REGGE BEHAVIOUR OF SYMMETRIC OCTET EXCHANGE AMPLITUDE FOR GLUON-GLUON SCATTERING

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(*Received September 14, 1980*)

The Regge behaviour of the symmetric octet amplitude for gluon-gluon scattering in the massive $SU(3)_c$ theory is studied. It is shown that the asymptotic behaviour of this amplitude is controlled by a moving Regge pole similarly to the antisymmetric octet amplitude which is asymptotically described by the reggeised gluon. The corresponding Regge trajectories are, however, different and physical origin of this difference is discussed.

PACS numbers: 12.40.—y, 12.40.Mm

1. *Introduction*

Much progress has recently been achieved in understanding asymptotic behaviour of scattering amplitudes in nonabelian gauge theories [1–7]. One of the basic results obtained in this field is reggeisation of gauge vector mesons. This means that the vector meson–vector meson scattering amplitude characterised by gauge vector meson quantum numbers in the t-channel has a moving Regge pole in a j-plane and the point ($t = m^2, j = 1$) lies on its trajectory (m stands for vector meson. mass).

Strong interactions are believed to be based on the $SU(3)_c$ group. The two body scattering amplitudes of gauge vector mesons (gluons) can be decomposed into eigenamplitudes corresponding to irreducible $SU(3)_c$ representations appearing in the product of two adjoint representations:

$$8\otimes 8 = 1\oplus 8_a\oplus 8_s\oplus \dots . \tag{1.1}$$

The adjoint octet representation appears twice in this product.

Gluons are characterised by generalized C-parity $C = -1$ [8]. Two octets in (1.1) differ by their generalized C-parities; 8_a has negative whereas 8_s has positive C, since they are obtained by F and D coupling of two gluon octets respectively [9].

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It was shown that the 8_a amplitude for gluon–gluon scattering with gluon quantum numbers in the t-channel, reggeises [1–7]. In this paper we study the 8_s amplitude and its j-plane singularities.

The main motivation for the detailed study of gluon–gluon amplitude just in this channel is its importance for understanding the origin of the Regge singularities in colour singlet and negative charge conjugation ($C = -1$) channel.

The singularities in the colour singlet channel are the most interesting ones as they can couple to hadrons. Two (octet) gluons can couple to the colour singlet and the $C = +1$ state. The corresponding multiperipheral-like equation generating bare Pomeron singularity in Quantum Chromodynamics was studied in Ref. [10]. In order to form the colour singlet and $C = -1$ state in the t-channel at least three gluons are needed with one pair coupled to the *symmetric* 8_s octet state. In this way the symmetric octet amplitude appears as a quasi two-body subamplitude in a three gluon channel with negative C and singlet $SU(3)_c$ quantum numbers. It is mainly for this reason that detailed understanding of gluon–gluon amplitude is important and the corresponding three-gluon equation has recently been formulated by us in Ref. [11].

In this paper we follow closely the method proposed by Lipatov, Kuraev and Fadin [4] considering the massive $SU(3)_c$ theory where gluon mass m is generated via the Higgs mechanism [12]. We study asymptotic behaviour of the scattering amplitudes in a leading logarithm approximation using perturbation theory. In Section 2 we construct explicitly an imaginary part of the elastic gluon–gluon amplitude, which we use in Section 3 to built up, via unitarity, the integral equation for full elastic amplitude. We find that its solution for the symmetric octet amplitude has a moving Regge pole, like the amplitude corresponding to antisymmetric octet. These two trajectories are, however, different. In Section 4 we discuss the role of Higs particles in the whole scheme and comment on the $m \to 0$ limit.

2. *Imaginary part of the elastic amplitude for gluon–gluon scattering*

We consider spontaneously broken $SU(3)_c$ gauge theory [4, 12] with 8 massive vector gluons; V^a, and with Higgs sector consisting of 8 scalar "colour" particles, S^a, and 2 "white" scalars; F and Z. For a $V-V$ inelastic process $a+b \to a'+b'+d_1+ \dots +d_n$ (for notation see Fig. 1) the reggeised amplitude [4] is:

