RELATIVISTIC, NONLOCAL EFFECTS AT LOW ENERGIES AND LOW MOMENTUM TRANSFER

By J. M. Namysłowski and P. Danielewicz

Institute of Theoretical Physics, Warsaw University*

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The relativistic effects due to the nonlocal character of the one particle exchange, derived from the light front field theory, are illustrated in a model of scalar nucleons exchanging a particle of mass $\mu = 138$ MeV. For the laboratory energy of the order of 0.1 GeV, and momentum transfer squared $|t| < (2\mu)^2$ we find a substantial change of the *shapes* of the phase shifts and the *shapes* of the differential cross section. These effects cannot be accommodated in an appropriate change of the coupling constant, if for the clarity of separating them we consider only one particle exchange as a driving force.

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To estimate the relativistic effects at low energies one would either compare the relativistic and nonrelativistic expressions for the kinetic energy, or use the smallness of the momentum transfer. For the laboratory energy E_{lab} of the order of 0.1 GeV, and scattering of particles with the nucleon mass, at low momentum transfer ($|t| < (2\mu)^2$, $\mu = 0.138$ GeV) one would expect negligible relativistic corrections. Indeed, on pure kinematical grounds, if $E_{lab} \in (0.02 \div 0.2)$ GeV, and q denotes the center of mass momentum, then the difference

$$[(q^2+m^2)^{1/2}-m]-\frac{1}{2}\,q^2m^{-1} \approx -\frac{1}{8}\,q^4m^{-3}$$

gives only an effect of $(0.3\div 3)\%$. Moreover, if forces are assumed to be local (at low energies), then for the small momentum transfer we expect to be far away from the inner region of interaction, where the relativistic effects are crucial, and therefore for the local forces we may have small relativistic corrections at low energies.

The aim of our paper is to see how large are these effects if we take such relativistic framework, which has exactly the same two-body free propagator as in the Lippmann–Schwinger equation, and in which the off-shell continuation of the potential is *uniquelly* defined by the field theory rules. Then, the answer to the question how large are the relativistic effects at low energies depends only on the non-local features of the driving force.

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

The relativistic scheme which we are using is the Weinberg [1] infinite momentum dynamics, written in terms of the light front [2] variables, which are then replaced by the projections of the four-momenta on tetrads (Vierbeins). This scheme was developed in Refs [3–7], and here we only mention its main properties: 1) all particles, including the intermediate one, are on their mass shells, while the off-shell continuation is in the "minus" component of the total momentum, 2) the rules of evaluating any diagram are similar to the old fashioned perturbation theory rules, with the replacement of the energy denominators by the "minus" component denominators, and with the new simplifying property that many of the old fashioned diagrams contribute zero (at each vertex there are conserved the perpendicular and the "plus" components of the total momentum), 3) the Weinberg equation is fully relativistic, nevertheless it is a three-dimensional integral equation, without the problem of the relative time (relative energy) variable, which arises in the Bethe–Salpeter equation, 4) the intermediate states correspond to the fixed number of particles, 5) the inverse of the two-body free propagator is a quadratic function of the relative momentum, allowing for the probabilistic interpretation of the relativistic bound state wave function, 6) there is a cancellation between the higher order irreducible diagrams (e.g. some parts of the ordinary box diagram are cancelled against the crossed box diagram), and 7) if we extend the scheme to the many body system, as it was done in Refs [4] and [6], then we have naturally the cluster decomposition property for the S matrix, arising from the additivity of the "minus" component momenta in the denominators evaluated within the Weinberg rules.

For the clear isolation of the relativistic effects at low energies, due to the nonlocality of the field theoretic potential, we consider a model of two scalar nucleons exchanging only one scalar meson of mass $\mu = 130$ MeV. Thus, the interaction Lagrangian is $2mg : \varphi^2\varphi_0 :$, where m is the nucleon mass, g is a dimensionless coupling constant, and φ, φ_0 describe the nucleon, meson fields, respectively. From this Lagrangian, the Weinberg rules, and the appropriate change of variables we get the driving force, corresponding to the one meson exchange in the Weinberg equation, as

$$\langle \bar{q}' \cos\theta' 0 | Y | \bar{q}'' \cos\theta'' \varphi'' \rangle = -2mg^2 M_0'^{-1/2}(A+\alpha)^{-1} M_0''^{-1/2}, \tag{1}$$

