleus.

Let us introduce the creation and the annihilation operators of an oscillator quanta $\vec{a}_j^\dagger = \frac{1}{\sqrt{2}}(\vec{x}_j - i\vec{p}_j)$, $\vec{a}_j = \frac{1}{\sqrt{2}}(\vec{x}_j + i\vec{p}_j)$ for all degrees of freedom $\vec{x}_j$. For these operators the usual boson commutation relations are valid.

Operators

$$J_+ = \tfrac{1}{2}\sum_j (\vec{a}_j^\dagger \cdot \vec{a}_j^\dagger), \quad J_- = \tfrac{1}{2}\sum_j (\vec{a}_j \cdot \vec{a}_j),$$
$$J_0 = \tfrac{1}{4}\sum_j [(\vec{a}_j^\dagger \cdot \vec{a}_j) + (\vec{a}_j \cdot \vec{a}_j^\dagger)] = \tfrac{1}{2}E, \tag{1}$$

which satisfy the relations:

$$[J_0, J_\pm] = \pm J_\pm, \quad [J_+, J_-] = -2J_0,$$
$$(J_\pm)^\dagger = J_\mp, \quad J_0^+ = J_0 \tag{2}$$

are the generators of the canonical transformation group, Sp(2, R), which is the dynamics group of a $(3A-3)$ -dimensional harmonic oscillator with the frequency ω.

The totally antisymmetrical wave functions, $|ANKL\beta\rangle$, of the translationally invariant shell model (TISM) [10, 11] form the basis in the space of the infinite unitary irreducible representation of the Sp(2, R) group. This representation belongs to the positive discrete series, D^J, with $J = \frac{1}{2}\left(K+\frac{3A-3}{2}\right)-1$ (for details see [7, 8, 12]). Here the wave functions of the TISM were constructed according to the so-called orthogonal scheme as in our previous paper [13]. N is the total number of the oscillator quanta, i.e., the total oscillator energy $E = N+\frac{3A-3}{2}$ in the $\hbar\omega$ units. The global momentum, K, appearing in the K-harmonics method [14], characterizes the irreducible representation, D^K, of the $O(3A-3)$ group, to which the wave function $|ANKL\beta\rangle$ belongs. L is the usual orbital angular momentum. Index β denotes all other quantum numbers, distinguishing the nuclear states in the TISM. β includes Young's scheme $[f]$ for the spatial part of the wave function, spin S, isospin T, total angular momentum I and other needed quantum numbers.

The generators (2) fulfil the relation:

$$[J_-, J_+^{\kappa}] = \kappa J_+^{\kappa-1}(E+\kappa-1) \tag{3}$$

from which it follows that the normalized function of the κ-multipole monopole excitation $|ANKL\beta\rangle$ $(N = 2\kappa+K)$ may be written as follows

$$|ANKL\beta\rangle = \left[\frac{(E_0-1)!}{\kappa!(E_0+\kappa-1)!}\right]^{1/2} J_+^{\kappa}|AKKL\beta\rangle. \tag{4}$$

Here $|AKKL\beta\rangle$ is the state with the minimal number of quanta $N_{\min} = K$, which are permitted for a fixed K, and $E_0 = K+\frac{3A-3}{2}$ is oscillator energy of this state. It should be noted that the following relations are fulfilled

$$J_+^{\nu}|ANKL\beta\rangle = \left[\frac{(\kappa+\nu)!(E_0+\kappa+\nu-1)!}{\kappa!(E_0+\kappa-1)!}\right]^{1/2} |AN+2\nu KL\beta\rangle,$$

$$J_-|AKKL\beta\rangle = 0. \tag{5}$$

We are interested in the spectroscopic amplitude, $S_{l,\mathscr{L}}^{1/2}$, of the decay of the nucleus A into the fragments A_1 and A_2. The wave functions of the TISM will be used with the same oscillator parameters, $\hbar\omega$, for the parent nucleus and the final fragments. Then the spectroscopic amplitude, $S_{l,\mathscr{L}}^{1/2}$, may be expressed in terms of the fractional parentage coefficients of the TISM as follows

$$S_{l,\mathscr{L}}^{1/2} = \sqrt{\frac{A!}{A_1!A_2!}}\langle ANKL\beta|A_1N_1K_1L_1\beta_1; nl, A_2N_2K_2L_2\beta_2\{\mathscr{L}\}\rangle, \tag{6}$$

where $n = N-N_1-N_2$ and l are the number of quanta and the orbital momentum of the relative motion of the fragments A_1 and A_2, respectively.

In a general case (i.e., for different frequencies, $\hbar\omega_i$) the wave function of the nucleus, A_i, will be the superposition of the states, $|A_iN_iK_iL_i\beta_i\rangle$, and the spectroscopic amplitude to be found will be a linear combination of the simple amplitudes (6). Therefore, the present analysis of the properties of the amplitude (6), will allow us to consider also more realistic situations.

