ASYMMETRIC NUCLEAR MATTER AND SKYRME FORCES*

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The properties of asymmetric nuclear matter are studied using several existing parametrizations of the Skyrme interaction. The dependence of the Fermi liquid parameters and of the compression modulus on the neutron excess is investigated. Apart from the standard case of nuclear matter at zero pressure, the case of highly asymmetric nuclear matter under high pressure, relevant to astrophysical applications, is also studied.

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1. Introduction

The knowledge of the properties of asymmetric nuclear matter is important for many reasons. Heavy nuclei have an appreciable neutron excess, characterized by the parameter $\alpha = (N-Z)/A$ which reaches the value $\alpha = 0.23$ for ^{238}U. In the case of hypothetical superheavy nuclei the neutron excess is expected to be even larger. Moreover, the knowledge of the properties of a cold, highly asymmetric nuclear matter with a very large neutron excess ($\varrho_n \gg \varrho_p$) is relevant to physics of neutron stars. Also, the calculation of the equation of state of hot, dense matter, relevant to physics of gravitational collapse, requires the knowledge of the properties of asymmetric nuclear matter.

In the last decade many properties of nuclei have been reproduced in self-consistent microscopic calculations using phenomenological effective nucleon-nucleon interaction [1–3]. The effective nucleon-nucleon interaction, the parameters of which have been determined by fitting a large number of properties of nuclei, can then be used to describe the properties of infinite nuclear matter in the Hartree-Fock approximation.

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The Skyrme type forces (or the Skyrme type energy density functional) lead to particularly simple calculations for nuclear systems in the unusual conditions corresponding to astrophysical applications. Skyrme forces have been used for example in the calculation of the equation of state of neutron star matter at the density below the standard nuclear matter density $\varrho_{NM} = 0.17\,\mathrm{fm}^{-3}$. One is dealing there with unusual nuclei with a very large neutron excess, immersed in a neutron-electron fluid [4]. Skyrme forces have also been used to calculate the properties of hot, dense matter encountered in the gravitational collapse of massive stars. One considers there a high temperature system consisting of hot nuclei, often with a large neutron excess (e.g., $\alpha \cong 0.5$), immersed in a hot fluid composed of neutrons, protons, alpha particles and electrons [5, 6].

In the present paper we study the properties of asymmetric nuclear matter using several available parametrizations of the Skyrme forces. In Section 2 we present the formulae for the Fermi liquid parameters, describing the quasiparticle interaction in asymmetric nuclear matter. The quantities related to compressibility of nuclear matter are studied in Section 3. Numerical results obtained using various parametrizations of the Skyrme forces are presented in Section 4. We study separately the standard case of asymmetric nuclear matter at the equilibrium density and that of a strongly asymmetric nuclear matter under a high external pressure, peculiar to the astrophysical applications. We discuss the effect of neutron excess on the compression modulus of nuclear matter. In Section 5 we compare the Skyrme force results with estimates obtained using existing Fermi liquid parameters for nuclear and neutron matter, calculated starting from the Reid soft-core nucleon-nucleon interaction. Section 5 contains also the main conclusions of the present paper.

2. *Fermi liquid parameters*

Let us consider nuclear matter of the density $\varrho = \varrho_n + \varrho_p$, the neutron excess being described by the neutron excess parameter $\alpha = (\varrho_n - \varrho_p)/\varrho$. The average Fermi momentum, k_F, is related to ϱ by $\varrho = 2k_F^3/3\pi^2$. The Fermi momenta for neutrons and protons may be expressed in terms of the average Fermi momentum k_F and the neutron excess parameter α as

$$k_n = k_F(1+\alpha)^{1/3},$$
$$k_p = k_F(1-\alpha)^{1/3}. \tag{1}$$

We consider spin-unpolarized asymmetric nuclear matter. Using standard representation of the Skyrme force we obtain the following formulae for the direct and exchange matrix elements in the plane wave basis (neglecting the spin-orbit term which does not play any role in the present discussion):