$$A_{2\to 2+n} = s\Gamma^{i_1}_{aa'} \frac{\left(\frac{s_1}{m^2}\right)^{\alpha(q_1{}^2)}}{q_1^2-m^2} \gamma^{d_1}_{i_1 i_2} \frac{\left(\frac{s_2}{m^2}\right)^{\alpha(q_2{}^2)}}{q_2^2-m^2} \gamma^{d_2}_{i_2 i_3} \cdots \frac{\left(\frac{s_{n+1}}{m^2}\right)^{\alpha(q_{n+1}{}^2)}}{q_{n+1}^2-m^2} \Gamma^{i_{n+1}}_{bb'}, \tag{2.1}$$

where the vertex functions [4] are given by (Fig. 2):

$$\Gamma^i_{aa'} = -ig\sqrt{2}\, f_{iaa'}\delta_{\lambda_a\lambda_{a'}} a_{\lambda_a},$$

$$\Gamma^i_{as} = g\sqrt{2}\, d_{ias} b_{\lambda_a},$$

$$\Gamma^i_{af} = g\frac{2}{\sqrt{3}}\,\delta_{ia} b_{\lambda_a}, \tag{2.2}$$

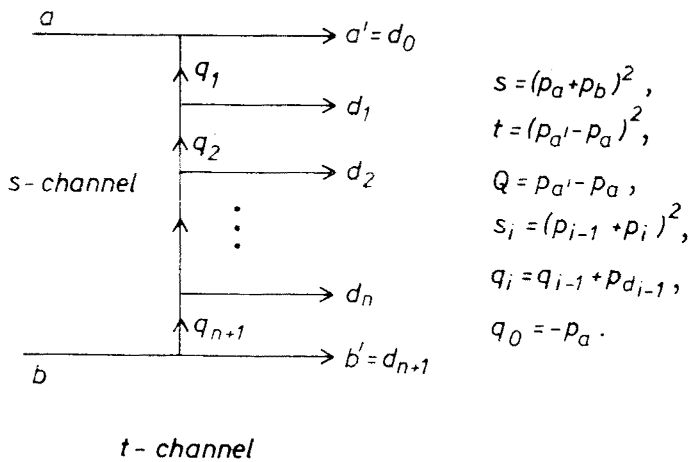

Fig. 1. Inelastic gluon–gluon amplitude — definition of kinematic variables

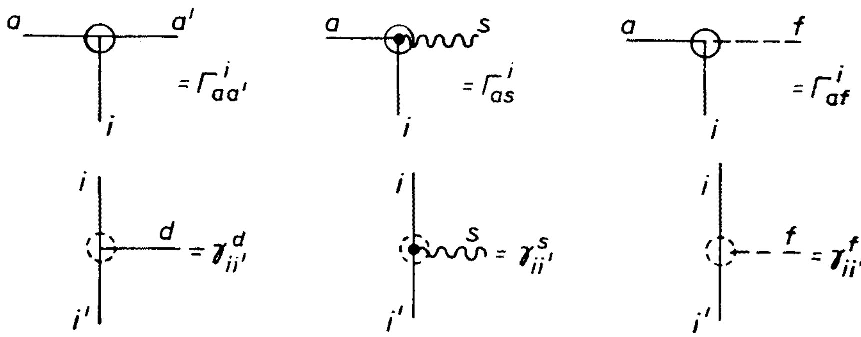

Fig. 2. Vertex functions for high energy amplitudes. The sings in circles indicate the group-theoretical factors appearing in vertex functions: $f_{iaa'}$, $d_{iaa'}$, and δ_{ai}

and f_{ijk}, d_{ijk} are SU(3)$_c$ structure constants, indices i, a, a' stand for vector gluons, indices s and f denote the S and F type Higgs scalars. By λ_a we denote the polarization of the particle a, polarization four-vectors will be denoted by $e^{\mu}_{\lambda_a}$, the constants a_{λ_a} and b_{λ_a} are defined as:

$$a_{\lambda_a} = -1+b_{\lambda_a},$$

$$b_{\lambda_a} = \tfrac{1}{2}\,\delta_{\lambda_a 3}, \tag{2.3}$$

so that a_{λ_a} equals -1 for transverse, or $-\frac{1}{2}$ for longitudinal polarizations.