Suppose we know the fractional parentage decomposition for the lowest vector, $|AKKL\beta\rangle$, which corresponds to the ground state shell model configuration of the nucleus A:

$$|AKKL\beta\rangle = \sum_{\substack{N_1K_1L_1\beta_1 nl\\ N_2K_2L_2\beta_2\mathscr{L}}} \langle AKKL\beta|A_1N_1K_1L_1\beta_1;\, nl,\, A_2N_2K_2L_2\beta_2\{\mathscr{L}\}\rangle$$
$$\times|A_1N_1K_1L_1\beta_1;\, nl,\, A_2N_2K_2L_2\beta_2\{\mathscr{L}\}\rangle.$$

Let us apply to both sides of this expression the operator, J_+, which is characterized by property

$$J_+(A) = J_+(A_1)+J_+(A_2)+J_+(\vec{\varrho}),$$

where $J_+(\vec{\varrho})$ is the operator of the monopole excitation in relative motion of the fragments A_1 and A_2. By using relation (4) we obtain the following expression for the general fractional parentage coefficient, which is included in the spectroscopic amplitude (6)

$$\langle ANKL\beta|A_1N_1K_1L_1\beta_1;\, nl,\, A_2N_2K_2L_2\beta_2(\mathscr{L})\rangle$$
$$= \sum_{\nu_1\nu_2\nu_3} \frac{\kappa!}{\nu_1!\nu_2!\nu_3!}\left[\frac{\kappa_1!\kappa_2!\kappa_3!(E_0-1)!}{\kappa!(E_0+\kappa-1)!(\kappa_1-\nu_1)!(\kappa_2-\nu_2)!}\right.$$
$$\times\left.\frac{(E_{1,0}+\kappa_1-1)!(E_{2,0}+\kappa_2-1)!(\varepsilon_0+\kappa_3-1)!}{(\kappa_3-\nu_3)!(E_{1,0}+\kappa_1-\nu_1-1)!(E_{2,0}+\kappa_2-\nu_2-1)!(\varepsilon_0+\kappa_3-\nu_3-1)!}\right]^{1/2}$$
$$\times\langle AKKL\beta|A_1N_1-2\nu_1K_1L_1\beta_1;\, n-2\nu_3l,\, A_2N_2-2\nu_2K_2L_2\beta_2(\mathscr{L})\rangle. \quad (7)$$

Here,

$$\kappa_1 = \tfrac{1}{2}(N_1-K_1),\quad \kappa_2 = \tfrac{1}{2}(N_2-K_2),\quad \kappa_3 = \tfrac{1}{2}(n-l),\quad \nu_1+\nu_2+\nu_3 = \kappa,$$
$$E_{i,0} = K_i+\frac{3A_i-3}{2},\quad \varepsilon_0 = l+\tfrac{3}{2}.$$

If both fragments are formed in the lowest states allowed by the Pauli exclusion principle, then $N_1 = K_1$ and $N_2 = K_2$. For this important case from expression (7) we find the connection between spectroscopic factors for the ground and monopole excited states of the nucleus A:

$$S_{l,\mathscr{L}}(ANKL\beta\to A_1K_1K_1L_1\beta_1+A_2K_2K_2L_2\beta_2)$$
$$= \frac{(E_0-1)!(\varepsilon_0+\kappa_3-1)!\kappa_3!}{\kappa!(E_0+\kappa-1)!(\kappa_3-\kappa)!(\varepsilon_0+\kappa_3-\kappa-1)!}S_{l,\mathscr{L}}(AKKL\beta\to A_1K_1K_1L_1\beta_1+A_2K_2K_2L_2\beta_2).$$
$$(8)$$

Particularly for the α-particle spectroscopic factor of the isoscalar giant resonance E0 of the nucleus $A(\kappa = 1, A_2 = 4)$ we have:

$$S_{\alpha,l}(AK+2KL\beta \rightarrow A_1K_1K_1L_1\beta_1)$$
$$= \frac{(K-K_1-l+2)(K-K_1+l+3)}{2(2K+3A-3)} S_{\alpha,l}(AKKL\beta \rightarrow A_1K_1K_1L_1\beta_1). \quad (9)$$

If A changes in the limit of one shell, then the number of quanta $n = K-K_1$, corresponding to the motion of the α-cluster, remains constant, but the denominator increases monotonically in the right side of expression (9). Therefore the ratio $\frac{S_{\alpha,l}(A^*)}{S_{\alpha,l}(A)}$ of the monopole isoscalar giant resonances E0 and the ground states of nuclei decreases on the whole with increasing A. As can be seen from Table I some irregularities are observed in the transition region between neighbouring shells. Specific examples, which have been considered in paper [3], also lead to this conclusion.