$$(\boldsymbol{k}\sigma\tau\boldsymbol{k}'\sigma'\tau'|V|\boldsymbol{k}\sigma\tau\boldsymbol{k}'\sigma'\tau') = t_0(1+x_0\delta_{\sigma\sigma'})+(t_1+t_2)\kappa^2+\tfrac{1}{6}t_3\varrho^d(1+x_3\delta_{\sigma\sigma'}),$$
$$(\boldsymbol{k}\sigma\tau\boldsymbol{k}'\sigma'\tau'|V|\boldsymbol{k}'\sigma'\tau'\boldsymbol{k}\sigma\tau) = [t_0+(t_1-t_2)\kappa^2+\tfrac{1}{6}t_3\varrho^d]\delta_{\sigma\sigma'}\delta_{\tau\tau'}+(t_0x_0+\tfrac{1}{6}t_3x_3\varrho^d)\delta_{\tau\tau'}. \tag{2}$$

We have set $\boldsymbol{\kappa} = \frac{1}{2}(\boldsymbol{k}-\boldsymbol{k}')$. All momenta are measured in units of $\hbar$. Using Eq. (2) we derive, in the Hartree-Fock approximation, the energy density E of the system as a functional of the distribution functions $n_{k\sigma\tau}$, $E = E[n_{k\sigma\tau}]$ (here, τ = n, p and $\sigma = \uparrow, \downarrow$). This enables us to write down the formulae for the quasiparticle energies

$$e_{k\sigma\tau} = \frac{\delta E}{\delta n_{k\sigma\tau}} \tag{3}$$

and for the quasiparticle interaction of the Landau theory of normal Fermi liquids [9–12]

$$f_{k\sigma\tau, k'\sigma'\tau'} = \frac{\delta^2 E}{\delta n_{k\sigma\tau} \delta n_{k'\sigma'\tau'}} . \tag{4}$$

The functional derivatives in Eqs. (3–4) are evaluated for the distribution function corresponding to the ground state, $n^0_{k\sigma\tau} = \theta(k_\tau - |\boldsymbol{k}|)$. The quasiparticle momenta in Eq. (4) should be taken on the corresponding Fermi surfaces, so that the quantity f depends only on the cosine of the angle between the quasiparticle momenta, $\cos\theta = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}}'$. Using Eq. (3) we calculate the quasiparticle effective mass on the Fermi surface.

$$m^*_\tau = \hbar^2 k_\tau \left(\frac{\partial e_{k\sigma\tau}}{\partial k} \right)^{-1}_{k = k_\tau} . \tag{5}$$

The explicit formula reads:

$$m/m^*_\tau = 1 + \frac{m}{4\hbar^2} [2(t_1 + t_2)\varrho + (t_2 - t_1)\varrho_\tau] \tag{6}$$

Note that since we are considering spin saturated nuclear matter, the quasiparticle energy and consequently the effective mass do not depend upon the spin variable σ. The spin averaged quasiparticle interaction may be described by a set of dimensionless Fermi liquid parameters $\mathscr{F}^{\tau\tau'}_l$ (we follow closely the notation of Ref. [13])

$$f^{\tau\tau'} \equiv \tfrac{1}{4} \sum_{\sigma\sigma'} f_{k\sigma\tau, k'\sigma'\tau'} = N_0^{-1} \sum_l \mathscr{F}^{\tau\tau'}_l P_l(\cos\theta), \tag{7}$$

where N_0 is the density of quasiparticle states calculated in symmetric ($\alpha = 0$) nuclear matter of the same density ϱ, $N_0 = 2k_F m^*/(\pi^2\hbar^2)$. For the Skyrme type effective forces all $\mathscr{F}^{\tau\tau'}_l$ with $l > 1$ vanish. The nonzero Fermi liquid parameters describing the spin independent part of the quasi-particle interaction are given by the formulae:

$$N_0^{-1} \mathscr{F}^{\tau\tau}_0 = \tfrac{1}{2}(1 - x_0)t_0 + \tfrac{1}{4}(3t_2 + t_1)k^2_\tau + \tfrac{1}{12} t_3 \varrho^d [1 - x_3 + \tfrac{1}{2} d(7 + 5x_3) + \tfrac{1}{2} d^2(1 - x_3)]$$
$$+ \tfrac{1}{12} t_3 d \varrho^{d-1}(1 + 2x_3)(d - 3)\varrho_\tau - \tfrac{1}{6} t_3 d(d-1)\varrho^{d-2}\varrho^2_\tau(\tfrac{1}{2} + x_3), \tag{8a}$$

