A GL(4,R) VERSION OF A GAUGE THEORY OF GRAVITATION

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Following an extension of the Utiyama and Kibble method, gravitational fields are introduced as gauge fields. The gauge group is the GL(4,R) which is derived from the group of general coordinate transformations. This group corresponds to a metric affine geometry. The equations of fields follow from a Lagrangian containing linear and quadratic invariants constructed from the gauge field tensor and an additional scalar field. Certain constraints lead to a hierarchy of gauge gravitational theories including the Einstein or U_4 theory as the most direct case.

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1. *Introduction*

The gauge field idea is a powerful principle for constructing theories of fundamental interactions. The successful theories of unified electromagnetic and weak interaction and of strong interaction are based on a gauge invariance with respect to the local SU(2)⊗U(1) and SU(3)- colour transformations which lead to the Weinberg-Salam theory and chromodynamics, respectively.

Also Einstein's gravitational theory can be considered as a gauge theory in the spirit of Yang and Mills [1] with an extension to external symmetries [2–4] (however, compare Ref. [30]). Utiyama [2] used a Lorentz group as the basic symmetry group. It was extended in Ref. [3, 4] to the Poincaré group. This approach resulted in the U_4 theory of gravitation advocated by Trautman [5] and by Hehl et al. [6]. Later, other authors constructed theories using wider groups like the Weyl group [7, 8] (as an extension of Ref. [4]). Agnese and Calvini [9] developed a formalism including an arbitrary N-parameter Lie group which they used for the investigation of a theory with a gauge group according to the conformal group. Recently, interest in conformal gravity is renewed [10, 11].

Further, there exists a series of papers [12–15] in which a GA(4, R) gauge gravitation theory is considered. The gauge group results from the relativistic extension of the SL(3, R)

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(connected with angular momentum excitations of hadrons) and the concept of hypermomentum [16].

Thus the question arises, "What is the largest possible group for applying the conventional gauge field formalism and what structures are preferred for free gravitational Lagrangians from the point of view of this group?" Using the group of general coordinate transformations one includes all other groups considered thus far. In the spirit of a local gauge the untractable group of general coordinate transformations can be reduced to the GA(4, R).

In this paper we consider the GL(4, R) symmetry group and derive a gauge field theory of gravitation which is easily reducible to Einstein's theory. For this reason we need to use a Lagrangian with a term linear in the gauge field tensor (curvature tensor). However, the inclusion of non-antisymmetric gauge fields corresponding to transformations other than Lorentz transformations forces us to include an additional quadratic term. In this manner our Lagrangian involves both linear and quadratic terms, which turned out to be the simplest way to take into account all gauge fields and to avoid the difficulties associated with the projective invariance of the curvature scalar. A choice of a pure quadratic Lagrangian like the Maxwell theory or chromodynamics is not suitable for constructing viable theories of gravitation [17–21].

But recent investigations by Hehl et al. ([14] and Refs. cited therein) indicate that the proper inclusion of all the gauge fields in both quadratic torsion and field strength terms overcomes the difficulties of pure quadratic Langrangians. Moreover, such an ansatz deletes the degeneracy of the torsion as well as the inequal footing of gauge potentials.

Here we shall try to develop a somewhat alternative theory. Further, there is the hope that theories after quantization are renormalizable if quadratic terms are taken into account [22, 23].

After a short review of the gauge field formalism by Agnese and Calvini [9] in Section II we give the step by step reduction of the group of general coordinate transformations to the GL(4, R) group (Section III). In Section IV a Lagrangian is constructed. With the choice of the GL(4, R) the theory includes 4 translational gauge fields h_i^μ, $(i = 1, \ldots, 4)$ 16 ratational gauge fields Γ_μ^A $(A = 1 \ldots 16)$ and an additional scalar field σ. By restrictions we are led directly to the U_4 theory (Section V).

In the framework of this formalism the Einstein or the U_4 theory is the only pure metric one. Non-metric theories, according to enlarged symmetry groups, involve additional constants and fields.

II. Gauge field approach to gravitation

Let us consider, following the approach of Ref. [9], the gravitational fields as universal gauge fields corresponding to a N-parameter Lie group which expresses a certain space-time symmetry.