TABLE I

Ratio of the α-particle spectroscopic factors for the giant resonance EO and for the ground states of the nuclei $\frac{S_{\alpha,l}(AK+2KL = 0\beta \rightarrow A_1K_1K_1L_1 = l\beta_1)}{S_{\alpha,l}(AKKL = 0\beta \rightarrow A_1K_1K_1L_1 = l\beta_1)}$ for $l = 0, 2, 4$

l	^{8}Be	^{12}C	^{16}O	^{20}Ne	^{24}Mg	^{28}Si	^{32}S	^{36}Ar	^{40}Ca
0	0,724	0,429	0,304	0,567	0,440	0,359	0,304	0,263	0,232
2	0,621	0,367	0,261	0,536	0,416	0,340	0,287	0,249	0,219
4	0,379	0,224	0,159	0,464	0,360	0,294	0,249	0,215	0,190

It is also seen from Table I that the ratio $\frac{S_{\alpha,l}(AK+2KL = 0\beta \rightarrow A_1K_1K_1L_1 = l\beta_1}{S_{\alpha,l}(AKKL = 0\beta \rightarrow A_1K_1K_1L_1 = l\beta_1}$ decreases with an increase in the orbital momentum l of the relative motion of the α-particle and the daughter nucleus A_1.

These results allow us to estimate the values of the spectroscopic factors $S_{\alpha,l}(A^*)$ for the giant resonances E0 because the spectroscopic factors for the ground states $S_{\alpha,l}(A)$ may be calculated by using the usual shell model methods [10, 11].

Let consider now the α-particle spectroscopic factors for the transition from the state of the giant resonance E0 of the nucleus A to the monopole excited state of the daughter nucleus A-4. From expression (7) it follows that

$$\langle AK+2KL\beta | A_1K_1+2K_1L_1\beta_1;\, nl,\, A_2K_2K_2L_2\beta_2(\mathscr{L})\rangle$$
$$= \sqrt{\frac{2K_1+3A_1-3}{2K+3A-3}} \langle AKKL\beta | A_1K_1K_1L_1\beta_1;\, nl,\, A_2K_2K_2L_2\beta_2(\mathscr{L})\rangle$$
$$+ \sqrt{\frac{(n-l)(n+l+1)}{2(2K+3A-3)}} \langle AKKL\beta | A_1K_1+2K_1L_1\beta_1;\, n-2l,\, A_2K_2K_2L_2\beta_2(\mathscr{L})\rangle. \quad (10)$$

As can be seen from this equation, the α-particle spectroscopic factors of the lowest state of the nucleus A for the transition to the monopole excited state of the nucleus, A–4, would be known in order to obtain the value $S_{\alpha,l}(AK+2KL\beta \rightarrow A_1K_1+2K_1L_1\beta_1)$. The technique of the calculation of these complicated spectroscopic factors is described in [11, 15].

But for the particular case $l = n$, only one component remains on the right side of formula (10) and we obtain the relation between $S_{\alpha,l=n}(AK+2KL\beta \rightarrow A_1K_1+2K_1L_1'\beta_1)$ and $S_{\alpha,l=n}(AKKL\beta \rightarrow A_1K_1K_1L_1\beta_1)$

$$\frac{S_{\alpha,l=n}(AK+2KL\beta \rightarrow A_1K_1+2K_1L_1\beta_1)}{S_{\alpha,l=n}(AKKL\beta \rightarrow A_1K_1K_1L_1\beta_1)} = \frac{2K_1+3A_1-3}{2K+3A-3}. \tag{11}$$

In particular for the ^{16}O nucleus we calculate:

$$\frac{S_{\alpha,l=4}(^{16}\mathrm{O}^*0^+ \rightarrow {}^{12}\mathrm{C}^*4^+)}{S_{\alpha,l=4}(^{16}\mathrm{O}0^+ \rightarrow {}^{12}\mathrm{C}4^+)} = \frac{49}{69}.$$

and

$$\frac{S_{\alpha,l=4}(^{16}\mathrm{O}^*0^+ \rightarrow {}^{12}\mathrm{C}4^+)}{S_{\alpha,l=4}(^{16}\mathrm{O}0^+ \rightarrow {}^{12}\mathrm{C}4^+)} = \frac{11}{69}.$$

Here symbols A, J^π and A^*J^π denote the lowest state of the nucleus A with the spin J and the monopole excited state, respectively.

In the general case for $l = n$ we have:

$$\frac{S_{\alpha,l=n}(AK+2KL\beta \rightarrow A_1K_1+2K_1L_1\beta_1)}{S_{\alpha,l=n}(AK+2KL\beta \rightarrow A_1K_1K_1L_1\beta_1)} = \frac{2K_1+3A_1-3}{2K-2K_1+3}, \tag{12}$$

i.e. the spectroscopic factors for the transition from the giant resonance E0 for nucleus A $(A > 8)$ to the monopole excited state of the nucleus A–4 are bigger than for the transition to the state of the lowest configuration for this nucleus.

Especially for the ^{16}O nucleus we obtain the result that

$$\frac{S_{\alpha,l=4}(^{16}\mathrm{O}^*0^+ \rightarrow {}^{12}\mathrm{C}^*4^+)}{S_{\alpha,l=4}(^{16}\mathrm{O}^*0^+ \rightarrow {}^{12}\mathrm{C}4^+)} = \frac{49}{11}.$$

The expressions obtained above also allow us to estimate the values of the overlap integrals between the wave functions of the giant resonances E0 and the cluster states [5].

The authors are grateful to Yu. I. Kharitonov, B. Kamys and L. A. Sliv for helpful discussions.

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