"UP-DOWN" AMBIGUITY FOR $\pi\pi$ S-WAVE PHASE SHIFTS IN THE REACTION $\pi^- p \rightarrow \pi^+\pi^- n$ at 63 GeV/c

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Production mechanism of the reaction $\pi^- p \rightarrow \pi^+\pi^- n$ at 63 GeV/c is analysed in the ϱ-mass region of the dipion mass. Arguments favouring "up" solution for S-wave $\pi\pi$ phase shift, corresponding to the low mass isoscalar resonance ε(800), are presented.

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1. *Introduction*

The main interest in the $\pi\pi$ phase shift analysis below 1 GeV has been in clarifying the $I = 0$ S-wave and the question of scalar resonances. An answer to it would have important bearing on the problem of four-quark states predicted in QCD [1]. A few years ago it seemed as though a general consensus had been reached in this matter [2]. Out of two physical solutions of set of equations relating observables to the S and P wave dipion production amplitudes only the so-called "down" solution has been accepted [3]. Its characteristics are the following

(*i*) Phase coherence between P^0 and P^-;

(*ii*) the S wave in ϱ region does not show any resonant behaviour.

The other, i.e. the "up" solution is characterized by relatively narrow ε having mass and width similar to that of the ϱ. P^0 and P^- are not coherent in phase which is a complicated function of momentum transfer. Results of $\pi^- p \rightarrow \pi^0\pi^0 n$ showing no evidence for narrow ε under ϱ were essential in rejecting this solution. However, the possibility that a narrow ε resonance exists approximately degenerate in mass and width with ϱ has been reconsidered recently by Donohue and Leroyer [4]. They show that the data on $\pi^+\pi^-$ production near the ϱ mass, including full set of joint moments in $\pi^+ p \rightarrow \pi^+\pi^-\Delta^{++}$ as well as the moments obtained with a polarized target in $\pi^- p \rightarrow \pi^+\pi^- n$, are consistent with the existence of such resonance. They observe also that only direct experimental evidence against it is the $\pi^0\pi^0$ mass spectrum of Ref. [5]. Considering the compilation of $\pi^0\pi^0$ results presented in Ref. [6] this evidence cannot be taken too seriously.

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In this paper we present a support for the Donohue and Leroyer point of view. This results from an analysis of production mechanism in the reaction $\pi^- p \to \pi^+\pi^- n$ at 63 GeV/c and energy dependence of the phase coherent solutions of the partial wave equations analysed as a function of $\pi\pi$ effective mass.

We analyse two subsets of a sample of about 230000 events of the reaction $\pi^- p \to \pi^+\pi^- n$ measured with a magnetic forward spectrometer set up in an unseparated π^- beam with a momentum of 63 GeV/c at SPS (WA3 experiment, Collaboration ACCMOR, Refs. [11, 12]). The first subset consists of $\sim$ 38000 events in ϱ mass region of the dipion mass $0.71 \leqslant m_{\pi\pi} \leqslant 0.83$ GeV with the momentum transfer below 1 GeV. The second subset consists of $\sim$ 55000 events with low momentum transfer $|t| < 0.2$ GeV in the dipion mass region from threshold to 1 GeV.

For the purposes of our analysis we use angular distribution moments of the dipion system in 13 bins of the momentum transfer (first subset) or in 20 MeV bins of the dipion effective mass (second subset).

We show that only solution analogical to the Estabrooks and Martin [3] "up" solution giving narrow ε under ϱ is consistent with the cut energy dependence observed in Fermilab energy pn charge exchange (CEX) data as well as the Regge pole phenomenology. Namely, we show that at 63 GeV/c the "down" (phase coherent) solution which does not show any resonant structure below 1000 MeV corresponds to t dependence of production amplitudes predicted by the Williams model [8]. As shown in Ref. [10] this model fails above 30 GeV in description of pn CEX data, and inclusion of a large A_2-cut contribution is necessary. On the other hand, simple Regge pole parametrization with complex $\pi\otimes P + A_2\otimes P$ cut seems to describe "up" production amplitudes at 63 GeV/c fairly well. We show also that "down" (phase coherent) solution at 63 GeV/c shows differences in relation to analogous solutions at 17 GeV/c whereas "non-coherent" solution does not depend on energy and therefore should be favoured.