where $M_0 \equiv 2(\bar{q}^2+m^2)^{\frac{1}{2}}$, $A = (\vec{q}'-\vec{q}'')^2+\mu^2$, and $\vec{q}'$, $\vec{q}''$ are shorthand notations for the projections of the space-like relative four-momenta q' and q'' on the appropriate tetrads. We note, that the Fourier transform of $-g^2A^{-1}$ is the standard Yukawa interaction $-(g^2/4\pi)r^{-1}\exp(-\mu r)$. The essential new terms in Eq. (1) are the so-called "minimal relativity" factors $(M_0'M_0'')^{-\frac{1}{2}}$ and the term α in the denominator, which changes effectively the μ^2 term in the denominator of the Yukawa interaction. Using Refs [1], [4] and [7] we get from the Weinberg rules the following expression for α

$$\alpha = 2|q'M_0'^{-1}\cos\theta' - q''M_0''^{-1}\cos\theta''|\,(q'^2+q''^2-\tfrac{1}{2}s-2m^2)$$
$$-q'M_0'^{-1}q''M_0''^{-1}(M_0'-M_0'')^2\cos\theta'\cos\theta'', \tag{2}$$

and we note the highly nonlocal behaviour, manifesting itself through the separate dependence on q' and q'', as well as through the energy dependence in terms of the Mandelstam variable s. The "minimal relativity" factors $(M'_0 M''_0)^{-\frac{1}{2}}$ are also responsible for the nonlocal behaviour of Y. The entire effect of the nonlocality of the driving force Y, given by Eq. (1), can be separated out in a form factor F, which is defined by the equation

$$Y = -g^2 A^{-1} F. \tag{3}$$

From Eq. (1) and (3) we get

$$F = [1-\alpha(A+\alpha)^{-1}] 2m (M'_0 M''_0)^{-1/2}. \tag{4}$$

The presence of this form factor F influences various physical quantities, as it will be illustrated in our paper.

The Weinberg interaction Y is the kernel in the following two-body relativistic equation

$$\langle \bar{q}' \cos\theta' 0 | T | q\ 1\ 0 \rangle = \langle \bar{q}' \cos\theta' 0 | Y | q\ 1\ 0 \rangle$$
$$-(2\pi)^{-3} m \int \bar{q}''^2 d\bar{q}'' d\Omega'' \langle \bar{q}' \cos\theta' 0 | Y | \bar{q}'' \cos\theta'' \varphi'' \rangle$$
$$\times (\bar{q}''^2 - \bar{q}^2 - i\varepsilon)^{-1} \langle \bar{q}'' \cos\theta'' \varphi'' | T | \bar{q}\ 1\ 0 \rangle. \tag{5}$$

The scattering amplitude T is normalized in such a way, that the invariant differential cross section is given by

$$\pi^{-1} \bar{q}^2 d\sigma/dt = (\tfrac{1}{2} m)^2 (2\pi)^{-2} |\langle q' | T | q \rangle|^2 |_{\bar{q}' = \bar{q}},$$

where $\bar{q}^2 = \frac{1}{4}s - m^2$, and $t = (q'-q)^2$.

Solving numerically Eq. (5), with the driving force given by Eq. (1), for energies below and above threshold ($s = 4m^2$) we can find the effects of the relativistic form factor F on: the binding energy, partial wave phase shifts, and the differential cross section. The form factor F decreases the attractive Yukawa interaction $-g^2 A^{-1}$, lowering down the values of the binding energy, phase shifts, and differential cross section, if we keep the coupling constant fixed. In Ref. [7] we studied the relativistic effects on the binding energy, and on the S-wave phase shift for different values of the mass of the exchanged particle and the coupling constant. We found the following relativistic effects: for the S-wave phase shift from 10% up to 30%, and for the binding energy from 60% up to 90%.

Looking at the curves in Ref. [7] one is tempted to say, that an appriopriate readjustment of the coupling constant could completely accomodate the whole relativistic effect. In the present paper we disprove this expectation, by showing that the *shapes* of the phase shifts, and also the *shapes* of the differential cross section, for low energies and low momentum transfer, are very sensitive to the presence or absence of the relativistic form factor F. Only at one energy, and at one value of the momentum transfer we can get a zero effect by adjusting the coupling constant, but then at other energies and other values of momenta transfer we get substantial differences (roughly an order of magnitude larger than the kinematical estimates made at the begining of this paper). To show this we consider several examples, keeping the mass of the exchanged meson $\mu = 138$ MeV fixed, and making different choices for the coupling constant g. Although the real nucleon-nucleon force is approximated by several meson exchanges, with different masses and coupling constants, we feel that for the clear separation of the relativistic effects at low energies, due to the nonlocal character of the one-particle exchange derived from the field theory, it is more appropriate to consider a model of only one light meson exchange.

