AN ANALYSIS OF THE REACTIONS $\pi^+ p \rightarrow K^+ \Sigma^+$ AND $K^- p \rightarrow \pi^- \Sigma^+$ *

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Differential cross-section and polarisation data for the pair of line reversed reactions $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$ is analysed in the range $p_{lab} \geqslant 7$ GeV/c and $|t| \leqslant 1$ (GeV/c)^2 using non degenerate $K^* - K^{**}$ trajectories. The K^* residues are taken to be peripheral in impact-parameter space while the K^{**} residue is taken as non-peripheral in nature. It is found that, contrary to popular belief, a satisfactory fit to the data can be obtained without invoking Regge cut terms. Comparison of our results with several models is made.

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Introduction

The pair of line reversed reactions $\pi^+ p \rightarrow K^+ \Sigma^+$ and $K^- p \rightarrow \pi^- \Sigma^+$ has been the subject of keen phenomenological scrutiny over the past decade. The theoretical analyses of these reactions involve, among others, simple Regge pole parametrisations [1], dual absorptive model based investigations [2] and rather tedious pole+cut parametrisations [3, 4] as well as amplitude analyses of a general nature utilising a variety of ideas as well as SU(3) constraints [5]. There are several reasons for theoretical interest in these reactions. These reactions are among the very few measured processes involving the mesonic hypercharge exchanges K^* and K^{**}. Quite naturally therefore they constitute an important source of information on the trajectories and residues of these exchanges. An extremely important reason for interest in these reactions resides in the interesting implications which the additional assumption of EXD of K^*–K^{**} exchanges has for these reactions. In its strong form K^*–K^{**} EXD with respect to these reactions leads to the rather interesting prediction of zero polarisations for both reactions and to the equality of their differential cross-sections at common energies and momentum transfers. The less restrictive assumption of weak K^*–K^{**} EXD however, while still predicting equal differential cross-sections, generates nonzero but mirror symmetric polarisations for these reactions. These implications of EXD for these reactions were first pointed out by Gilman [6]. Yet another reason

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for interest in these reactions lies in the information they furnish on the extent to which SU(3) constraints are satisfied in hypercharge exchange reactions since SU(3) relates the amplitudes for these reactions to those for $\pi^- p \to K^0\Lambda$, $K^- p \to \pi^0\Lambda$, etc. Furthermore SU(3) also relates the amplitudes involving K^*–K^{**} exchange to amplitudes of processes in which the ρ and A_2 are exchanged. These latter relationships have led Martin et al. [5] to formulate interesting relations between CEX and hypercharge exchange reaction polarisations. In view of the preceding remarks it is not surprising that this pair of line reversed reactions has received so much theoretical attention inspite of a comparative scarcity of data.

The main features of the experimental data for $p_{lab} \geqslant 7$ GeV/c can be summarised as follows [7, 8]:

(*i*) The $\frac{d\sigma}{dt}$ data for both reactions is peaked in the forward direction. This can be interpreted as implying the dominance of the nonflip amplitude in the smaller $|t|$ region (say $|t| \lesssim 0.4\,(\text{GeV}/c)^2$).

(*ii*) The differential cross-sections have a mild structure in the form of a shoulder around $|t| \simeq 0.5\,(\text{GeV}/c)^2$. This shoulder is present even in the 70 GeV/c data for $\pi^+ p \to K^+\Sigma^+$ (measurements for its line reversed partner have not so far been made at this energy). There is no definite agreement on the mechanism to which the origin of this structure may be attributed — the structure producing mechanism is generally model dependent.

(*iii*) Associated with the onset of structure in $\frac{d\sigma}{dt}$ around $|t| \simeq 0.5\,(\text{GeV}/c)^2$ is a marked change in the slope.

(*iv*) The $\frac{d\sigma}{dt}$ data for $p_{lab} \geqslant 7$ (GeV/c) indicates that the differential cross-sections for the two reactions, though not exactly equal, are rather close.

(*v*) Polarisation measurements for $p_{lab} \geqslant 7$ GeV/c are consistent with mirror symmetry (although the errors on the data do permit violations of exact mirror symmetry).