2. *S and P wave production amplitudes at 63 GeV/c*

Let us remind the Estabrooks and Martin [3] method for amplitude analysis of the process $\pi^- p \to \pi^+\pi^- n$ in the mass region dominated by S and P dipion production amplitudes. Unnatural parity exchange amplitudes are assumed to be spin-coherent, in fact to be pure flip in the s-channel as expected from absorbed one pion exchange (OPE)[1]. These assumptions imply

$$\frac{d\sigma}{dt} = |S|^2+|P_0|^2+|P_+|^2+|P_-|^2,$$

$$\sqrt{5\pi}\,\langle Y_0^2\rangle = |P_0|^2-\tfrac{1}{2}(|P_+|^2+|P_-|^2),$$

[1] This assumption is violated by the A_1-exchange, known to be present at 17 GeV/c, however, its effects do not change salient features of solutions obtained in a simplified pure flip model [7]. Moreover, we can expect less A_1 at 63 GeV/c than was found in 17 GeV/c data.

$$-\sqrt{\frac{10\pi}{3}}\langle Y_2^2\rangle = \tfrac{1}{2}(|P_+|^2-|P_-|^2),$$

$$\sqrt{\frac{10\pi}{3}}\langle Y_1^2\rangle = |P_-|\,|P_0|\cos\varphi,$$

$$\sqrt{\pi}\,\langle Y_0^1\rangle = |P_0|\,|S|\cos\Delta,$$

$$\sqrt{2\pi}\,\langle Y_1^1\rangle = |P_-|\,|S|\cos(\varphi-\Delta). \qquad (2.1)$$

The four moduli and the two angles may then be determined from the six observables. Equations (2.1) in general have three discrete solutions one of which is unphysical. We use these equations in order to determine the S and P wave production amplitudes at 63 GeV/c using two sets of data as an input:

(i) Moments of angular $\pi\pi$ distribution in t bins in the range 0.005–1.0, integrated over $m_{\pi\pi}$ range 0.71–0.83 GeV;

(ii) Moments of angular $\pi\pi$ distribution in mass bins in the range 0.4–1.0 GeV integrated over t from $t_{\min}$ to 0.2 GeV^2.

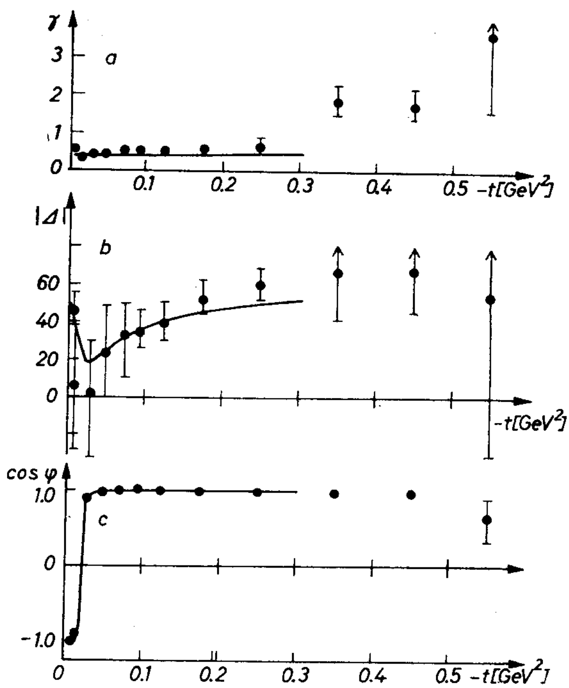

Fig. 1. Phase coherent solution ($\cos\varphi \approx 1$) of the partial wave equations (2.1) as a function of momentum transfer t in ϱ-mass region. Full line represents approximately analogous solution at 17.2 GeV/c [3]. In Fig. 1b only modulus Δ is shown for coherent solution (the sign is not well determined); a) $\gamma = |S|/|P_0|$ — the ratio of moduli of the S and P_0 wave amplitudes; b) Δ = Arg (S) − Arg (P_0) — relative phase of S and P amplitudes; c) $\cos\varphi = \cos(\mathrm{Arg}(P_-)-\mathrm{Arg}(P_0))$ — cosine of the relative phase of P_- and P_0 amplitudes

