CONFIGURATION OF A NEUTRON STAR WITH AXIALLY POLARIZED SPIN

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It is shown in the framework of the affine theory of gravitation that the surface of a neutron star differs slightly from sphere because of the contact spin-spin interaction of polarized neutrons.

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Among reasons leading to the deviation of the form of a neutron star from sphere Kerlick in his paper [1] indicated the neutrons' spin axial polarization originating in the intensive magnetic field. He also estimated the difference between the longitudinal and transversal (with respect to the direction of the polarization) dimensions of a standard neutron star, basing his considerations on the results obtained by Wald [2].

In our opinion the way of the investigation undertaken in [1] is not sufficiently correct, because Wald in his work [2] considered spin–gravitational interaction using for different special cases the Papapetrou equations [3]. Solutions of the equations describe the motion of test particles with spin or the force affecting them in the gravitational field of non-test bodies which can also possess their proper angular momentum. Collective effects of spin–spin interaction are described in the most logical way by the U_4 theory of gravitation, in which the spin of matter is considered to be the source of the space-time torsion. So we propose to consider the problem in the framework of the U_4 theory.

The exact solution of the self-consistent system of field equations and equations of motion and state would be the most interesting case. But such a solution is not known so we shall consider the equations in the approximation of weak gravitational and torsional fields. Moreover, since pure torsional effects are in question, it seems natural to abandon rotational non-static terms in the metric describing gravitational field inside the star. In the Appendix we show that these assumptions and physical conditions of the chosen model are not contradictory.

Let the star consist of incompressible ideal fluid with spin. Then its energy-momentum tensor is

$$\eta T^{ik} = v^k[(\varepsilon+p)v^i\eta+v_jDs^{ij}]-\eta pg^{ik} \tag{1}$$

and axial polarization of spin $S^{ij} = 2S\delta_1^{[i}\delta_2^{j]}$ determines the gravitational field

$$ds^2 = e^{2\tau(\vec{r})}dt^2 - e^{2\lambda(\vec{r})}(dx^2+dy^2+dz^2). \tag{2}$$

Here $\varepsilon = \text{const}$, $S = \text{const}$ and p are respectively the energy density, spin density and pressure, $s^{ij} = S^{ij}v^k\eta_k$, η_k is the 3-form dual to the basic 1-form θ^k, η is the element of 4-dimensional volume, D is the operator of the exterior covariant derivative (all notations correspond to those of paper [4], see also [5]). The vacuum velocity of light is chosen to be equal to unity.

The configuration of the star determined by the surfaces of equal pressure follows from the equations of motion of the fluid

$$D(T^{ik}\eta_k) = C_{kj}{}^{i}T^{mk}\theta^j \Lambda \eta_m + \tfrac{1}{2} R_{klj}{}^{i}\theta^j \Lambda s^{kl}, \tag{3}$$

which generalize for the U_4 manifolds the Papapetrou equations. Here $C_{kj}{}^{i} \neq 0$ is the torsion tensor. The algebraic connection between the torsion and spin density of the source assume in the natural form $C^i{}_{jk} = \chi\, v^i\, S_{jk}$; χ is the spin–torsion coupling constant, 4-velocity vector $v^i = \delta^i_0$ for the particles of matter being at rest in the chosen frame of reference. Calculating the connection 1-form and the U_4 curvature tensor components we find from Eq. (3)

$$p_{,A} + \tau_{,A}(\bar{\varepsilon} + \bar{p} + \chi S^2) = 0, \tag{4a}$$

$$p_{,3} + \tau_{,3}(\bar{\varepsilon} + \bar{p}) = 0, \tag{4b}$$

where

$$A = 1, 2; \quad \bar{p} = p - \frac{\chi S^2}{4}, \quad \bar{\varepsilon} = \varepsilon - \frac{\chi S^2}{4}.$$

Note, that the difference in form of Eqs (4a) and (4b) is due to the first term of the right hand side in Eq. (3), i.e. the deviation from sphere in this case is the result of the spin–spin torsional interaction, but not of the spin–gravitational interaction which is described in GR by the last term of Eq. (3); as a matter of fact the latter was used in [1].

Near the surface of the star $p \ll \varepsilon$; after the substitution of the expression for τ (see the Appendix) Eqs (4) have the solution

$$p = p_0 - \frac{\kappa}{12}\varepsilon^2 \left\{ \left(1 + \frac{\chi S^2}{2\varepsilon}\right)(x^2+y^2) + \left[1 - \frac{\chi S^2}{2\varepsilon}\left(1 + \frac{3}{2}\frac{\chi}{\kappa}\right)\right] z^2 \right\}, \tag{5}$$

which means that the surfaces of equal pressure are the z-prolonged rotational ellipsoids. Determining the constant of integration p_0 at the pole ($p = 0$, $x = y = 0$, $z = z_0$) we find the expressions for the coordinate values of semi-axis

$$x_0 = \left[1 - \frac{\chi S^2}{4\varepsilon}\left(1 + \frac{3}{2}\frac{\chi}{\kappa}\right)\right] R, \quad z_0 = \left[1 + \frac{\chi S^2}{4\varepsilon}\left(1 + \frac{3}{2}\frac{\chi}{\kappa}\right)\right] R, \tag{6}$$

where $R = \frac{1}{2}(x_0+z_0)$ is the mean radius. Eq. (6) yields the formula for the deviation of the shape of the star from sphere

$$\delta l = 2\left(\int_0^{z_0} e^{\lambda} dz - \int_0^{x_0} e^{\lambda} dx\right) \simeq 2(z_0-x_0)+\frac{\chi^2 S^2}{6} R^3. \tag{7}$$