Features (*iv*) and (*v*) of the data have generally been interpreted as implying approximate weak EXD of K^* and K^{**} trajectories. This interpretation owes its existence to the general belief that the equality of differential cross-sections and mirror symmetry of polarisations in these reactions can be attained only by weak K^*–K^{**} EXD considerations. It has only been pointed out very recently by Saleem et al. [9] that it is possible to obtain equal differential cross-sections and mirror symmetric polarisations for this line reversed pair even when weak K^*–K^{**} EXD is broken. These features of the data can in fact be reproduced by associating one of the two trajectories with one of the two amplitudes and the other trajectory with the remaining amplitude in each process. The line reversal properties of K^* and K^{**} exchanges then automatically yield equal differential cross--sections and mirror symmetric polarisations regardless of what equations one uses for the corresponding trajectories. From this point of view the departure of differential cross--sections from equality and of polarisations from exact mirror symmetry can be attributed to the "contamination" of either one, or both, of the amplitudes by the otherwise non-contributing trajectory. In the course of this note we will exploit this observation in fitting P and $\frac{d\sigma}{dt}$ for both reactions in the region $p_{lab} \geqslant 7$ GeV/c and $|t| \leqslant 1$ (GeV/c)^2. We will show that it is indeed possible to fit the data in the aforementioned region by using non--degenerate trajectories alongwith a phenomenological choice of residue functions.

Parametrisation

The reactions $\pi^+ p \to K^+\Sigma^+$ and $K^- p \to \pi^-\Sigma^+$ are processes of the type $0+{}^1/_2 \to 0+{}^1/_2$. Consequently there are two independent amplitudes for each of these reactions corresponding to no helicity flip and helicity flip. However the two amplitudes for one reaction are related by line reversal symmetry and isospin considerations to the corresponding amplitudes for the other reactions. Denoting the nonflip amplitude by T_0 and the flip amplitude by T_1 these relations can be written as

$$T_0(K^-p) = -\frac{q_{K^-p}}{q_{\pi^+p}} T_0(\pi^+p), \tag{1}$$

$$T_1(K^-p) = \frac{q_{K^-p}}{q_{\pi^+p}} T_1(\pi^+p), \tag{2}$$

where the amplitudes on the LHS refer to the reaction $K^-p \to \pi^-\Sigma^+$ and the amplitudes on the RHS to the reaction $\pi^+p \to K^+\Sigma^+$.

Let us now turn to the parametrisation of $T_0(\pi^+p)$ and $T_1(\pi^+p)$. We will assume that the amplitude $T_0(\pi^+p)$ is dominated in the region of the diffraction peak by K** exchange with a minor K* contribution. To simplify matters the flip amplitude T_1 shall be assumed to receive a contribution only from K* exchange. In such a parametrisation the difference in the $\frac{d\sigma}{dt}$ values for the two reactions will arise from the presence of a K* contribution in T_0. Thus we have:

$$T_0(\pi^+p) = \gamma_T^{(0)}(t)\xi_+(t)\left(\frac{s}{s_0}\right)^{\alpha_T(t)} + \gamma_V^{(0)}(t)J_0(R\sqrt{-t})\xi_-(t)\left(\frac{s}{s_0}\right)^{\alpha_V(t)}, \tag{3}$$

$$T_1(\pi^+p) = \gamma_V^{(1)}(t)J_1(R\sqrt{-t})\xi_-(t)\left(\frac{s}{s_0}\right)^{\alpha_V(t)}, \tag{4}$$

where the tensor trajectory K** has been taken as non-peripheral in nature. The subscripts V and T label quantities associated with the K* and K** exchanges respectively. It may also be noted that we have assumed that the phase-energy relation is valid even for non-flip helicity amplitude. The fact that phase-energy relation is not violated for non-flip amplitudes is also supported by the measurements of differential cross sections for the reaction $K_L^0\, p \to K_S^0 p$ between 4 and 14 GeV/c in the range $0.1 \leqslant -t \leqslant 2$ (GeV/c)^2 [10]. Absorbing

