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INSTABILITY OF A RECENTLY PROPOSED MIT VACUUM

By K. Zalewski

Institute of Nuclear Physics, Cracow*

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It is pointed out that a recently proposed QCD vacuum model, where space is filled with empty MIT bags, is unstable with respect to changes of shape of the bags. The argument assumes that the exact quantitative results obtained for parallelepipeds apply qualitatively also to bags of other shapes and that certain infinite terms can be safely ignored. Both these assumptions seem, however, unavoidable to make the original formulation of the model plausible.

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In this note we point out that a model vacuum, proposed recently [1] to bridge the gap between perturbative QCD and MIT bags, does not correspond to a minimum of energy and will collapse because of a degree of freedom ignored for technical reasons in Ref. [1].

According to Ref. [1] the QCD vacuum (ground state) has a foam-like structure with the bubbles being little MIT bags filled with perturbative vacuum. On the surface of each bag the boundary conditions are

$$\boldsymbol{n} \cdot \boldsymbol{E} = 0, \quad \boldsymbol{n} \times \boldsymbol{B} = 0 \tag{1a}$$

for gluon fields ($\boldsymbol{n}$ is the normal vector) and

$$\bar{\psi} \cdot \boldsymbol{n}\gamma\psi = 0 \tag{1b}$$

for quark fields. The space between bags, if any, is assumed irrelevant in the sense that the total energy is minimized by minimizing the energy within the bags. For simplicity all the bags have been assumed spherical, though, in principle, their shapes should be found by minimizing the total energy. Minimizing the total energy with respect to the bubble radius R, Ref. [1] includes two effects: the Casimir forces, which tend to expand the bubble,

* Address: Instytut Fizyki Jądrowej, Kawiory 26a, 30-057 Kraków, Poland.

and intragluon interactions, which hopefully work in the opposite direction. Assuming for the interaction energy the Ansatz

$$E_I(R) = -\frac{b\alpha_s(\Lambda R)}{R}, \tag{2}$$

where b is a positive constant, Λ is the QCD scale parameter and α_s the QCD running coupling constant, one indeed finds a stable equilibrium point at some $R = R_0$ [1]. This model leads to the phenomenology of MIT bags. On the other hand its parameters are calculable in terms of the fundamental parameters of QCD. Therefore the model, if correct, opens the possibility of calculating the phenomenological parameters of MIT bags from first principles. Some results along this line have already been reported in Ref. [1].

Two assumptions, necessary to justify the model, are important for the interpretation of our further analysis. There is no good reason why the bubbles should be spherical — actually it is not even possible to fill space completely with spheres of equal size. The hope is that the result for spheres is qualitatively correct also for other "reasonable" shapes. There is some support for this point of view, if the shapes are not too different. Lukosz [2] studying the Casimir energy for the electromagnetic field in the presence of perfect conductors pointed out the similarity between his result for the surface of a cube of edge length $2R$ ($0.0916/2R$) and the well-known (cf. e.g. [3]) result for a sphere of radius R ($0.0924/2R$). Since Maxwell equations in vacuum are invariant with respect to the exchanges $\boldsymbol{E} \to \boldsymbol{B}$, $\boldsymbol{B} \to -\boldsymbol{E}$, the Casimir energy for photons with perfect conductor boundary conditions coincides with that for noninteracting gluons (of one colour) with boundary conditions (1a). If the shapes are very different, the argument is known to fail. Thus e.g. the Casimir force tends to expand a sphere, but to compress a parallel plate condenser (cf. e.g. [3]). In our work we assume that an ellipsoid with semiaxes $L/2$, $L/2$, $\lambda L/2$ may be replaced by a parallelepiped of dimension $L \times L \times \lambda L$ without *qualitatively* changing the results.

The finite results for the Casimir energy are usually obtained by rejecting some infinite terms. For parallelepipeds the presence of infinite terms is due to the sharp edges, as has already been known for some time [4]. For the insides of spheres this phenomenon, due to other reasons, has been also pointed out [5]. A detailed study shows [6] that these infinities result from the unphysical idealization implied by the boundary conditions (1a). The procedure accepted is to ignore the infinite terms. In our work we add only the assumption that in this respect parallelepipeds are no worse than insides of spheres. Incidentally, when both the inside and the outside of a sphere are included and both are free of other conducting surfaces (which is not the case in the present vacuum model), it has been shown that all infinities cancel. This result does not seem relevant to the present problem.