For the neutron star we have $S = \frac{\hbar}{2}\frac{\varepsilon}{m_N}$ with $m_N \simeq 10^{-24}$ g (mass of a neutron); then, introducing the velocity of light in explicit form, we obtain the final expression

$$\delta l = \frac{1}{8}\frac{\chi\varepsilon}{c^4 m_N^2} \hbar^2 R\left(1+\frac{3}{2}\frac{\chi}{\kappa}+\frac{\chi\varepsilon}{6c^2} R^2\right). \tag{8}$$

If we choose the neutron star with typical parameters: mass $\sim 10^{33}$ g, $R \sim 10^6$ cm, then under the assumption of maximal spin polarization in the framework of the Einstein–Cartan theory, where the torsion–spin relation is determined by the Einstein gravitational constant ($\chi = \kappa$), we have $\delta l \simeq 10^{-34}$ cm, which surprisingly is of the same order as the estimate given in [1]. But there is also another possibility. Let $\delta l \simeq 10^3$ cm, so that the gravitational radiation would become the main reason of the energy loss of the star resulting in deceleration of its rotation [6]. In this case the value of the torsional–spin constant becomes $\chi \simeq 10^{12}$ g^{-1} cm^{3}s^{-2}, which still satisfies the used weak field approximation and is 20 orders less than the similar coupling constant of the theory of strong gravity: $\sim 10^{32}$ g^{-1} cm^3 s^{-2} (see e.g. [7]).

APPENDIX

1. In the computations we needed:

a) the basic 1-forms

$$\theta^0 = e^{\tau}dt, \quad \theta^1 = e^{\lambda}dx, \quad \theta^2 = e^{\lambda}dy, \quad \theta^3 = e^{\lambda}dz,$$

b) the components of the connection 1-form

$$\omega_{01} = \tau_{,1}e^{-\lambda}\theta^0 - \frac{\chi}{2}S\theta^2, \quad \omega_{12} = \frac{\chi}{2}S\theta^0 - \lambda_{,2}e^{-\lambda}\theta^1 - \lambda_{,1}e^{-\lambda}\theta^2,$$

$$\omega_{02} = \tau_{,2}e^{-\lambda}\theta^0 + \frac{\chi}{2}S\theta^1, \quad \omega_{13} = -\lambda_{,3}e^{-\lambda}\theta^1 + \lambda_{,1}e^{-\lambda}\theta^3,$$

$$\omega_{03} = \tau_{,3}e^{-\lambda}\theta^0, \quad \omega_{23} = -\lambda_{,3}e^{-\lambda}\theta^2 + \lambda_{,2}e^{-\lambda}\theta^3,$$

c) the components of the curvature tensor

$$R_{1201} = -\chi e^{-\lambda}(\tfrac{1}{2}S_{,1}+S\tau_{,1}), \quad R_{1202} = -\chi e^{-\lambda}(\tfrac{1}{2}S_{,2}+S\tau_{,2}),$$

$$R_{1203} = -\chi e^{-1}(\tfrac{1}{2}S_{,3}+S\tau_{,3}),$$

d) the U_4 theory field equations

$$-R_{00} \simeq \Delta\tau \simeq \tfrac{1}{2}(\kappa\varepsilon - \chi^2 S^2),$$

$\Delta\lambda \simeq -\frac{1}{2}\kappa\varepsilon$ which is the $R_{\alpha\alpha}$, $\alpha = 1, 2, 3$, when $\tau \simeq -\lambda$ ($\chi^2 S^2 \ll \kappa\varepsilon$); the simplest non--spherical solution of these equations is

$$\tau = -\lambda = \frac{\kappa}{12}\varepsilon(x^2+y^2) + \left(\frac{\kappa}{12}\varepsilon - \frac{\chi^2 S^2}{4}\right) z^2.$$

2. Possibility of using the weak field method. The expression for τ in the interior of the star is

$$\tau \simeq \frac{\kappa}{12c^2}\varepsilon r^2 - \frac{\chi^2 S^2}{4c^6} z^2.$$

Near the surface of the star $\tau \simeq 10^{-3} - \chi^2 \cdot 10^{-40}$ i.e. it is a rather small quantity if $\chi \ll 10^{20}$ g^{-1} cm^3 s^{-2}. It is easy to show that all derivatives of τ are also small at the surface. The same can be shown for λ.

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