LETTERS TO THE EDITOR

GLUEBALLS IN QCD SUM RULES

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We review an approach to gluonium based on the QCD sum rules. Emphasis is put on a new mass scale implied by the sum rules. Some manifestations of it might have already been seen in η' physics

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In this note we review some of the QCD predictions for glueball properties elaborated in the original papers of the same authors [1]. (These papers contain a detailed list of references).

1. The general framework remains the same that we are pursuing for several years now — the QCD sum rules. The basic idea underlying the sum rules is that asymptotic freedom is violated first by the interaction of quarks and gluons with vacuum fields. The formation of resonances is a phenomenological manifestation of this interaction. The formal framework is provided by operator expansion and by the introduction of various vacuum-to-vacuum matrix elements, such as $\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle$ ($G^a_{\mu\nu}$ is the gluon field strength tensor). These vanish, by construction, in perturbation theory and measure the strength of nonperturbative vacuum fields.

To get insight into gluon physics consider the two-point functions induced by various gluon currents. For example, the proper correlator to study scalar glueballs is

$$S(Q^2) = i\int dx e^{iqx}\langle 0|T\{j_s(x), j_s(0)\}|0\rangle,$$

where

$$j_s = \alpha_s G^a_{\mu\nu}G^a_{\mu\nu}.$$

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The corresponding sum rule reduces to

$$\frac{1}{\pi M^2}\int \mathrm{Im}\, S(s)\exp(-s/M^2)ds/s = \frac{2M^2}{\pi^2}\alpha_s^2(M)$$

$$+\langle 0|(\alpha_s/\pi)G^2|0\rangle M^{-2}(32\pi^2/b-4\pi\alpha_s(M))+O(M^{-4}), \tag{1}$$

where M^2 is an "external" variable and we keep only the contribution which survives at $M^2 \to \infty$ and the first power correction (split into two pieces for the reasons clarified below). Actual calculation extends to a few higher terms as well.

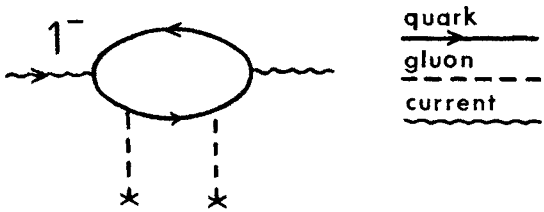

Fig. 1a. The correction due to the gluon vacuum condensate for vector quark currents

2. Confining ourselves here to a qualitative analysis alone, we note first that the sum rules imply a larger mass scale for the violation of asymptotic freedom than that we are accustomed to from the known quark resonances.

Let us recall to the reader that in the ϱ-meson sum rules the correction due to the gluon vacuum condensate comes from Fig. 1a and reaches 10% of the asymptotic contribution at $M^2 = 0.6\ \mathrm{GeV}^2$. In the case of the gluon current the corresponding graph is that of Fig. 1b (it gives rise to the second term in brackets, Eq. (1)) and the mass scale brought by this interaction is

$$M^2_{\text{gluon condensate}} \sim 5\ \mathrm{GeV}^2. \tag{2}$$

The factor of ~ 10 difference in the critical value of M^2 is even more important since the sum rules deal with power corrections of high orders.

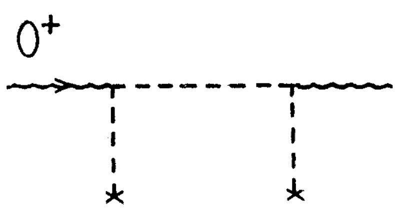

Fig. 1b. The correction due to the gluon vacuum condensate for scalar gluon currents

The origin of the difference can be readily traced. Indeed, the gluon currents communicate with the vacuum gluon fields easier than quark currents. Formally, the power correction in the gluon channel is associated with a Born type graph while in the quark channel we have a loop graph. Numerical smallness associated with the loop integration makes the difference.

3. However, the leading power correction in Eq. (1) comes from another source. Let us consider $S(Q^2 = 0)$ which can be rewritten as

$$S(Q^2 = 0) = \frac{d}{d(-1/4g_0^2)} \frac{1}{16\pi^2} \langle 0|\bar{G}^2|0\rangle,$$

where $\bar{G} = g_0 G$ and g_0 is the bare coupling constant. On the other hand, the g_0 dependence is fixed by the renormalization group

$$\langle 0|\bar{G}^2|0\rangle = \text{const}\,[M_0 e^{-8\pi^2/bg_0^2}]^4,$$

where M_0 is the regulator mass and, therefore,

$$S(Q^2 = 0) = \frac{32\pi^2}{b} \langle 0| \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu}|0\rangle. \tag{3}$$

This low-energy theorem can be translated into the power correction at large M^2 since the asymptotic behaviour of any two-point function is know from asymptotic freedom. The corresponding mass scale for asymptotic freedom violation is

$$M^2_{\text{direct fluctuations}} \sim 16\ \text{GeV}^2 \tag{4}$$

— the largest one we have ever met.

3. We have labelled this new scale by the subscript "direct fluctuations" since looking for the origin of the correction at high M^2 one immediately recognizes that it should be related to the graph of Fig. 2 where all the momentum brought by the current is transferred to the vacuum. A model for direct fluctuations is provided by instantons. To evaluate the latter contribution one substitutes the (classical) instanton field for all the gluon legs and integrates over the instanton density.