Figures 1 and 2 show two physical solutions for set (2.1) of equations analysed as a function of the momentum transfer. Its errors reflect the statistical errors on observables used as input to the equations. Figures 3 and 4 show two physical solutions of equations (2.1) analysed as a function of mass.

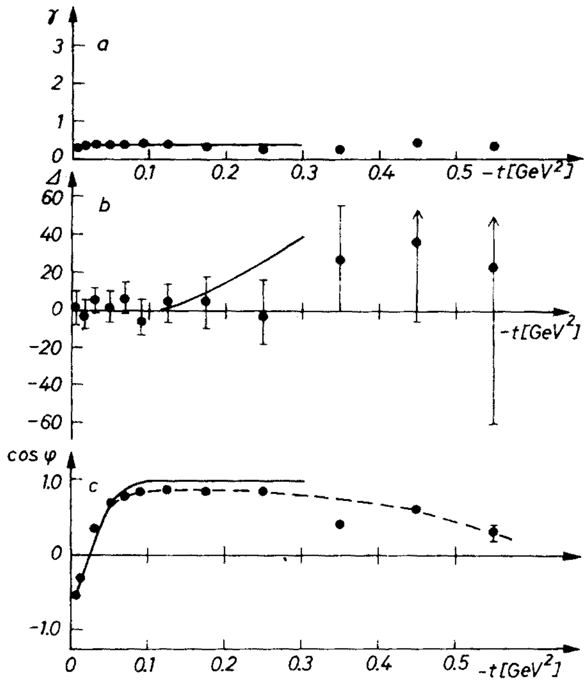

Fig. 2. Second, non-coherent solution of the partial wave equations (2.1) as a function of momentum transfer. Full line represents approximately analogous solution at 17.2 GeV/c [3]. Broken line is the result of the fit to 63 GeV/c data of the model described in Section 3; a), b), c) — the same as in Fig. 1

The following observations can be made

(i) Solutions in Figs 1 and 3 show the phase coherence between P^0 and P^-, therefore they correspond to the "down" solutions of Estabrooks and Martin [3]. Solutions in Figs 2 and 4 both show considerable phase difference between P^0 and P^- being thus analogues of the "up" solutions of Estabrooks and Martin.

(ii) Solutions shown in Fig. 1 show very little variation with energy, especially cos φ dependence, as in fact predicted by the Williams model [8] (full line represents the appropriate solution of Estabrooks and Martin at 17.2 GeV/c). The ratio $|S|/|P_0|$ increases steadily above $|t| = 0.1$. This fact itself makes physical interpretation of this solution very doubtful because S should be OPE dominated precisely as P_0.

(iii) cos φ dependence shown in Fig. 2 changes with energy. Whereas at 17 GeV/c P_0 and P_- are almost phase coherent above $|t| = 0.1$, at 63 GeV/c they are in whole t region incoherent. $\gamma = |S|/|P_0|$ stays constant up to high t values, as required by the OPE model.

(*iv*) The "down" (phase coherent) solution at 63 GeV/*c* represented by points in Fig. 3 shows differences in relation to analogous solution at 17 GeV/*c* (represented by circles). At 63 GeV/*c* in ϱ mass region we have more S wave.