An action remains invariant with respect to the following coordinate transformations

$$x^\mu \to x^{\mu'} = x^\mu + \varepsilon^A (T_A)^\mu{}_\nu x^\nu + \alpha^\mu, \tag{1}$$

(with the Lie group generators T_A ($A = 1, \dots N$) obeying the usual commutation relations) and

$$\varepsilon^A \to \varepsilon^A(x^\mu), \qquad \alpha^\mu \to \alpha^\mu(x^\nu) \tag{1'}$$

by converting the Lagrangian scalar into a density

$$\mathscr{L} \to h\mathscr{L}, \tag{2}$$

(with a suitable function h) and by replacing the field derivatives by gauge covariant derivatives

$$\partial_\mu \to D_\mu \,. \tag{3}$$

In general, the flat space coordinates as arguments of the field functions ϕ_Σ are replaced by curvilinear coordinates. Thus, the fields ϕ_Σ may be defined with respect to an orthogonal tetrad field $h^i_{\ \mu}$. Latin indices correspond to local components, and Greek indices denote the world components.

The covariant derivatives are defined by

$$\begin{aligned} D_k\phi_\Sigma &= h^\mu{}_k D_\mu \phi_\Sigma \\ &= h^\mu{}_k(\partial_\mu \phi_\Sigma + \Gamma_\mu{}^A (T_A)_\Sigma{}^A \phi_A). \end{aligned} \tag{4}$$

Quantities whose laws of transformation under the action of (1) are

$$H_\Sigma \to H'_\Sigma = H_\Sigma + \varepsilon^A (T_A)_\Sigma{}^A H_A + \xi^\mu{}_{,\nu}(\Sigma_\mu{}^\nu)_\Sigma{}^A H_A, \tag{5}$$

obey a "mixed coordinate-gauge" covariant derivative

$$\begin{aligned} D_k H_\Sigma &= h^\mu{}_k D_\mu H_\Sigma \\ &= h^\mu{}_k(\partial_\mu H_\Sigma + \Gamma_\mu{}^A (T_A)_\Sigma{}^A H_A + \Gamma^\alpha_{\beta\mu}(\Sigma_\alpha{}^\beta)_\Sigma{}^A H_A. \end{aligned} \tag{6}$$

In this way the affine connections $\Gamma^\alpha_{\beta\mu}$ are introduced. With the conventionally adopted ad hoc postulate

$$D_\nu h^\mu{}_k = 0 \tag{7}$$

and using Eqs. (4) and (6), we can find a link between gauge fields and connections

$$\Gamma^\alpha_{\beta\nu} = h_\beta{}^e (\Gamma^A_\nu (T_A)_e{}^k h^\alpha{}_k - k^\alpha{}_{e,\nu}). \tag{8}$$

As usual, the gauge invariant quantities are found by considering the commutator of two subsequent derivatives

$$[D_\mu, D_\nu] H_\Sigma = F_{\mu\nu}{}^A (T_A)_\Sigma{}^A H_A - C_{\mu\nu}{}^\alpha D_\alpha H_\Sigma + R_{\mu\nu}{}^\alpha{}_\beta (\Sigma_\alpha{}^\beta)_\Sigma{}^A H_A. \tag{9}$$

Applying Eq. (9) to $h^\mu{}_k$ and taking into account the postulate (7), we get

$$F_{\mu\nu}{}^A (T_A)_\Sigma{}^A = -R_{\mu\nu}{}^\alpha{}_\beta (\Sigma_\alpha{}^\beta)_\Sigma{}^A. \tag{10}$$

So the elements of a gauge invariant Lagrangian are the tertrads $h^{\mu}{}_{k}$, the field strength tensor $F^{A}_{\mu\nu}$ (or the curvature tensor $R_{\mu\nu}{}^{\alpha}{}_{\beta}$ according Eq. (10))

$$\tfrac{1}{2} F_{\mu\nu}{}^{A} = \partial_{[\nu}\Gamma^{A}_{\mu]} - f^{A}_{BC}\Gamma^{B}_{[\mu}\Gamma^{C}_{\nu]} \tag{11}$$

and the torsion tensor

$$\tfrac{1}{2} C_{\beta\nu}{}^{\alpha} = \Gamma^{\alpha}_{[\beta\nu]}. \tag{12}$$

III. The gauge group

For an explicit construction of the free gauge field Lagrangian we have to choose a gauge group. Our aim is to include a gauge group as general as possible and to construct a theory which yields other theories in considering smaller groups (like the Lorentz group, Weyl group, and a conformal group). Let us consider the group of general infinitesimal transformation of coordinates in a flat space.