The corresponding vertex functions [4] are (Fig. 2):

$$\gamma^d_{ii'} = \gamma^d_{ii'}(q_i, q_{i'}) = \begin{cases} -igf_{idi'}e^{\nu}_{\lambda_d}\cdot P_\nu(q_i, q_{i'}) & \text{for } d = V, \\ mgd_{idi'} & \text{for } d = S, \\ mg\sqrt{\tfrac{2}{3}}\,\delta_{ii'} & \text{for } d = F, \end{cases} \tag{2.4}$$

where

$$P_\nu(q, q') = -(q_\perp + q'_\perp)_\nu - \left(\frac{s_2}{s} + 2\,\frac{q^2 - m^2}{s_1}\right) p_{a\nu} + \left(\frac{s_1}{s} + \frac{q'^2 - m^2}{s_2}\right) p_{b\nu} \tag{2.5}$$

and the trajectory $\alpha(q^2)$ has a form:

$$\alpha(q^2) = \tfrac{3}{2}\,\frac{g^2}{(2\pi)^3}(q^2 - m^2)\int \frac{d^2k}{(k^2 - m^2)\,((k-q)^2 - m^2)}\,. \tag{2.6}$$

The imaginary part of the elastic V–V amplitude can be calculated via unitarity condition

$$\operatorname{Im} A_{2\to 2} = \tfrac{1}{2}\sum_{n=0}^{\infty}\int (2\pi)^4\delta(p_a + p_b - p'_a - p'_b - \sum_{i=1}^{n} p_i)$$

$$\times\,\frac{d^3p'_a}{2E'_a(2\pi)^3}\,\frac{d^3p'_b}{2E'_b(2\pi)^3}\prod_{i=1}^{n}\frac{d^3p_i}{2E_i(2\pi)^3}\,A_{2\to 2+n}A^*_{2\to 2+n}, \tag{2.7}$$

where $A_{2\to 2+n}$ amplitude is given by (2.1). Since all incoming and outgoing particles belong to the $\mu = 8$ representation of the SU(3)$_c$ group, the elastic amplitude can be decomposed into the amplitudes with the t-channel exchange of singlet ($\mu = 1$), symmetric and anti-

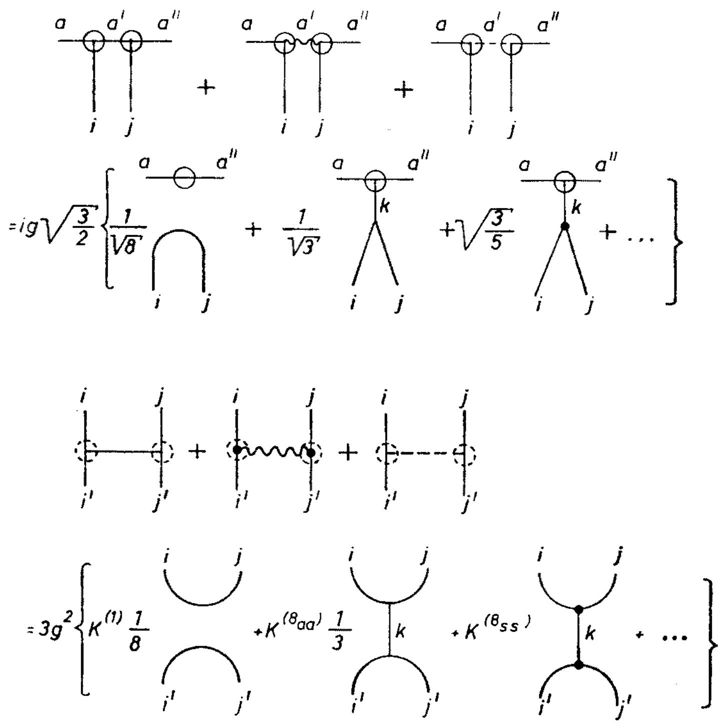

Fig. 3. t-channel vertices projection

symmetric octets ($\mu = 8_{ss}$, $\mu = 8_{aa}$), and also higher representations quantum numbers exchanged in the t-channel. In order to perform this decomposition we have to calculate two vertex functions product (Fig. 3) appearing in the unitarity condition (2.7):