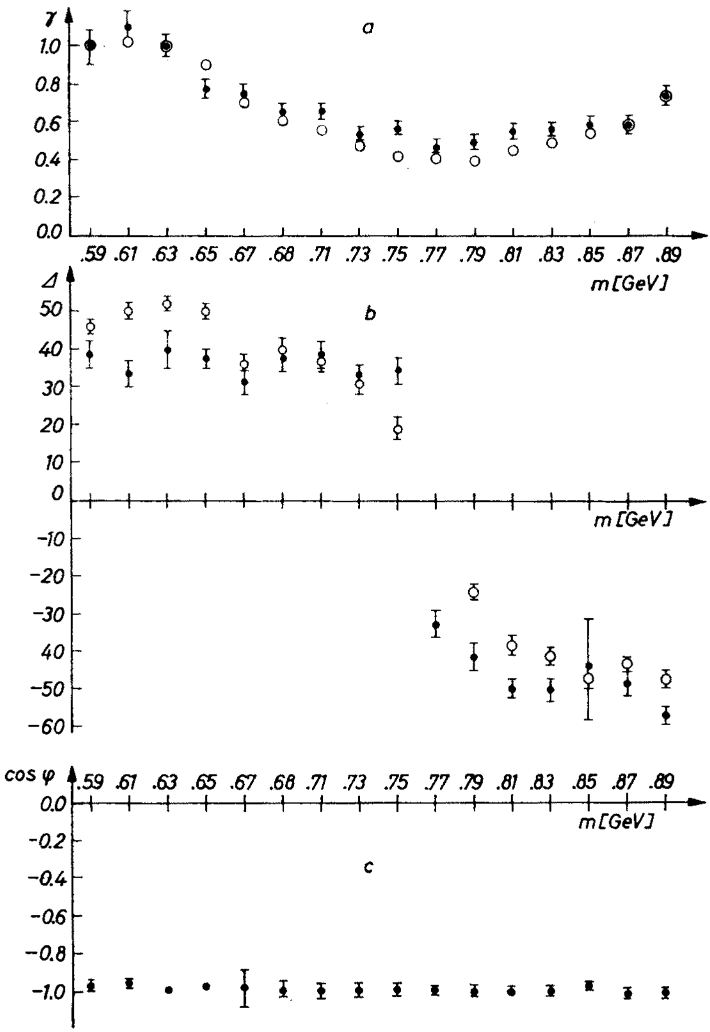

Fig. 3. Phase coherent or "down" solution of the partial wave equations (2.1) as a function of $\pi\pi$ effective mass for $-t < 0.2$. Circles represent analogous solution at 17.2 GeV/*c* [3]; a), b), c) — the same as in Fig. 1

(*v*) The "up" solution in Fig. 4 (not phase coherent) does not change with energy. The S wave follows closely the P_0 wave both in intensity (constant $\gamma = |S|/|P_0|$) and in phase (SP relative phase ≈ 0). Obviously this solution has to be interpreted as a resonance in $I = 0$ S wave.