This group per se does not have the same meaning as the groups mentioned above. But, perhaps, it covers all external symmetries of interest.

According to Ogievetski's theorem [24] the infinite number of generators is reduced to a finite number of generators: each generator can be represented by a linear combination of repeated commutators of generators of the conformal and special linear groups, respectively.

In this manner we need only consider the generators of the groups C_{15} and SL(4, R) which have in coordinate representation the explicit form

$$\begin{aligned} P_{\mu} &= \partial_{\mu} \text{ (translations)}, \\ L_{\mu\nu} &= 2x_{[\mu}\partial_{\nu]} \text{ (Lorentz transformations)}, \\ D &= x^{\mu}\partial_{\mu} \text{ (dilatations)}, \\ K_{\mu} &= 2x_{\mu}x^{\nu}\partial_{\nu} - x^{\nu}x_{\nu}\partial_{\mu} \text{ (special conformal transformations)}, \\ M_{\mu\nu} &= x_{\mu}\partial_{\nu} - \tfrac{1}{4}\eta_{\mu\nu}x^{\varrho}\partial_{\varrho} \text{ (special linear transformations)}. \end{aligned} \tag{13}$$

The generated transformations in coordinate space are

$$\begin{aligned} P &: \delta x^{\mu} = \bar{\alpha}^{\mu}, \\ L &: \delta x^{\mu} = \varepsilon^{\mu}{}_{\nu}x^{\nu}, \qquad \varepsilon^{(\mu\nu)} = 0, \\ D &: \delta x^{\mu} = \alpha x^{\mu}, \\ K &: \delta x^{\mu} = x^{\nu}x_{\nu}\beta^{\mu} - 2x^{\mu}x^{\nu}\beta_{\nu} \\ M &: \delta x^{\mu} = \alpha^{\mu}{}_{\nu}x^{\nu} - \tfrac{1}{4}\alpha x^{\mu}. \end{aligned} \tag{14}$$

Hence, the matrix representations are

$$\begin{aligned} (P_{\mu})^{\varrho\tau} &= \text{no 4-dim representation}, \\ (L_{\mu\nu})^{\varrho\tau} &= 2g^{\varrho}_{[\mu}g^{\tau}_{\nu]}, \end{aligned}$$

$$(D)^{\varrho}{}_{\tau} = g^{\varrho}_{\tau},$$

$$(K_{\mu})^{\varrho\tau} = x^{\sigma}(L_{\sigma\mu})^{\varrho\tau} + x_{\mu}(D)^{\varrho\tau},$$

$$(M_{\mu\nu})^{\varrho\tau} = g^{\varrho}_{\mu} g^{\tau}_{\nu} - \frac{1}{4}\,\eta_{\mu\nu}\eta^{\varrho\tau}. \tag{15}$$

The usual gauge field approach is based on linear transformations. For a proper treatment of the conformal group we had to use a 6-dimensional space-time wherein the transformations (14) possess linear representations. But we restrict ourselves to the 4-dimensional approach. To do so in Eq. (15) the special conformal generator is represented by linear combinations of Lorentz and dilatation generators with linear space-time dependent parameters [9]. This is possible because we are interested in local gauges; that means the group parameters are allowed to be space-time dependent. For this procedure also compare [10].

Thus, we have only the 16 generators of the general linear group GL(4, R) and the 4 translation generators. Denoting the GL (4, R) generators by $T_{\mu\nu}$, we can write

$$T_{[\mu\nu]} = \frac{1}{2}\,L_{\mu\nu},$$

$$T_{\mu\nu}\eta^{\mu\nu} = T = D,$$

$$T_{(\mu\nu)} - \frac{1}{4}\,\eta_{\mu\nu}T = T_{\{\mu\nu\}} = M_{(\mu\nu)}. \tag{16}$$

Each generator of the group of general coordinate transformations can be constructed by the Ogievetski theorem using only the GL(4, R) and the 4 translation generators and linear combinations with no more than linear space-time dependent coefficients.