$$N_0^{-1} \mathscr{F}^{\tau\tau}_1 = -\tfrac{1}{4}(3t_2 + t_1)k^2_\tau, \tag{8b}$$

$$N_0^{-1} \mathscr{F}^{np}_0 = \tfrac{1}{2}(2 + x_0)t_0 + \tfrac{1}{4}(k^2_n + k^2_p)(t_2 + t_1)$$
$$+ \tfrac{1}{12} t_3 \varrho^d [2 + x_3 + \tfrac{1}{2} d(5 + x_3) + \tfrac{1}{2} d^2(1 - x_3)] + \tfrac{1}{12} t_3 d(d-1)\varrho^{d-2}\varrho_n\varrho_p(1 + 2x_3), \tag{8c}$$

$$N_0^{-1} \mathscr{F}^{np}_1 = -\tfrac{1}{2} k_n k_p (t_1 + t_2). \tag{8d}$$

The Fermi liquid parameters satisfy the relations

$$\mathcal{F}_l^{pp}(\varrho, \alpha) = \mathcal{F}_l^{nn}(\varrho, -\alpha),$$

$$\mathcal{F}_l^{np}(\varrho, \alpha) = \mathcal{F}_l^{np}(\varrho, -\alpha). \tag{8'}$$

These symmetry properties stem from the charge independence of the nuclear hamiltonian. In the limiting case of symmetric nuclear matter we obtain, using standard notation [12],

$$\mathcal{F}_l^{nn}(\varrho, 0) = \mathcal{F}_l^{pp}(\varrho, 0) = F_l + F_l', \tag{9a}$$

$$\mathcal{F}_l^{np}(\varrho, 0) = F_l - F_l'. \tag{9b}$$

3. Compressibilities

The compression modulus of nuclear matter is defined as

$$K = 9\,\frac{\partial P}{\partial \varrho}, \tag{10}$$

where P is the pressure in the system. At $T = 0$ K the pressure P is related to energy per particle $\mathscr{E}_\alpha(\varrho) = E/\varrho$ by

$$P = \varrho^2\,\frac{\partial \mathscr{E}_\alpha}{\partial \varrho} \tag{11}$$

and hence in general

$$K = 18\,\frac{P}{\varrho} + 9\varrho^2\,\frac{\partial^2 \mathscr{E}_\alpha}{\partial \varrho^2}. \tag{12}$$

K is a function of ϱ and α which we denote hereafter by $K_\alpha(\varrho)$. At the value of ϱ which corresponds to the minimum of $\mathscr{E}_\alpha(\varrho)$ (at fixed α, this value of ϱ will be hereafter referred to as $\varrho_{0\alpha}$, with $\varrho_{00} \equiv \varrho_0$) one obtains a standard formula

$$K_\alpha^{eq} \equiv K_\alpha(\varrho_{0\alpha}) = 9\left(\varrho^2\,\frac{\partial^2 \mathscr{E}_\alpha}{\partial \varrho^2}\right)_{\varrho=\varrho_{0\alpha}} = \left(k_F^2\,\frac{\partial^2 \mathscr{E}_\alpha}{\partial k_F^2}\right)_{k_F=k_{F\alpha}}, \tag{13}$$

where $\varrho_{0\alpha} = 2k_{F\alpha}^3/(3\pi^2)$. The compression modulus $K_\alpha(\varrho)$ may be shown to be related to the Fermi liquid parameters $\mathcal{F}_0^{\tau\tau'}$ by the general formula [14]

$$K_\alpha(\varrho) = \frac{3\hbar^2 k_F^2}{m^*}\left\{\frac{m^*}{2}\,[(1+\alpha)^{5/3}/m_n^* + (1-\alpha)^{5/3}/m_p^*] + \tfrac{1}{4}\,[(1+\alpha)^2\mathcal{F}_0^{nn} + (1-\alpha)^2\mathcal{F}_0^{pp} + 2(1-\alpha^2)\mathcal{F}_0^{np}]\right\}. \tag{14}$$

Charge symmetry of nuclear hamiltonian implies $K_\alpha(\varrho) = K_{-\alpha}(\varrho)$ and in particular $K_\alpha^{eq} = K_{-\alpha}^{eq}$. In the case of a small neutron excess, peculiar to standard nuclear physics, one may expand both K_α^{eq} and $K_\alpha(\varrho)$ in α, keeping only terms quadratic in α. In this approximation

$$K_\alpha^{eq} = K_0^{eq} + K_{sym}^{eq}\alpha^2, \tag{15a}$$

$$K_\alpha(\varrho) = K_0(\varrho) + K_{sym}(\varrho)\alpha^2. \tag{15b}$$

We shall see in Section 4 that, surprisingly, the quadratic approximation (15a) works extremely well even at $\alpha = 0.3$.