$$\sum_{a'} \Gamma^i_{aa'}\Gamma^{j*}_{a''a'} = ig\sqrt{\tfrac{3}{2}}\left\{\Gamma_{aa''}\frac{\delta_{ij}}{\sqrt{8}} + \Gamma^k_{aa''}\frac{f_{kij}}{\sqrt{3}} + \tilde{\Gamma}^k_{aa''}\sqrt{\tfrac{3}{5}}\,d_{kij} + \dots\right\}, \tag{2.8}$$

with the singlet and symmetric octet vertex functions defined as:

$$\Gamma_{aa''} \equiv (-ig\sqrt{2}\,\delta_{\lambda_a\lambda_{a''}})(a_{\lambda_a} + \tfrac{7}{9}b^2_{\lambda_a})\sqrt{\tfrac{3}{2}}\,\delta_{aa''},$$

$$\tilde{\Gamma}^k_{aa''} \equiv (-ig\sqrt{\tfrac{3}{2}}\,\delta_{\lambda_a\lambda_{a''}})\left(\sqrt{3}\,a^2_{\lambda_a} + \frac{1}{3\sqrt{3}}\,b^2_{\lambda_a}\right)\sqrt{\tfrac{3}{5}}\,d_{aa''k}, \tag{2.9}$$

and

$$\sum_d \gamma^d_{ii'}(q, q')\gamma^d_{jj'}(q-Q, q'-Q)$$

$$= 3g^2\left\{K^{(1)}(q, q', Q)\frac{\delta_{ij}\delta_{i'j'}}{8} + K^{(8aa)}(q, q', Q)\frac{f_{ijk}f_{ki'j'}}{3} + K^{(8ss)}_{(q,q',Q)}\,\tfrac{3}{5}\,d_{ijk}d_{ki'j'}\right\}. \tag{2.10}$$

The kernels $K^{(\mu)}(q, q', Q)$ are given by

$$K^{(\mu)}(q, q', Q) = K^{(\mu)}(Q^2) - \tilde{K}^{(\mu)}(q, q', Q), \tag{2.11}$$

where $K^{(\mu)}(Q^2)$ does not depend upon the integration variables in (2.7) and $\tilde{K}^{(\mu)}(q, q', Q)$ is singular for $m \to 0$ when integrated over d^2q or d^2q',

$$K^{(8aa)}(Q^2) = Q^2 - m^2, \quad K^{(8ss)}(Q^2) = Q^2 - \tfrac{13}{9}\,m^2,$$

$$\tilde{K}(q, q', Q) \equiv \tilde{K}^{(8aa)}(q, q', Q) = \tilde{K}^{(8ss)}(q, q', Q)$$

$$= \frac{(q^2 - m^2)((q'-Q)^2 - m^2) + (q'^2 - m^2)((q-Q)^2 - m^2)}{(q-q')^2 - m^2}. \tag{2.12}$$

As we shall see ((3.5)–(3.7)) the singular part $\tilde{K}$ of the kernels (2.12), which is the same for 8_{aa} and 8_{ss} amplitudes but different for singlet [10], is responsible for the reggeisation of these amplitudes. The nonsingular parts, however, differ in mass terms, this indicates that 8_{aa} and 8_{ss} trajectories will not be degenerate.

3. *Integral equation for the t-channel amplitudes*

The sum in (2.7) can be rewritten [4] in terms of the t-channel amplitudes:

$$\sum_n A_{2\to 2+n}A^*_{2\to 2+n} = \Gamma_{aa'}A^{(1)}(s, t)\Gamma^k_{bb'} + \Gamma^k_{aa'}A^{(8_{aa})}(s, t)\Gamma^k_{bb'}$$

$$+ \tilde{\Gamma}^k_{aa'}A^{(8_{ss})}(s, t)\tilde{\Gamma}^k_{bb'} + \dots, \tag{3.1}$$

where

$$A^{(\mu)}(s,t) = -\sum_{n=0}^{\infty}\left(\prod_{i=1}^{n+1}\frac{\left(\frac{s_i}{m^2}\right)^{\alpha(q_i^2)+\alpha((q_i-Q)^2)}}{(q_i^2-m^2)((q_i-Q)^2-m^2)}\right.$$

$$\left.\times 3g^2K^{(\mu)}(q_1,q_2,Q)3g^2K^{(\mu)}(q_2,q_3,Q)\ldots 3g^2K^{(\mu)}(q_n,q_{n+1},Q)\right). \tag{3.2}$$