Thus, we are going to base our approach on the GL(4, R) group. In principle, we have coordinate transformations (resulting from local translations) together with GL(4, R) gauge transformations which we could consider as affine (GA(4, R)) gauge transformations. Lord [25] considered such combined transformations whereas the GL(4, R) part is related to tetrad deformations. Considering the affined group rather than the linear group, there would be a change for the gauge kinematics as noted in [6, 14].

As mentioned in the introduction, the hypermomentum concept, introduced in [12, 16], also leads to the GL(4, R) group. It is a very general approach from our point of view because it includes local all space-time symmetries. For a physical foundation compare further [13, 15].

IV. The construction of a Lagrangian

In analogy with electrodynamics we should use as a Lagrangian a quadratic function of the field strength (curvature) and of the torsion. Such a choice is successful in the Weinberg-Salam theory and chromodynamics. In the theory of gravitation the Schwarzschild behaviour and the weak-field approximation of Einstein's theory can be reproduced (see Ref. [14]). But having in mind a post-Newtonian deviation from the Einstein theory, the non-direct recovering of the U_4 limit (in principle an extra derivative is necessary) as well as the non-invariance against scale transformations (the Lagrangian is not of Weyl weight zero), it seems reasonable to choose an alternative Lagrangian.

The simplest Lagrangian is (see e.g. Kibble [4])

$$\mathscr{L} = \sqrt{-g}\, F_{\mu\nu}{}^{ij} h^{\mu}{}_{i} h^{\nu}{}_{j}, \tag{17}$$

where one uses for the function h in (2) the quantity

$$h = \sqrt{-g},$$

$$g = \det \| g_{\mu\nu} \| = \det \| h_{\mu}{}^{e} h_{\nu e} \|. \tag{18}$$

Our field strength tensor follows from relation (11) by taking into account the results from Section III

$$\tfrac{1}{2} F_{\mu\nu}{}^{ij} = \partial_{[\nu} \Gamma_{\mu]}{}^{ij} - \Gamma_{[\mu}{}^{ik} \Gamma_{\nu]}{}^{ej} \eta_{ke}. \tag{19}$$

Due to the anti-symmetry of the lower indices one finds

$$F_{\mu\nu}{}^{(ij)} h^{\mu}{}_{i} h^{\nu}{}_{j} = 0. \tag{20}$$

According to Eq. (20) the Lagrangian (17) involves only $F_{\mu\nu}{}^{[ij]}$. To see, which gauge fields are included in the Lagrangian (17), we decompose the field strength (19) into symmetric and anti-symmetric parts

$$F_{\mu\nu}{}^{ij} = F_{\mu\nu}{}^{(ij)} + F_{\mu\nu}{}^{[ij]},$$

$$\tfrac{1}{2} F_{\mu\nu}{}^{(ij)} = \partial_{[\nu} \Gamma_{\mu]}{}^{(ij)} - 2\Gamma_{[\mu}^{(\{i|e\}} \Gamma_{\nu]}^{[k|j])} \eta_{ke},$$

$$\tfrac{1}{2} \Gamma_{\mu\nu}{}^{[ij]} = \partial_{[\nu} \Gamma_{\mu]}^{[ij]} - 2\Gamma_{[\mu}^{[[i|e]} \Gamma_{\nu]}^{[k|j]]} \eta_{ke} - 2\Gamma_{[\mu}^{[\{i|e\}} \Gamma_{\nu]}^{\{k|j\}]} \eta_{ke}. \tag{21}$$