In what follows, we shall mainly study the α-dependence of compressibility and of the Fermi liquid parameters at a given value of pressure P: $\mathscr{F}_l^{\tau\tau'}(P, \alpha)$ and $K_\alpha(P)$. In particular, the saturation properties of asymmetric nuclear matter will correspond to $P = 0$ (e.g., $K_\alpha^{eq} = K_\alpha(P = 0)$) while those relevant to astrophysical applications will be calculated at the values of $P > 0$, encountered in the calculations of supernova matter.

4. *Results*

Our calculations have been performed for five different parametrizations of the Skyrme force. The parameters of the Skyrme forces used in the present study are shown in Table I. The forces SIII, SIV [15] and Ska [16] were used in extensive self-consistent calculations

TABLE I

Numerical values of the parameters of the Skyrme type interactions used in the present work

	t_0 (MeV · fm^3)	x_0	t_1 (MeV · fm^5)	t_2 (MeV · fm^5)	t_3 (MeV · fm^{3+3d})	x_3	d
SIII	−1128.75	.45	395.0	−95.0	14000.0	1.0	1
SIV	−1205.6	.05	765.0	35.0	5000.0	1.0	1
SKA	−1602.78	−.02	570.88	−67.7	8000.0	−.286	1/3
SkM	−2645.0	.09	385.0	−120.0	15595.0	0.0	1/6
LR	−1057.3	.288	235.9	−100.0	14463.5	226	1

of a large number of properties of atomic nuclei [3]. The Skyrme type force of Ref. [17] (hereafter referred to as the SkM force) and that of Ref. [5] (hereafter referred to as the LR one) have been used in astrophysical applications, involving very often strongly asymmetric nuclear matter with a very large neutron excess. However, these parametrizations are also consistent with the properties of symmetric nuclear matter at the equilibrium density (see Table IV).

Let us consider first the standard case of asymmetric nuclear matter in equilibrium, i.e. at the density $\varrho_{0\alpha}$ corresponding to the minimum of $\mathscr{E}_\alpha(\varrho)$ ($P = 0$). We choose $\alpha \lesssim 0.3$ which is relevant to physics of heavy nuclei. In Table II we present the values of effective masses and Fermi liquid parameters, calculated at $P = 0$ for several values of α. The behaviour of m_τ^* is quite similar for all forces. At $P = 0$ and $\alpha \leqslant 0.1$ m_n^* increases and m_p^* decreases with increasing α. This is connected with the sign of the quantity $t_2 - t_1$ which remains the same for all the forces considered (see formula (6) and Table I). Note that for Skyrme forces the variation of $1/m_\tau^*$ with α at *fixed* ϱ is linear and that at $\alpha \leqslant 0.1$ and $P = 0$ the additional α-dependence of m_τ^* coming from the decrease of $\varrho_{0\alpha}$ with increasing α is negligible ($\varrho_0 - \varrho_{0\alpha} \sim \alpha^2$). For the SIII, SkM and LR forces this qualitative behaviour of m_τ^* does not change when one passes to $0.1 \leqslant \alpha \leqslant 0.3$. In the case of the SIV and Ska forces the effect of the decrease of $\varrho_{0\alpha}$ makes m_p^* practically constant for $0.1 \leqslant \alpha \leqslant 0.3$.