In our first example we take such g, that $-g^2 A^{-1}$ used as a driving force in the Lippman–Schwinger equation gives for the deuteron binding energy 2.224 MeV. Then, the presence of the form factor F lowers down that energy to the value 0.81 MeV.

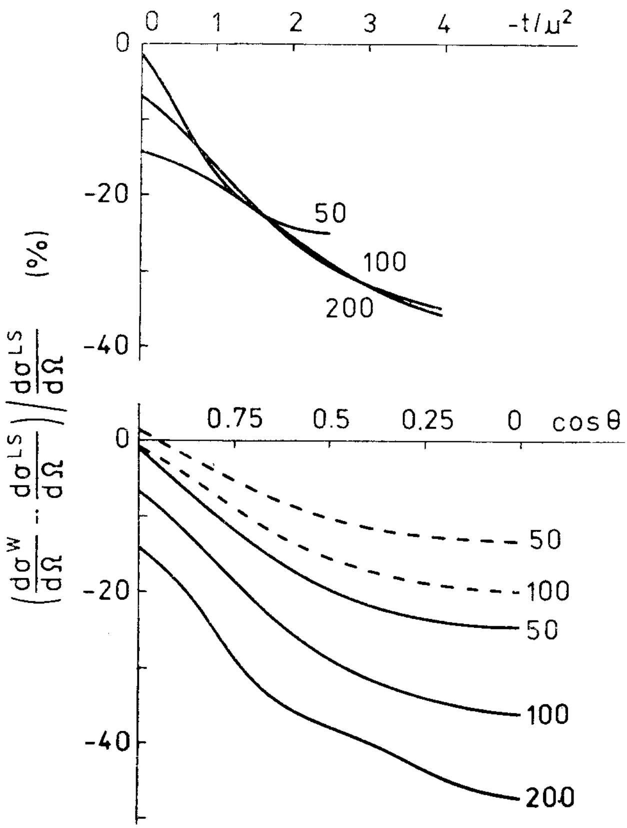

Fig. 1. The relative difference of the differential cross sections for the same value of the coupling constant. The numbers associated with curves denote the laboratory energy in MeV. The broken curves correspond to the local modification

The differential cross sections, evaluated with the F present, and F absent (denoted as $d\sigma^{W}/dt$ and $d\sigma^{LS}/dt$, wi*h W and LS for Weinberg, and Lippmann–Schwinger, respectively) differ largely as seen in Fig. 1. The largest difference reaches 40%. Note, that even for the highest considered $E_{lab} = 0.2$ GeV, the relativistic correction estimated from the kinematical argument is only 3%. The large size of the relativistic corrections in Fig. 1 is due to the nonlocality contained in the form factor F. To verify this point we consider a local modification of the attractive Yukawa force $-(g^2/4\pi)r^{-1}\exp(-\mu r)$ by the repulsive term $+0.2(g^2/4\pi)r^{-1}\exp(-3\mu r)$. Such correction modifies the original Yukawa interaction at $r = \mu^{-1}$ by 3% (like the kinematical correction), and by 20% near $r \approx 0$. The results induced by this local modification are shown in Fig. 1 as the broken curves. We notice that these effects are much smaller than those produced by the presence of the relativistic form factor F. Moreover, if we make the partial wave analysis, and plot in Fig. 2 the phase

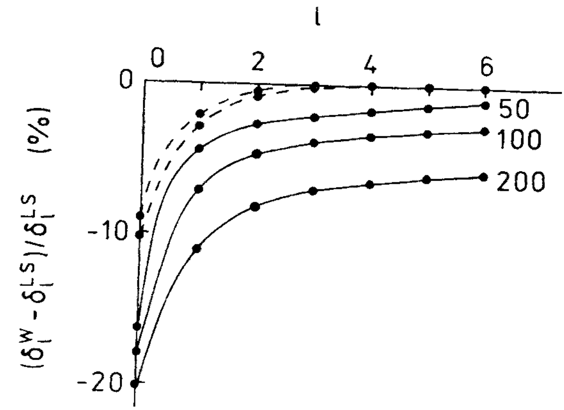

Fig. 2. The relative difference of the partial wave phase shifts as function of the angular momentum l, for the same value of the coupling constant. The numbers, and the broken curves as in Fig. 1

shifts as function of the angular momentum l, then we find, that for the D, F, G, H, and L waves the local modification gives practically no effect (as expected), while the presence of the form factor F makes a substantial change even for the highest partial wave. This again emphasizes the nonlocal character of the relativistic corrections produced by the form factor F.