Since we want to investigate the j-plane singularities of $A^{(\mu)}(s,t)$ amplitudes, we shall make use of the following equation:

$$F_\omega^{(\mu)}(Q^2) = -\frac{2}{\pi}\int_1^\infty d\left(\frac{s}{m^2}\right)\left(\frac{s}{m^2}\right)^{-\omega-1} A^{(\mu)}(s,t),$$

$$\omega = j-1, \tag{3.3}$$

where $F_\omega^{(\mu)}(Q^2)$ is the j-representation for $A_{2\to2}^{(\mu)}(s,t)$ amplitude.

In order to find $F_\omega^{(\mu)}(Q^2)$ we define the function $f_\omega^{(\mu)}(k,k-Q)$:

$$F_\omega^{(\mu)}(Q^2) \equiv \frac{1}{\omega}\left[\frac{3}{2}\frac{g^2}{(2\pi)^3}\right]\int\frac{d^2k}{(k^2-m^2)((k-Q)^2-m^2)}K^{(\mu)}(Q^2)f_\omega^{(\mu)}(k,k-Q). \tag{3.4}$$

Taking into account relation (3.2) we obtain for $f_\omega^{(\mu)}(k,k-Q)$:

$$[\omega-\alpha(k^2)-\alpha((k-Q)^2)]f_\omega^{(\mu)}(k,k-Q)$$

$$= \frac{\omega}{K^{(\mu)}(Q^2)}+\left[\frac{3}{2}\frac{g^2}{(2\pi)^3}\right]\int\frac{d^2q}{(q^2-m^2)((q-Q)^2-m^2)}K^{(\mu)}(k,q,Q)f_\omega^{(\mu)}(q,q-Q). \tag{3.5}$$

This equation can be easily solved for $\mu = 8_{aa}$ and also for $\mu = 8_{ss}$ because the singular part of the kernel $K^{(\mu)}((q,q',Q)$ reproduces the trajectories standing on the left-hand-side of (3.5), so that

$$f_\omega^{(8aa)}(k,k-Q) = \frac{\omega}{Q^2-m^2}\,\frac{1}{\omega-\alpha(Q^2)}, \tag{3.6}$$

$$f_\omega^{(8ss)}(k,k-Q) = \frac{\omega}{Q^2-\frac{13}{9}m^2}\,\frac{1}{\omega-\tilde{\alpha}(Q^2)}, \tag{3.7}$$

where

$$\tilde{\alpha}(Q^2) \equiv \left[\frac{3}{2}\frac{g^2}{(2\pi)^3}\right](Q^2-\tfrac{13}{9}m^2)\int\frac{d^2q}{(q^2-m^2)((q-Q)^2-m^2)}. \tag{3.8}$$

For $F_\omega^{(\mu)}(Q^2)$ Eqs. (3.6)–(3.7) read

$$F_\omega^{(8_{aa})}(Q^2) = \frac{\alpha(Q^2)}{(Q^2-m^2)(\omega-\alpha(Q^2))}, \tag{3.9}$$

$$F_\omega^{(8_{ss})}(Q^2) = \frac{\tilde{\alpha}(Q^2)}{(Q^2-\frac{13}{9}m^2)(\omega-\tilde{\alpha}(Q^2))}. \tag{3.10}$$

As we see the symmetric octet amplitude does reggeise, i.e. it has a moving Regge pole $j = 1+\tilde{\alpha}(Q^2)$, however its trajectory differs from 8_{aa} amplitude by a mass parameter. The 8_{aa} and 8_{ss} amplitudes differ by their generalized C-parity and hence also by signature. The corresponding signature factors are $\exp\left(-i\frac{\pi}{2}\alpha\right)/\sin\frac{\pi}{2}\alpha$ and $i\exp\left(-i\frac{\pi}{2}\tilde{\alpha}\right)/\cos\frac{\pi}{2}\tilde{\alpha}$ for 8_{aa} and 8_{ss} octets respectively. Because of this fact in the leading logarithm [3, 5] approximation even signature 8_{ss} amplitude is one power of $\ln s$ down with respect to odd signature 8_{aa} amplitude.