TABLE II

The Fermi liquid parameters calculated at equilibrium density of asymmetric nuclear matter ($P = 0$) for $0 \leqslant \alpha \leqslant 0.3$

α/force		SIII	SIV	Ska	SkM	LR
m_n^*/m	0.0	0.763	0.471	0.608	0.789	0.911
	0.1	0.776	0.480	0.622	0.805	0.925
	0.2	0.789	0.492	0.640	0.824	0.939
	0.3	0.804	0.508	0.662	0.846	0.954
m_p^*/m	0.0	0.763	0.471	0.608	0.789	0.911
	0.1	0.751	0.463	0.600	0.775	0.899
	0.2	0.739	0.463	0.596	0.764	0.887
	0.3	0.729	0.464	0.597	0.758	0.877
$\mathscr{F}_0^{nn}$	0.0	1.17	−0.04	0.40	0.74	1.78
	0.1	0.94	−0.03	0.41	0.66	1.59
	0.2	0.705	−0.04	0.39	0.56	1.39
	0.3	0.465	−0.07	0.35	0.44	1.16
$\mathscr{F}_0^{pp}$	0.0	1.17	−0.04	0.40	0.74	1.78
	0.1	1.40	−0.06	0.37	0.81	1.93
	0.2	1.62	−0.11	0.31	0.87	2.06
	0.3	1.83	−0.18	0.22	0.90	2.15
$\mathscr{F}_0^{np}$	0.0	−0.56	−0.53	−0.92	−1.20	−0.66
	0.1	−0.56	−0.54	−0.95	−1.22	−0.68
	0.2	−0.58	−0.60	−1.01	−1.27	−0.74
	0.3	−0.62	−0.69	−1.13	−1.37	−0.84

In contrast to this regular behaviour of the effective masses, the character of the α-dependence of $\mathscr{F}_0^{\tau\tau'}$ may change when one passes from one force to another. In particular, both the sign and size of

$$(\Delta\mathscr{F}_0)_{eq} \equiv \mathscr{F}_0^{nn}(P = 0, \alpha) - \mathscr{F}_0^{pp}(P = 0, \alpha)$$

may depend on the force used. At $\alpha = 0.3$ the quantity $(\Delta\mathscr{F}_0)_{eq}$ is 0.11 and 0.13 for the SIV and Ska forces and is −1.36, −0.46 and −0.99 for the SIII, SkM and LR forces, respectively.

The values of the quantities K_0^{eq} and K_{sym}^{eq}, calculated for all the Skyrme forces studied, are shown in Table III. The quadratic approximation, $K_0^{eq} + K_{sym}^{eq}\alpha^2$, gives a very precise description of the dependence of K_α^{eq} on α. This is illustrated in Table IV. The values of K_{sym}^{eq} for all the Skyrme forces used are quite similar. Namely, -504 MeV $< K_{sym}^{eq} < -441$ MeV for the SIII, SIV, Ska and LR forces, and $K_{sym}^{eq} = -357$ MeV for the SkM force.

The difference between the symmetry term in the compression modulus calculated at the saturation density of symmetric nuclear matter, $K_\alpha(\varrho_0)$ ($K_{sym}(\varrho_0)$ defined in Eq. (15b))

TABLE III

The equilibrium compression modulus K_α^{eq} for the SIV force at several values of α. The values in brackets have been calculated using corresponding quadratic approximation, Eq. (15a)

α	K_α^{eq} (MeV)	
0.0	324.5	(324.5)
0.2	304.5	(304.3)
0.3	279.7	(279.1)
0.4	245.7	(243.9)

TABLE IV

Saturation properties of symmetric nuclear matter obtained using the Skyrme type interactions studied in the present paper

Force	SIII	SIV	Ska	SkM	LR
k_{F_0}(fm^{-1})	1.291	1.307	1.320	1.334	1.320
a_{sym} (MeV)	28.2	31.2	32.9	30.75	29.3
K_0^{eq} (MeV)	355.4	324.5	263.15	216.6	370.4
$K_{sym}(\varrho_0)$ (MeV)	-334.3	244.3	369.3	147.25	-47.7
δK_{sym} (MeV)	-121.7	-748.5	-810.4	-504.2	-437.5
K_{sym}^{eq} (MeV)	-456.0	-504.2	-441.1	-356.9	-483.2

and that calculated from the equilibrium ($P = 0$) compression modulus, K_α^{eq} (K_{sym}^{eq} defined in Eq. (15a)) deserves an additional comment. The difference between K_{sym} and K_{sym}^{eq} is due to terms which arise from the equilibrium condition:

$$\left.\frac{d\mathscr{E}_\alpha(\varrho)}{d\varrho}\right|_{\varrho=\varrho_{0\alpha}} = 0. \tag{16}$$