We propose to check, how the deformation of spherical bubbles into ellipsoids affects the total energy. Since the Casimir energy for ellipsoids is not known, we will replace spheres by cubes and ellipsoids by parallelepipeds $L \times L \times \lambda L$. According to the physical picture put forward in Ref. [1] the energy density inside the bubble is

$$\varepsilon(\lambda, L) = L^{-4}(a_g(\lambda) + a_q(\lambda) - b(\lambda, L\Lambda)). \tag{3}$$

This follows from dimensional arguments: a_g and a_q correspond to Casimir energies of gluons and quarks and b to the interaction energy. By assumption, the only dimensional parameter besides L is the QCD scale Λ and it enters only the interaction energy. A method for calculating $a_g(\lambda)$ is given in Ref. [4]. The result is shown in Fig. 1. We have not calcu-

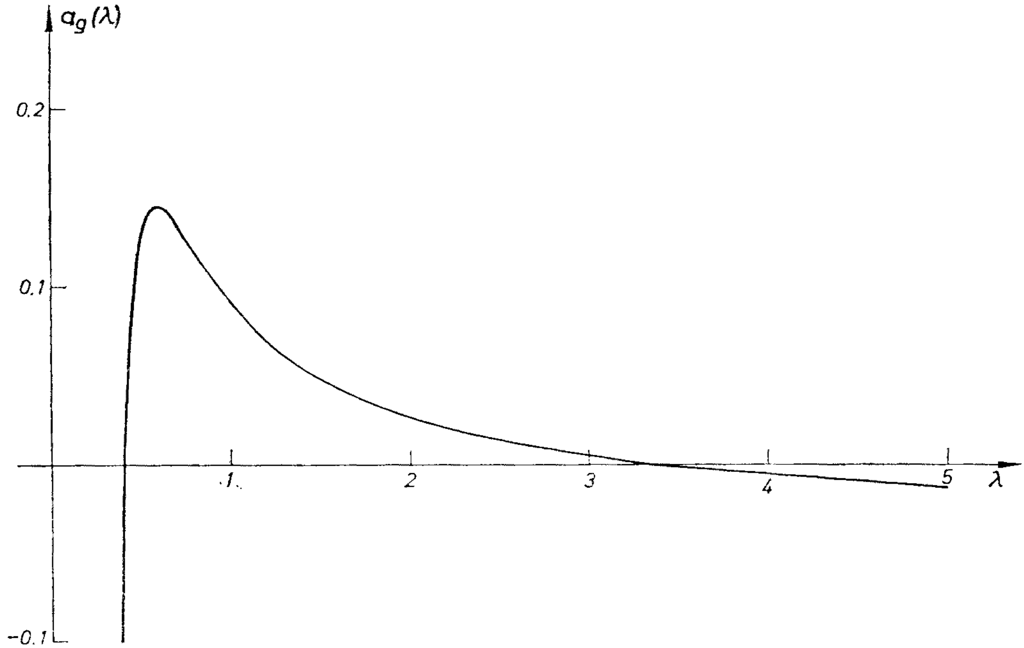

Fig. 1. λ-dependence of the Casimir energy density for gluons: $\varepsilon_g(\lambda, L) = a_g(\lambda)L^{-4}$

lated $a_q(\lambda)$, where the starting formula for the eigenfrequencies is unknown, but it is plausible that qualitatively it does not change the λ dependence of the Casimir energy. Indeed, it is known that [7]

$$a_q(\lambda) \approx -\frac{7\pi^2}{2880}\lambda^{-4} \quad \text{for} \quad \lambda \ll 1. \tag{4}$$

On the other hand, for a sphere of radius R [1, 8]

$$\varepsilon_q(R) \approx \frac{3}{4\pi}\frac{0.27}{R^4}. \tag{5}$$

Both results are in qualitative agreement with the results shown in Fig. 1, if the sphere corresponds to a cube with edge length $2R$ as it did for gluons. We will assume that $a_g(\lambda)L^{-4}$ represents qualitatively correctly the total Casimir energy density. We plot and discuss the energy density and not the energy, following Ref. [1], where the number of bubbles is variable, while the total volume of all the bubbles is fixed. Thus the ground state corresponds to the minimum of the energy density. The result shown in Fig. 1 implies that for fixed λ the Casimir pressure acts outwards, and has a chance of compensating the interaction forces, only for values of λ not too far from one. Neglecting the quark contribution, only bubbles with $0.4 \leqslant \lambda \leqslant 3.4$ could be formed. The inclusion of quarks will change somewhat these limits, but the qualitative result is likely to remain valid. Besides changes at fixed λ, also changes of λ are possible. As seen from the figure a significant gain in Casimir energy density can be achieved by making λ small. On the other hand, if λ is large, it is more likely to increase than to decrease. Thus, unless $b(\lambda, L\Lambda)$ has an unexpectedly strong λ dependence, the bubbles will become flat, or needleshaped, and disappear, even if they are formed by some fluctuation. We conclude that the bubble structure is unstable. After this work has been completed, we have been informed [9] that parallelepipeds had also been studied with similar results by H. Hanson, K. Johnson and C. Peterson.

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