4. *Summary*

The Higgs particles enter explicitly into our reggeisation scheme. At first they generate mass of gluons, which regularises the integrals (2.6) and (3.8) defining trajectories $\alpha(Q^2)$. Secondly they couple to gluons when the physical amplitudes are calculated. In order to obtain the kernels $K^{(\mu)}(q, q', Q)$ (2.12) we used the crossing relations [13] connecting different representations amplitudes in s and t channels:

$$A_s^{(\mu_{\gamma\beta})} = \sum_{\mu'\gamma'\beta'} (\mu_{\gamma\beta}|C|\mu'_{\gamma'\beta'})A_t^{(\mu'\gamma'\beta')}. \tag{4.1}$$

Because of the following equality between the crossing matrix elements

$$(8_{aa}|C|8_{aa}) = (8_{ss}|C|8_{aa}), \tag{4.2}$$

the singular parts of the kernels $K^{(8_{aa})}$ and $K^{(8_{ss})}$ are identical and therefore both amplitudes reggeise (see comment under (3.5)). The other elements of the crossing matrix projecting exchange of the Higgs particles onto antisymmetric and symmetric octet states are not equal i.e.

$$(8_{aa}|C|8_{ss}) \neq (8_{ss}|C|8_{ss}). \tag{4.3}$$

The nonsingular parts of the kernels $K^{(\mu)}$ are therefore different and, correspondingly, the 8_a (i.e. gluon) and 8_s trajectories are also different. It is worthwile to notice that the octet channels are the only ones which have the moving Regge singularities in the leading logarithm approximation. Amplitudes corresponding to other representations have fixed (branch point) singularities, since the singular parts of the kernels $K^{(\mu)}$ are different from the singular part of $K^{(8)}$ for $\mu \neq 8$.

To summarize: we have shown that the amplitude corresponding to the symmetric octet has a moving Regge pole although its trajectory differs from the gluon one.

We have also traced physical origin of this difference. The singular parts of the kernels

$K^{(\mu)}$, responsible for reggeisation ((2.12) and (3.5)), come from the gluon exchange in the crossed (s) channel (Figs. 1 and 3) and corresponding elements of the crossing matrix are the same for 8_{ss} and 8_{aa} amplitudes. The nonsingular part of $K^{(\mu)}$ on the other hand comes from gluon *and* Higgs scalar exchanges. The crossing matrix elements are different for 8_s and 8_a amplitudes and, in consequence, the trajectories differ by a mass term. In other words, possible exchange degeneracy of octet trajectories is broken by the Higgs sector.

Unfortunately, as it nevertheless should be expected, the $m \to 0$ limit has no sense at this stage since the effective kernel

$$K^{(\mu)}_{\text{eff}}(k, q, Q) \equiv K^{(\mu)}(k, q, Q) + \left[\frac{2}{3}\frac{(2\pi)^3}{g^2}\right](q^2-m^2)((q-Q)^2-m^2) \times [\alpha(q^2)+\alpha((q-Q)^2)]\delta^{(2)}(k-q), \tag{4.4}$$

which appears in (3.5) is infrared divergent. Corresponding Regge trajectories are also divergent in this limit i.e. for genuine QCD with massles gluons.

Infrared singularities are not confined to octet channels only. They appear in all other representations, except colour singlets [10, 11, 14]. In the case of singlet amplitudes the effective kernel $K^{(1)}_{\text{eff}}$ is infrared finite — this being achieved by exact cancellation of the singularities of the kernel $K^{(1)}$ by the infrared singularities of gluon trajectories (see the Refs. [10, 11, 14] for details). Thus, although the octet amplitudes are themselves infrared singular in the limit $m \to 0$ their reggeisation properties are essential for ensuring infrared finitness of colour singlet amplitudes. The equality of kernels $K^{(8_{ss})}$ and $K^{(8_{aa})}$ which leads to reggeisation in the symmetric octet channel is crucial for cancellation of infrared divergences in the negative C and colour singlet amplitude [11].

One of us (J. K.) is grateful to Dr. J. Bartels for useful discussions.

Note added in proof

After this paper had been completed we noticed that the symmetric octet amplitude was also discussed by J. B. Bronzan and R. L. Sugar, *Phys. Rev.* **D17**, 2813 (1978) in the framework of Reggeon field theory. Our result is in agreement with theirs.

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