For small values of α one can approximate $\mathscr{E}_\alpha(\varrho)$ by:

$$\mathscr{E}_\alpha(\varrho) = \mathscr{E}_0(\varrho) + a_{sym}(\varrho)\alpha^2, \tag{17}$$

where a_{sym} is the usual symmetry energy. At equilibrium, we have:

$$\mathscr{E}'_\alpha(\varrho_{0\alpha}) = \mathscr{E}'_0(\varrho_{0\alpha}) + a'_{sym}(\varrho_{0\alpha})\alpha^2 = 0, \tag{18}$$

where the primes denote derivatives with respect to density ϱ. Expanding $\mathscr{E}'_0(\varrho_{0\alpha})$ up to first order in $\delta\varrho_\alpha = \varrho_{0\alpha} - \varrho_0$, one gets

$$\mathscr{E}'_0(\varrho_{0\alpha}) = \mathscr{E}''_0(\varrho_0)\delta\varrho_\alpha = \frac{K_0^{\rm eq}}{9\varrho_0^2}\,\delta\varrho_\alpha.$$

Combining this with Eq. (18), one obtains

$$\delta\varrho_\alpha = -\frac{9\varrho_0^2}{K_0^{\rm eq}}\,a'_{\rm sym}(\varrho_0)\alpha^2. \tag{19}$$

Note that $\delta\varrho_\alpha$ is quadratic in α as it should be due to charge symmetry. One can expand in a similar way the compression modulus

$$K_0(\varrho_{0\alpha}) = K_0^{\rm eq} + K'_0\delta\varrho_\alpha = K_0^{\rm eq} + \delta K_{\rm sym}\alpha^2, \tag{20}$$

where we have defined

$$\delta K_{\rm sym} = -\frac{9\varrho_0 K'_0}{K_0^{\rm eq}}\,a'_{\rm sym}. \tag{21}$$

TABLE V

Fermi liquid parameters calculated at $P = 1$ MeV fm^{-3} for several values of α

α/force		SIII	SIV	Ska	SkM	LR
m_n^*/m	0.0	0.739	0.438	0.571	0.758	0.902
	0.2	0.767	0.457	0.600	0.795	0.932
	0.5	0.810	0.504	0.669	0.865	0.980
	0.7	0.842	0.554	0.729	0.917	1.013
m_p^*/m	0.0	0.739	0.438	0.571	0.758	0.902
	0.2	0.713	0.428	0.555	0.729	0.874
	0.5	0.675	0.430	0.557	0.706	0.841
	0.7	0.650	0.449	0.579	0.710	0.827
$\mathscr{F}_0^{nn}$	0.0	1.538	0.143	0.619	0.973	2.339
	0.2	1.013	0.150	0.634	0.796	1.918
	0.5	0.232	0.060	0.509	0.470	1.190
	0.7	−0.290	−0.081	0.311	0.195	0.626
$\mathscr{F}_0^{pp}$	0.0	1.538	0.143	0.619	0.973	2.339
	0.2	2.073	0.083	0.536	1.121	2.700
	0.5	2.911	−0.131	0.282	1.265	3.058
	0.7	3.525	−0.389	0.023	1.267	3.059
$\mathscr{F}_0^{np}$	0.0	0.209	−0.170	−0.543	−0.711	0.104
	0.2	0.215	−0.222	−0.608	−0.757	0.058
	0.5	0.254	−0.518	−0.960	−1.022	−0.220
	0.7	0.323	−0.907	−1.371	−1.376	−0.633

Hence K_{sym} and K_{sym}^{eq} are related by

$$K_{sym}^{eq} = K_{sym} - \frac{9\varrho_0^2 K_0'}{K_0^{eq}} a_{sym}', \tag{22}$$

where all the quantities are calculated at the saturation density of symmetric nuclear matter, ϱ_0. The crucial importance of the term δK_{sym}, which results from the shift in the equilibrium density when passing from $\alpha = 0$ to a small nonzero α may be seen in Table IV. The term δK_{sym} compensates for the drastic differences between the values of $K_{sym}(\varrho_0)$ calculated using different Skyrme forces, and leads to quite similar values of K_{sym}^{eq}. Note that K_{sym}^{eq} is the quantity which appears in the phenomenological parametrization of the compression modulus of finite nuclei used in Ref. [23].