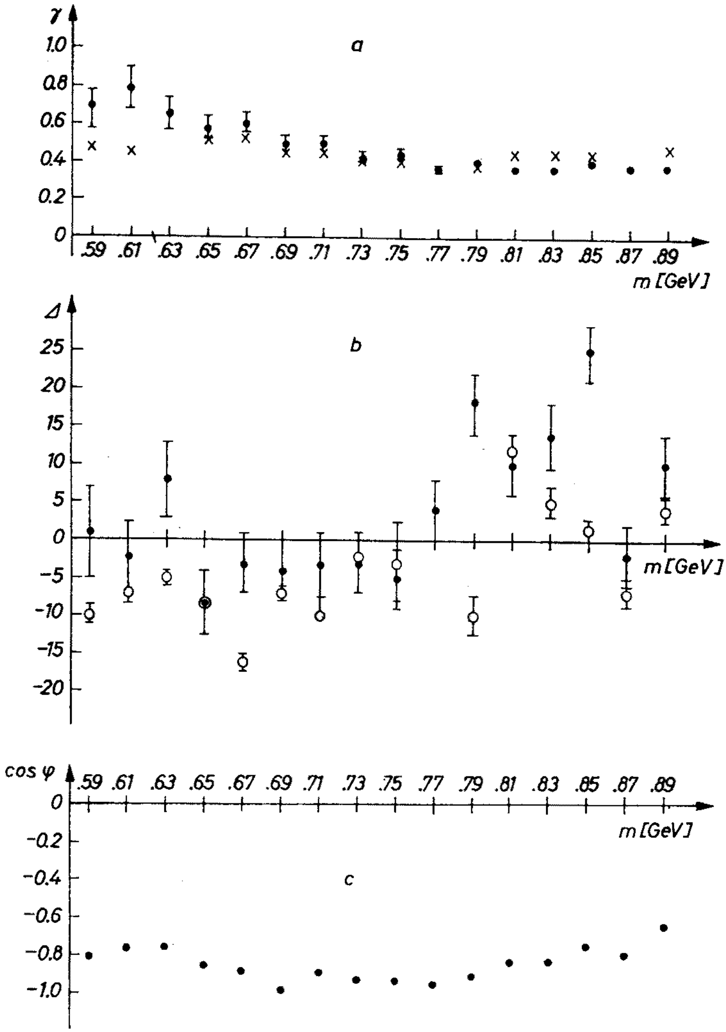

Fig. 4. Second, non-coherent or "up" solution of the partial wave equations (2.1) as a function of $\pi\pi$ effective mass. Circles represent analogous solution at 17.2 GeV/c [3]; a), b), c) — the same as in Fig. 1

Considering observations (*iv*) and (*v*) the "up" solution seems to be slightly favoured at 63 GeV/c. However, it is the cos φ t-dependence shown in Fig. 2 which we consider the main reason for choosing the "up" solution in Fig. 4 as a physical one, as will be explained in the next Section.

3. *t dependence of S and P wave production amplitudes*

The "up-down" ambiguity could be easily solved if the production mechanism in the reaction $\pi^-p \to \pi^+\pi^-n$ were precisely known. This is certainly not the case, particularly when the cut dominated P_- amplitude is concerned. However, the high energy data on pn CEX reaction [10] shed a new light on the cut composition problem, allowing us to choose the solution fitting into general picture of the cut phenomenology. The idea is that at sufficiently high energy the complex A_2-cut component should eventually show up in the relative phase between P_0 and P_-.

A simple phenomenological prescription of Williams [8] has been successfully used for almost a decade in interpretation of np CEX and related processes such as $\gamma p \to \pi^+ n$, $\pi p \to \varrho n$ below 20 GeV/c. The Williams prescription can be treated as an approximation of the Regge pole absorption model in which (*i*) cut contributions are dominated by the Pomeron cut with trajectory not too much different from that of the pole, (*ii*) the cut contributes only to $n=0$ amplitudes (n is net helicity flip), (*iii*) logarithmic energy dependence of pole/cut ratio can be disregarded, what is certainly true for energies below 20 GeV.

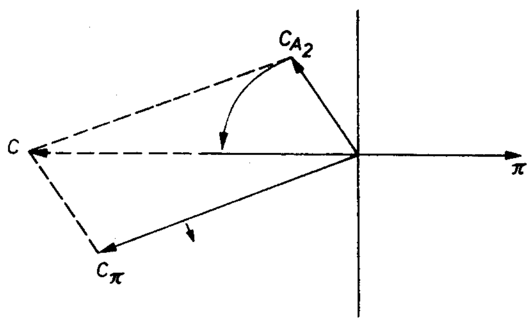

Fig. 5. Complex plane representation of the different contributions to the production amplitudes for the process $\pi^- p \to \pi^+ \pi^- n$

The specific predictions of this model for $\pi^- p \to \varrho^0 n$ are

(*i*) cosine of phase between P_0 and P_- production amplitudes (in helicity frame) is almost step function [9] passing through zero around $t \simeq -0.03$ and assuming values ± 1 on the left and right hand sides of this point;

(*ii*) the cut contribution has the same energy dependence as the pion pole contribution, i.e. for natural parity exchange component $\varrho^+ \frac{d\sigma}{dt}$ intercept of effective trajectory is close to zero.