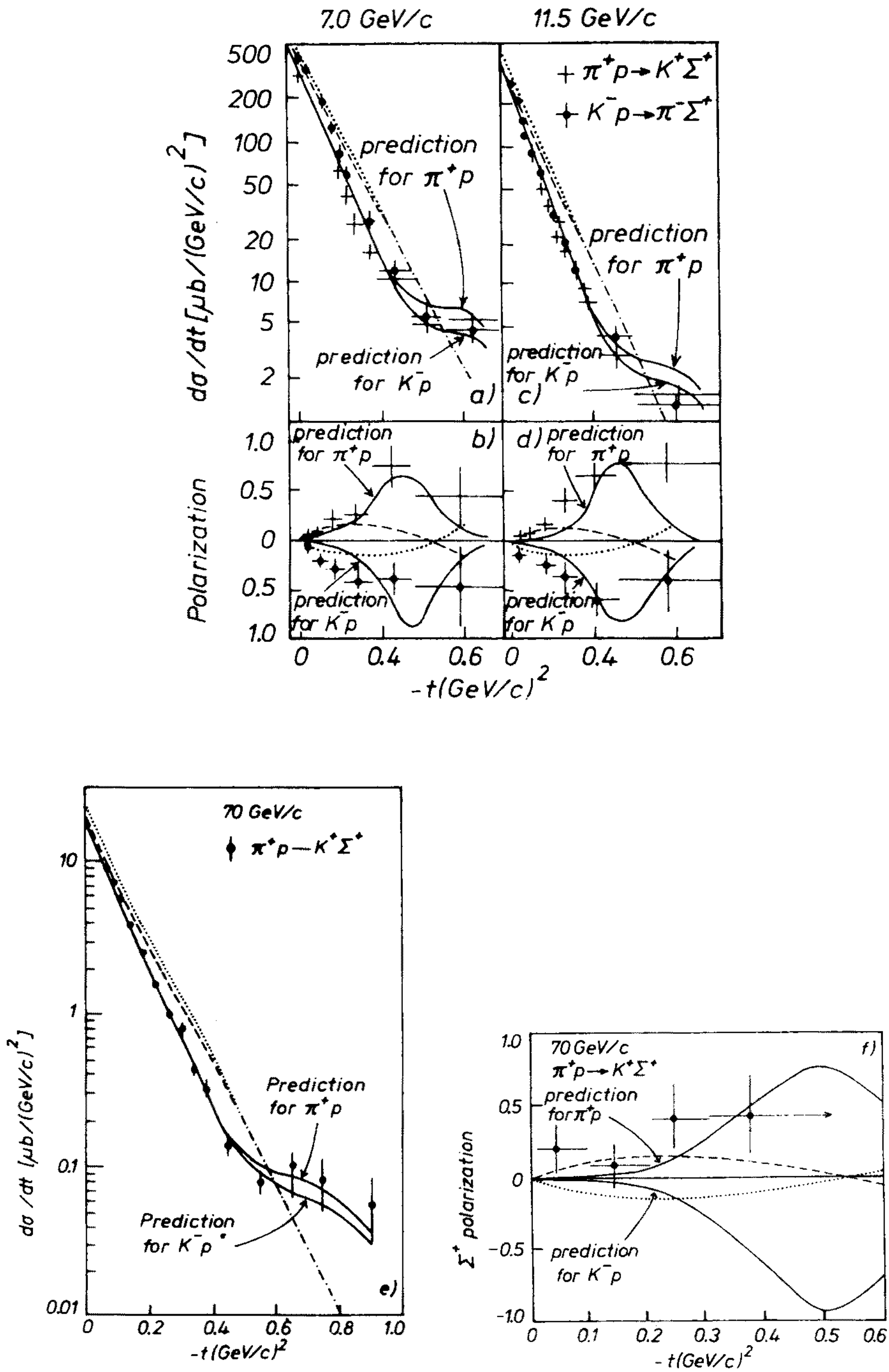

Fig. 1a-f. Fits to differential cross-section and polarization data at various momenta. Our fit is shown with solid lines while the dashed and dotted curves are the fits of Ward (Ref. [4])

the functions $\sin\frac{\pi\alpha_T}{2}$ and $\cos\frac{\pi\alpha_V}{2}$ occurring in the denominators of signature factors $\xi_+(t)$ and $\xi_-(t)$ respectively in the residues, we find that good agreement with data is obtained by the following phenomenological choice of the various residues:

$$\gamma_T^{(0)}(t) = 49.31e^{2.44t},$$

$$\gamma_V^{(0)}(t) = \gamma_V^{(1)}(t) = 0.11e^{-13.32t-7.6t^2}.$$

As usual $s_0 = 1\ (\mathrm{GeV}/c)^2$. The trajectory equations have been taken as

$$\alpha_T(t) = 0.375+0.678t,$$

$$\alpha_V(t) = 0.485+0.6t.$$

The use of these nondegenerate trajectories has been necessitated by the somewhat different energy dependence of the reactions in the $|t| \lesssim 0.4\ (\mathrm{GeV}/c)^2$ region and the region beyond. The energy dependence for $|t| \lesssim 0.4$ is given by the K^{**} trajectory primarily while further