Let us consider now the case of a highly asymmetric nuclear matter under a high external pressure. Such a situation is encountered in the calculations of the properties of hot dense matter in gravitational collapse of massive stars [6]. In Table V we give the values of the Fermi liquid parameters m_τ^* and $\mathscr{F}_0^{\tau\tau'}$ calculated at $P = 1$ MeV fm^{-3}, for several values of the neutron excess parameter α. The chosen value of pressure is typical to central part of collapsing stellar core at the final stage of gravitational collapse [6]. The behaviour of m_n^* is similar for all forces. As at $P = 0$, m_n^* increases with increasing α. The value of m_p^* decreases with increasing α for the SIII, SkM and LR forces or remains practically constant for the SIV and Ska forces. On the contrary, the behaviour of $\mathscr{F}_0^{\tau\tau'}$ as a function of α may qualitatively change when one passes from one force to another.

5. *Discussion and conclusions*

In this section we compare the results obtained with Skyrme forces with those obtained using quasiparticle interactions based on more realistic nucleon-nucleon potentials. Then we summarize the conclusions.

In order to make contact with existing results obtained using realistic nucleon-nucleon potentials, let us consider asymmetric nuclear matter at *fixed density* ϱ close to ϱ_{NM}. The calculation shows, that for all Skyrme forces used in the present paper the effective mass of a neutron, m_n^*, increases and that of a proton, m_p^*, decreases when neutron excess increases, the density ϱ being kept constant. This qualitative behaviour is in agreement with that obtained by Sjöberg [18] who calculated the quasiparticle effective masses at $\alpha \approx 0.9$, approximating the quasiparticle interaction by the Brueckner reaction matrix derived from the Reid soft-core nucleon-nucleon interaction. Let us notice that a similar behaviour is obtained in the self-consistent calculations of the effective masses of hole states in asymmetric nuclear matter within the framework of the lowest order Brueckner theory [19].

The quasiparticle interaction in asymmetric nuclear matter has been calculated up to now only in the extreme case of pure neutron matter ($\alpha = 1$) [20]. The spin averaged quasiparticle interaction in pure neutron matter, hereafter denoted as $\tilde{f}$, reads

$$\tilde{f} \equiv \tfrac{1}{4} \sum_{\sigma\sigma'} \tilde{f}_{k\sigma,k'\sigma'} = \tilde{N}_0^{-1} \sum \tilde{F}_l P_l(\cos\theta). \tag{23}$$

Here, $\tilde{N}_0$ is the density of quasiparticle states and $\tilde{F}_l$ are the Fermi liquid parameters, calculated in pure neutron matter. We have $f^{nn}(\varrho, \alpha = 1) = \tilde{f}(\varrho)$. Let us approximate the α-dependence of $\mathscr{F}_0^{\tau\tau'}(\varrho, \alpha)$ *at fixed* ϱ by a simplified linear form,

$$\mathscr{F}_0^{nn} = F_0+F_0'+B_1\alpha, \quad \mathscr{F}_0^{pp} = F_0+F_0'-B_1\alpha,$$

implied by Eq. (8) [13]. At fixed $\varrho \cong \varrho_{NM}$ this formula gives a qualitative agreement with numerical values of $\mathscr{F}_0^{\tau\tau}$ for the SkM and LR forces even at $\alpha \cong 0.9$. We may then expect that the following approximate relation will hold, at least qualitatively:

$$\mathscr{F}_0^{nn} \cong F_0+F_0'+\alpha\left(\tilde{F}_0\frac{N_0}{\tilde{N}_0}-F_0-F_0'\right). \tag{24}$$

Note that $N_0/\tilde{N}_0 = 2^{2/3}m^*/\tilde{m}^*$, where m^* and $\tilde{m}^*$ are the neutron effective masses, when $\alpha = 0$ and $\alpha = 1$, respectively. Hence, in such an approximation $B_1 = 2^{2/3}m^*/\tilde{m}^*\tilde{F}_0 - F_0 - F_0'$. Using the values of $\tilde{m}^*$ and $\tilde{F}_0$ obtained in Ref. [20] and the values of m^*, F_0 and F_0' obtained in Ref. [21] (both calculated using the Reid soft-core nucleon-nucleon interaction) we obtain at $k_F = 1.35\ \mathrm{fm}^{-1}$, corresponding to the saturation density ($P = 0$) for the model of nuclear matter used in Refs. [20, 21], a rough estimate $B_{1(\mathrm{Reid})} = 0.44$.