As already mentioned, these predictions are very well met by the data below 20 GeV/c. However, it has been shown [10] that the success of the Williams model is purely accidental. If the s^{α_π} energy dependence of the cut is modified by $1/\ln s$ factor (as reguired in absorption models) a large high lying cut can be used to boost the α_{eff} to just below zero. Very high energy data should show that A_2 cut wins eventually. As pointed out in Ref. [10] this is precisely what we observe in pn CEX data at Fermilab energies. $p_{\text{lab}}^2 \frac{d\sigma}{dt}$ $(t \approx -0.005)$ dominated by cut contribution shows 300% increase between 10 and 280 GeV. The authors of Ref. [10] parametrized $\pi \otimes P$ and $A_2 \otimes P$ cuts in such a way that total cut strength is nearly constant up to 30 GeV and starts to rise at slow pace above that energy.

An apparent phase coherence between P^0 and P^- helicity amplitudes observed in $|t|$ range of 0.1–0.3 for both solutions can be a result of an accident as well. One can achieve approximate phase coherence over a limited range of t, in a way shown in Fig. 5. But, since $A_3 \otimes P$ and $\pi \otimes P$ cuts have different energy dependences, one is bound to loose coherence eventually.

Indeed, at 63 GeV/c the second solution for the t dependence of P^0P^- phase is in qualitative agreement with two component cut contribution to P^-. In order to put it in quantitative terms we performed simple model fit to the data at 17 and 63 GeV/c. We parametrize s-channel helicity amplitudes as follows[2]:

$$P^0_{+-} = \pi^0_{+-},$$

$$S_{+-} = 0.4 P^0_{+-},$$

$$P^-_{+-} = \frac{2\sqrt{-t}}{m_\varrho} \pi^0_{+-} + \pi_{cut} + A_{2cut},$$

$$P^+_{+-} = \pi_{cut} + A_{2cut} + A_{2+-},$$

where

$$\pi^0_{+-} = \beta_\pi \frac{\sqrt{-t}}{\mu^2 - t} e^{-i\pi\alpha_\pi/2} e^{b_\pi{}^0 t},$$

$$A_{2\pm} = -\beta_{A_2} t e^{-i\pi\alpha_{A_2}/2} e^{b_{A_2} t},$$

$$\pi_{cut} = \beta_{\pi cut} e^{-i\pi\alpha_{\pi cut}/2} e^{b_{\pi cut} t},$$

$$A_{2cut} = \beta_{A_2 cut} e^{-i\pi\alpha_{A_2 cut}/2} e^{b_{A_2 cut} t}$$

and trajectory parameters are shown in Table I.

We fit the t dependence of six moments of angular distribution in the $|t|$ range 0.0–0.6. Addition of A_2 cut to parametrization of the data improves description of the data signif-

TABLE I

Parameters of the trajectories used in the model

Trajectory	Intercept	Slope
π	−0.014	0.7
A_2	0.43	0.7
π_{cut}	0.0	0.35
A_{2cut}	0.43	0.35

[2] For denoting helicity frame production amplitudes we use convention $L^m_{\lambda\lambda'}$ where L is orbital momentum of dipion system, m — its helicity ($m = 0$, $m = \pm$ for natural and unnatural exchange combinations of helicity 1 and −1) λ, λ' are the helicities of initial and final nucleons. In this simple model we disregard all nonflip contributions which produce second order effects.

TABLE II

Values of the model parameters resulting from the fit

Parameter	17 GeV/c	63 GeV/c
β_π	4.49	1.32
β_{A_2}	24.8	12.96
$\beta_{\pi\ \mathrm{cut}}$	−4.37	−1.03
$\beta_{A_2\ \mathrm{cut}}$	−1.37	−0.76
b_π^0	4.3	4.72
b_{A_2}	3.0	4.2
$b_{\pi\ \mathrm{cut}}$	1.6	3.4
$b_{A_2\ \mathrm{cut}}$	2.4	3.1

icantly reducing χ^2 [3]. Obviously, this is due to the existence of the second non-coherent solution. cos φ as a function of $|t|$ resulting from this model is shown in Fig. 2. We can see that this model describes fairly well the data.