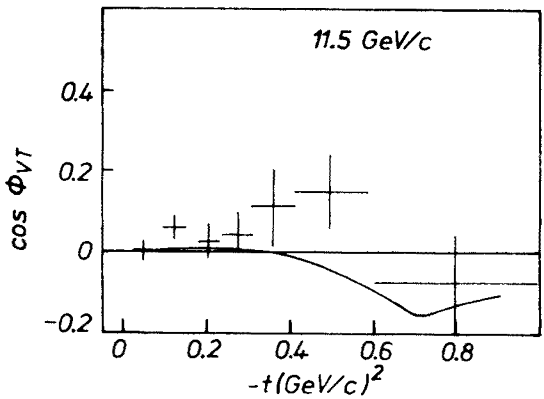

Fig. 2. Our prediction (solid curve) for $\cos\phi_{VT}$ at 11.5 GeV/c. Data from Ballam et al. [14]

out in $|t|$ the K^* contributions take over. Comparison between experiment and theory has been shown in Figs. 1a, c, e. Data was taken from Refs. [11, 12]. Our model also yields a very good agreement with the polarisation data as can be seen from Fig. 1b, d, f. The polarisation data was taken from Ref. [11, 12]. In Fig. 2 we also show a comparison of our calculated values for $\cos\phi_{VT}$ against experimental data [13] where

$$\cos\phi_{VT} = \frac{\frac{d\sigma}{dt}(K^-p) - \frac{d\sigma}{dt}(\pi^-p)}{\frac{d\sigma}{dt}(K^-p) + \frac{d\sigma}{dt}(\pi^+p)}. \tag{5}$$

Our theoretical values are not inconsistent with the data.

Discussion

The reactions $K^-p \to \pi^-\Sigma^+$ and $\pi^+p \to K^+\Sigma^+$ have been analysed by several groups over the past few years (Ref. [7, 8] constitute excellent reviews of the situation concerning these reactions). An important investigation of these reactions was conducted by Navelet and Stevens [15] in the light of the Regge model. They used a parametrisation in which the K*–K** contributions were weakly EXD. Their parametrisation contained, in addition to K*–K** Regge pole terms, Regge cuts as well. However comparison with the sum of

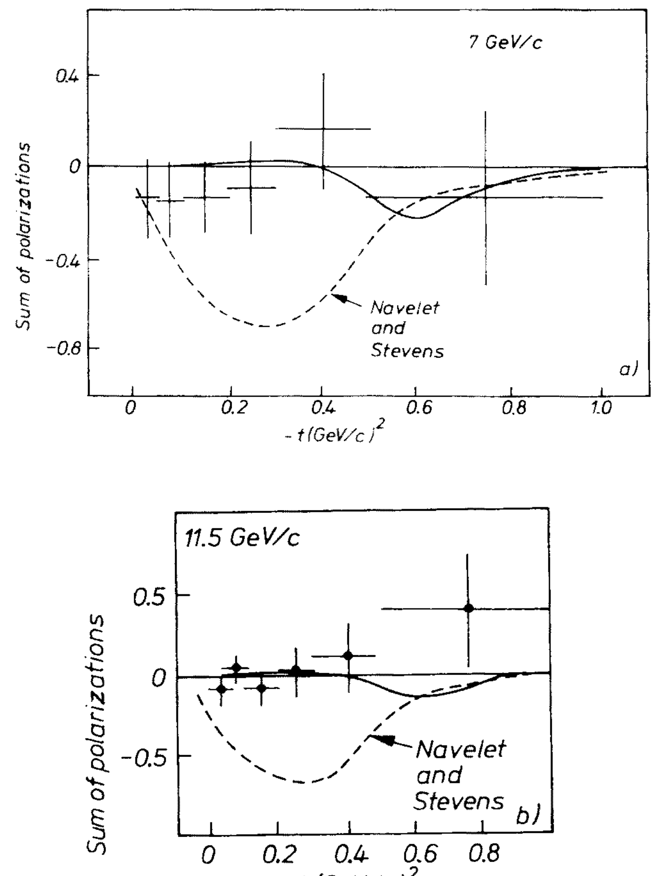

Fig. 3a, b. Our predictions (solid curves) for $P(K^-p)+P(\pi^+p)$ compared with those of Navelet and Stevens [16] (dashed curves) at 7 and 11.5 GeV/c