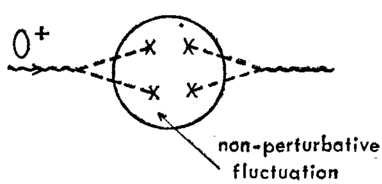

Fig. 2. Direct fluctuations responsible for the leading power corrections in the case of (pseudo) scalar gluonic currents

Moreover, this instanton contribution is an exact answer for the graphs considered as far as M^2 is large. Indeed, the instantons dominate over all other nonperturbative fluctuation at a short distance. Unfortunately, these distances are indeed short and the instanton contribution cannot be extrapolated into the vital region of $M^2 \sim$ (several GeV^2). The reason is that the instanton density itself is modified by the vacuum condensate

$$d_{\rm eff}(\varrho) = d_0(\varrho) \exp\left\{\frac{\pi^4\varrho^4}{8\alpha_s^2(\varrho)} \langle 0| \frac{\alpha_s}{\pi} G^2|0\rangle\right\}.$$

As a result, the naive instanton calculus should be abandoned at such high M^2 as $M^2 \gtrsim 25$ GeV^2, where the instanton contribution is totally negligible.

4. Altough we cannot literally rely on the instantons we would like to speculate that they give the right message on the quantum numbers of the currents which are contaminated by the direct-instanton-like contributions. For example, the gluon tensor current vanishes for instanton field and we would like to guess that there is no large "direct" contribution of Fig. 2 altogether.

In this way we come to a new classification scheme and try to follow the difference in mass scales of asymptotic freedom breaking and of the corresponding resonance spectrum.

There are essentially three scales for M^2, all of them mentioned above, namely, ~ 0.6 GeV^2, ~ 5 GeV^2 and ~ 16 GeV^2. The first one is realized in the quark vector and axial vector channels, the second one is relevant to the (pseudo) scalar quark and to the gluon tensor channels; and, finally, the largest scale is realized in the gluon scalar and pseudoscalar channels.

There is a note of caution, however: the scale of violation of asymptotic freedom is not necessarily realized as the scale of the lowest resonance. There could be a relatively light but "powerful" resonance as well. For example, the pion in the pseudoscalar quark channel does violate asymptotic freedom as early as ~ 2 GeV^2 but is nearly massless. A more quantitative analysis shows that scalar glueball can also be relatively light, $M^2 \sim$ (1–2) GeV^2. But as far as the pseudoscalar glueball is concerned we do expect it to be heavy, $M^2 \sim$ (6–16) GeV^2.

Note also that the recent discovery of the so called E meson in the radiative decays of J/ψ could well result in identification of a tensor gluonium. Indeed, its mass falls close to what is expected and a tensor meson is welcome by the duality argument. We look forward to determination of quantum numbers of the E meson.

5. It is amusing that a manifestation of the new mass scale might have been already seen. Indeed, the η' mass is known to vanish in the large N_c limit

$$m_{\eta'}^2 \sim 1/N_c$$

but it is still larger than m_ϱ^2. The puzzle seems to be resolved now since η' gets its mass through mixing with a pseudoscalar glueball. For the mass of the latter we expect (6–16) GeV^2 and η' is light in this mass scale. This could be the reason behind successful quark model predictions for η' as well. Moreover, following the line of argument outlined above we come to a new mass relation for

$$m_{\eta'}^2 f_{\eta'}^2 = \frac{18}{b} \langle 0| \frac{\alpha_s}{\pi} G_{\mu\nu}^a G_{\mu\nu}^a|0\rangle, \qquad b = \tfrac{11}{3} N_c - \tfrac{2}{3} N_f, \tag{5}$$

where $f^2_{\eta'}$ is defined as $\langle 0|\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+s\gamma_\mu\gamma_5 s|\eta'\rangle = if_{\eta'}\, k_\mu$. The relation turns out to be true up to a factor of 2 (neither $f^2_{\eta'}$ nor $\langle G^2\rangle$ is known to better accuracy).

Apart from the smallness of the η' mass in the natural scale we use at this point the similarity (in the imaginary world with no light quarks) of the two-point functions associated with the scalar current,, $j_s = \alpha_s G^\alpha_{\mu\nu} G^\alpha_{\mu\nu}$, and the pseudoscalar current, $j_p = \alpha_s G^\alpha_{\mu\nu}\tilde{G}^\alpha_{\mu\nu}$. The similarity is implied by the classification scheme elaborated in the paper [1] ($j_p = ij_s$ for the instanton field).

To make contact with the well known mass relations due to Witten and Veneziano we note that the value of $\langle G^2\rangle$ would be rather strongly modified by the removal of light quarks from the theory

$$\langle 0|G^2|0\rangle_{\text{no light quarks}} = (2-3)\,\langle 0|G^2|0\rangle_{\text{real world}}. \tag{6}$$

The estimate is based on the low energy theorem

$$\frac{d}{dm_q}\left\langle \frac{\alpha_s}{\pi}\, G^a_{\mu\nu}G^a_{\mu\nu}\right\rangle = -\frac{24}{b}\,\langle\bar{q}q\rangle \tag{7}$$

(which holds as far as the quark mass is small) and on some simple-minded matching of the light and heavy quark expansions.

One of the surprises implied by Eq. (7) is that $\langle G^2\rangle$ is changed by a factor of about 2 by shifting the strange quark mass from the SU(3) symmetric point, $m_s = 0$, to its present value.

The derivation of Eq. (7) is very similar to that of Eq. (3) and we omit the details here.

REFERENCES

[1] A. I. Vainshtein, V. I. Zakharov, V. A. Novikov, M. A. Shifman, Preprint ITEP 87–88 (1980).