Table II shows the parameters resulting from fits at 17 and 63 GeV/c. We see that between 17.2 and 63 GeV the relative pion cut contribution drops by 0.8 and the relative A_2 cut contribution increases by 1.8. This result is in rough agreement with the parametrization of Ref. [10]

$$R \otimes P = \frac{s^{\alpha_R(0)}}{(a_R + b_R \ln s)^2}$$

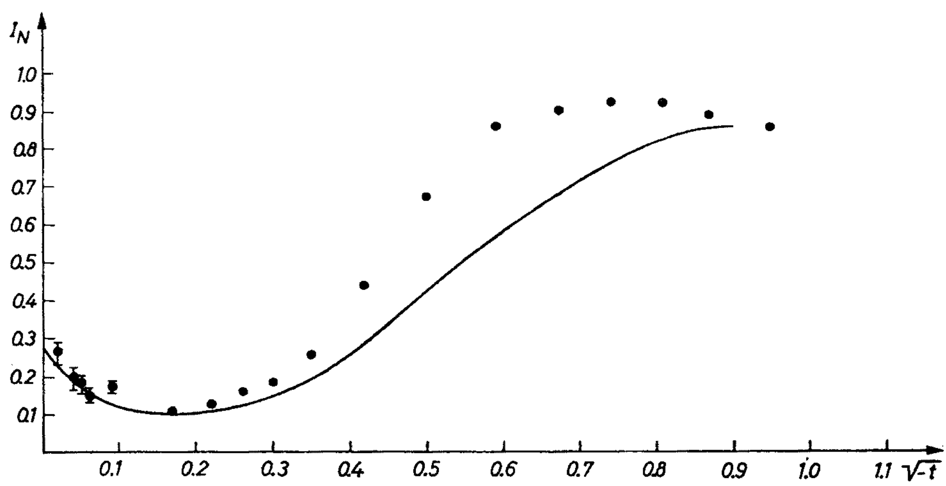

Fig. 6. Natural parity exchange relative intensity as a function of $\sqrt{-t}$ at 17.2 GeV/c (full line) and 63 GeV/c (points with errors)

[3] This fact was observed many years ago by Kimmel and Reya [9].

giving corresponding drop and increase 0.86 and 1.4 for $\pi\otimes P$ and $A_2\otimes P$ respectively. Effective cut strength remains almost unchanged as in fact the data require. In Fig. 6 we compare natural parity exchange relative intensity as a function of $\sqrt{-t}$ at 17 and 63 GeV/c. The 63 GeV/c data do not show significant increase in the small t region.

The analysis of the t-dependence of the angular distribution moments in the reaction $\pi^-p \to \pi^+\pi^-n$ at 63 GeV/c leads us to the following conclusions

(*i*) There exists a solution for which the Williams model [8] apparently works;

(*ii*) The second, non-coherent solution seems to be consistent with the standard Regge pole absorption model and pn CEX data at Fermilab energies. Choosing it as a physical one we are bound to favour the "up" solution, i.e. the non coherent solution of the set of equations (2.1) analysed as a function of mass.

4. *Conclusions*

The analysis of the reaction $\pi^-p \to \pi^+\pi^-n$ at 63 GeV/c in the ϱ region of the dipion mass gives the results similar to those at 17 GeV/c except for some features indicating the presence of the A_2-cut in P_- amplitude. Motivated by demand of consistency with the Regge absorption model and pn CEX data at Fermilab energies, we favour the non-coherent, i.e. "up" solution as the physical one. Obviously the final solution of the "up-down" ambiguity has yet to come for example from $\pi^-p \to \pi^0\pi^0n$ or $\pi^+p \to \pi^0\pi^0\Delta^{++}$ background free high statistics data.

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