Finally, let us consider the case of nuclear matter compressibilities, and in particular the quantities displayed in the four bottom rows in Table IV. Drastic differences between $K_{sym}(\varrho_0)$ are essentially balanced by those in δK_{sym}, leading to quite similar values of K_{sym}^{eq}. The relative differences in the values of K_{sym}^{eq} for the SIII, SIV, Ska and LR forces are smaller than 20%. The quantities $K_{sym}(\varrho_0)$ and δK_{sym} can be also estimated by using the Fermi liquid parameters calculated from the Reid soft-core nucleon-nucleon potential. Neglecting the α-dependence of $\mathscr{F}_1^{\tau\tau'}$ and $\mathscr{F}_0^{np}$, and making the linear approximation for $\mathscr{F}_0^{\tau\tau}$ we obtain, using the methods described in Ref. [14], the following expression:

$$K_{sym} = \frac{3\hbar^2k_F^2}{m^*}\left[\tfrac{5}{9}-\tfrac{5}{27}(2F_1'-F_1)+F_0'+B_1\right]. \tag{25}$$

The quantity δK_{sym} is given by the Eq. (20b). The derivatives of K_0 and a_{sym} which enter the expression for δK_{sym} can be expressed in terms of the derivatives of F_0, F_0' and m^* using the standard formulae

$$K_0 = \frac{3\hbar^2k_F^2}{m^*}(1+F_0), \tag{26a}$$

$$a_{sym} = \frac{\hbar^2k_F^2}{3m^*}(1+F_0'). \tag{26b}$$

With the help of these formulae we calculated K_{sym} and δK_{sym} at $k_F = 1.35\ \mathrm{fm}^{-1}$ (the saturation ($P = 0$) Fermi momentum for the model of nuclear matter assumed in Ref. [21]), using the values of F_0, F_0', m^* obtained in Ref. [21] and taking that of F_1' from a calculation of Sjöberg [22]. The derivatives of m^*, F_0 and F_0' have been calculated numerically from the plots presented in Ref. [21]. Such an approximate procedure yields the following estimates:

$$K_{\mathrm{sym(Reid)}} \cong 230 \text{ MeV},$$

$$\delta K_{\mathrm{sym(Reid)}} \cong -120 \text{ MeV}. \tag{27}$$

This gives an estimate $K^{\mathrm{eq}}_{\mathrm{sym(Reid)}} \cong 110$ MeV, in a drastic disagreement with the values of this quantity obtained using the Skyrme forces. This value is also in conflict with the one which seems to be required to explain the variations of the compression modulus K_A with neutron and proton numbers (Refs. [7, 8, 23]).

We summarize now the main conclusions of the present paper. The various existing parametrizations of the Skyrme forces lead to similar values of parameters describing the bulk properties of nuclei connected with neutron excess (symmetry energy, symmetry term in the compression modulus of asymmetric nuclear matter, $K^{\mathrm{eq}}_{\mathrm{sym}}$). However, they do not predict a unique variation of the Fermi liquid parameters with neutron excess. It appears therefore that such a dependence will be hard to extract from a mere analysis of bulk properties. A more refined analysis or a more fundamental approach is clearly needed to do so. As we mentioned, the values of $K^{\mathrm{eq}}_{\mathrm{sym}}$, obtained for the Skyrme forces are in agreement with our present knowledge of the empirical value of this quantity, obtained in the analysis of the breathing modes frequencies. However, as discussed in Refs. [8, 23] such an analysis is not free of ambiguities and the agreement just mentioned should not be taken too seriously. It is nevertheless interesting to notice that the simple estimate of $K^{\mathrm{eq}}_{\mathrm{sym}}$, based on the existing models of the Fermi liquid parameters derived from the Reid soft-core potential, is in conflict with the empirical value and with the estimate based on the Skyrme forces.

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