polarisations of the two reactions indicates severe qualitative disagreement with the data. In Fig. 3a, b we show a comparison of our results with those of Navelet and Stevens for $P(K^-p)+P(\pi^+p)$ at 7 and 11.5 GeV/c. Here we would also like to point out that the total number of free parameters used by Navelet and Stevens is 21. Ward [4] has recently generalised the model of Navelet and Stevens in an attempt to obtain better agreement with data. However the agreement between the parametrisation of Ward [4] and the experimental data is not satisfactory except for very small values of $|t|$. Thus for instance the model predicts zero polarisation for both reactions around $|t| \simeq 0.6\,(\mathrm{GeV}/c)^2$ while experimentally the polarisation for both processes is found to have a large magnitude at this t-value. Similarly the theoretical differential cross-sections in the parametrisation of Ward [4] deviate substantially from the experimental values for $|t| \lesssim 0.3\,(\mathrm{GeV}/c)^2$. This can be noted from Fig. 1a-c where a comparison of our fit and that of Ward [4] with the differential cross-section data is shown. The comparison with polarisation data is shown in Fig. 1b, d, f. Loos and Mathews [2] have attempted to analyse these reactions using the DAM ansatz for the *imaginary* parts of the amplitudes with the flip phases given by the Regge pole phase rule. However the non-flip phases are parametrised as polynomials in t instead of being given by the usual Regge pole phase rule. The resulting amplitudes however do not agree with the $\frac{d\sigma}{dt}$ data for $K^-p \to \pi^-\Sigma^+$ for $|t| \geqslant 0.2\,(\mathrm{GeV}/c)^2$. It should also be emphasised that 13 parameters were used in their parametrisation and Loos and Mathews [2] inserted an additional parameter (apart from the trajectory) in the power of s to obtain the correct energy dependence. Very recently Saleem et al. [9] have parametrised the amplitudes for these reactions using the simple Regge pole model. In their model the non-flip amplitude contains only K^{**} exchange and the flip amplitude only K^* exchange. The trajectories used were non degenerate and the residues were smooth functions of t. This parametrisation is in good agreement with data. There is little difference between the $\frac{d\sigma}{dt}$ values of our present parametrisation and that of Saleem et al. [9]. However while the parametrisation of Saleem et al. [9] yields exactly equal differential cross-sections for the two reactions our, parametrisation yields somewhat different cross-section values for the two reactions the difference becoming perceptible for $|t| \geqslant 0.6\,(\mathrm{GeV}/c)^2$ as may be noted from Fig. 1a, c, e. The polarisations in the parametrisation of Saleem et al. [9] however are mirror symmetric while in our case there is some departure from exact mirror symmetry. The data is consistent with both the models although our model gives a somewhat better agreement with polarisation. More accurate measurement in the $|t| \geqslant 0.5\,(\mathrm{GeV}/c)^2$ region might permit a choice between the two models.

There has been a controversy regarding the nature of tensor exchanges. Minami and Terada [14] have analysed the reactions $K^-p \to \eta\Lambda(\Sigma)$ in which K^* and K^{**} are exchanged and have stated that K^{**} may be non-peripheral in nature although, according to them, there is a possibility that K^* and K^{**} amplitudes with peripheral structure may interfere destructively and lose the dominance of the peripheral impact parameters. Barger et al. [16] have asserted that contrary to the conclusions of Harari's dual absorptive model tensor-exchange amplitude has non-peripheral structure. Girardi et al. [17–19] have discussed this point in detail and have opined that the tensor (A_2, f, K^{**}) exchange amplitudes have universal bahaviour i.e. they only depend on the reaction through their coupling, their structure being determined by their helicity content and the nature of exchange. Our analysis supports the conclusion drawn by these authors.

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