In the second example we choose such coupling constants g and $\tilde{g}$, that both $-g^2A^{-1}$ and $-\tilde{g}^2A^{-1}F$ give the same deuteron binding energy, equal to 2.224 MeV. Then, the S wave phase shift evaluated in the Weinberg scheme is still lower than in the Lippmann–-Schwinger equation, but the higher wave phase shifts are larger for the W than for the LS case, due to the increase of the coupling constant. Note that $\tilde{g} > g$ is necessary to overcome the action of the form factor F, and to bring the deuteron binding energy to the value 2.224 MeV. The final result is shown in Fig. 3, where at a particular value of the momentum transfer we get a zero effect. However, the slopes of the curves in Fig. 3 are comparable with the slopes of the appropriate curves in Fig. 1. This shows, that the presence or absence of the form factor F makes an essential effect on the *shape* of the differential cross section, and therefore the relativistic effects can not be accomodated in the value of the coupling constant.

In our third example we take such g and $\tilde{g}$, that the S wave phase shift (in which the effect is the strongest) are forced to be equal at $E_{lab} = 0.1$ GeV for both driving forces $-g^2A^{-1}$, and $-\tilde{g}^2A^{-1}F$. The coupling constant g is such, that the binding energy of deuteron evaluated with the driving force $-g^2A^{-1}$ is equal to 2.224 MeV. Then, already at

E_{ab} = 0.05 GeV, and at E_{lab} = 0.2 GeV the S wave phase shifts differ by +3%, and −3%, respectively. The P, D, F, G, H and L wave phase shifts at E_{lab} = 0.1 GeV differ by 20%, and the forward differential cross sections differ by 27%. The deuteron binding energy evaluated with the driving force having the form factor F would have to be 47%

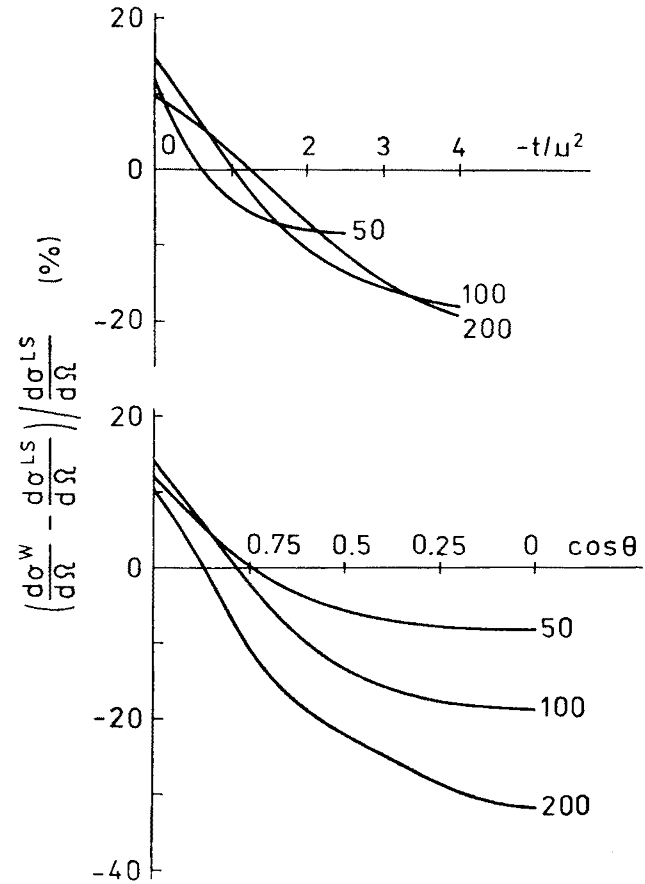

Fig. 3. The relative difference of the differential cross sections for such g and $\tilde{g}$, that both $-g^2A^{-1}$ and $-\tilde{g}^2A^{-1}F$ bind deuteron at 2.224 MeV

larger than 2.224 MeV if the S wave phase shifts are required to be the same at E_{lab} = 0.1 GeV.

All of the above examples show that the relativistic effects caused by the form factor F are much larger than either the simple effects estimated on pure kinematical grounds, or the effects due to a local modification of the original interaction. The whole range of energies and momentum transfer is influenced by the presence of the form factor F, because of its nonlocal character. The *shapes* of the phase shifts and the differential cross sections are sensitive to the presence or absence of the form factor F, and working even at low energies and low momentum transfer we cannot incorporate the substantial relativistic corrections in the value of the coupling constant.

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