In this manner the Lagrangian (17) includes only the dynamics of the antisymmetric gauge fields. It is our aim to construct a theory which is based on the full gauge group GL(4, R). Therefore, we need to introduce an additional term into the Lagrangian in order to really include the full set of gauge fields $\Gamma_{\mu}{}^{ij}$. A minimal inclusion of the symmetric field strength is given by adding the Yang–Mills term

$$F_{\mu\nu}{}^{(ij)} F^{\mu\nu}{}_{(ij)}.$$

A choice of a Lagrangian which is closely related to Einstein's or the U_4 theory and which avoids a homogeneous quadratic form is the following

$$\mathscr{L} = \sqrt{-g}\, (\sigma^2 F_{\mu\nu}{}^{[ij]} h^{\mu}\, i\, h^{\nu}{}_{j} + a F_{\mu\nu}{}^{(ij)} F^{\mu\nu}{}_{(ij)}). \tag{22}$$

The form of our Lagrangian (22) is suggested by the form of Eq. (21). To get an action which is a scalar with the dimensional number $d = 0$ we must include an additional field σ ($d = -1$). As usual, whenever masses appear in the matter Lagrangian we must replace

$$m \to \mu\sigma.$$

So, masses are to transform in the following way under the action of transformations (1)

$$m \to m' = m + \delta m,$$

$$\delta m = \delta(\mu\sigma) = \mu\delta\sigma = -\varepsilon\mu\sigma = -\varepsilon m \tag{23}$$

(for this procedure see e.g. [9, 17]). Clearly, the introduction of the additional σ field could be used as an argument against this ansatz. By omitting the σ field the scale invariance would be broken (as in the Hehl et al. Lagrangians).

To avoid the inconsistency of this Brans–Dicke-like ansatz with observations, the σ-field can be anchored by a suitable potential to have a fixed value (compare remarks in Ref. [27]). The advantage of the σ field consists in a dimensionless coupling constant, e.g. it admits the Weinberg-Salam concept in gravity. Using Eq. (10) and the results from Section III we have

$$F_{\mu\nu}{}^{ij} = -R_{\mu\nu}{}^{\alpha}{}_{\beta}h_{\alpha}{}^{i}h^{\beta j}. \tag{24}$$

From this an alternative form of our Lagrangian (22) follows

$$\mathscr{L} = \sqrt{-g}\left(\sigma^2 R^{\nu\alpha}{}_{\nu\alpha} + \frac{a}{2}\left[R_{\mu\nu\nu\varrho}R^{\mu\alpha\nu\varrho} + R_{\mu\alpha\nu\varrho}R^{\alpha\mu\nu\varrho}\right]\right), \tag{25}$$

where the curvature tensor has the form (cf. Eq. (9))

$$\tfrac{1}{2} R_{\mu\nu}{}^{\alpha}{}_{\beta} = \partial_{[\nu}\Gamma^{\alpha}_{|\beta|\mu]} - \Gamma^{\gamma}_{\beta[\mu}\Gamma^{\alpha}_{|\gamma|\nu]}. \tag{26}$$

Apart from the σ field, the Lagrangian (25) is similar to the one proposed by Mansouri and Chang [28] and Mansouri [10]. But they used a $F^{ij}{}_{\mu\nu}F_{ij}{}^{\mu\nu}$ term instead of the $F^{(ij)}{}_{\mu\nu}F_{(ij)}{}^{\mu\nu}$ term (as expressed in our notation). From the principle of simplicity our choice is favoured because it includes all gauge fields in a most simple manner.

V. Discussion

In this section we want to show what gravitational theories are obtained by considering certain restrictions of our Lagrangian.

Let us see, how the gauge fields enter the affine connections. For this reason we write down

$$g_{\mu\nu,\varrho} = h_{\mu}{}^{i}h_{i\nu,\varrho} + h_{\nu}{}^{i}h_{i\mu,\varrho} \tag{27}$$

and insert postulate (7). A straightforward calculation gives

$$\begin{aligned}\Gamma^{\alpha}_{\mu\nu} &= \tfrac{1}{2}g^{\alpha\varrho}(g_{\varrho\nu,\mu} + g_{\varrho\mu,\nu} - g_{\mu\nu,\varrho}) \\ &\quad + (\Gamma^{\alpha}_{[\mu\nu]} + \Gamma^{\tau}_{[\varrho\mu]}g_{\nu\tau}g^{\varrho\alpha} + \Gamma^{\tau}_{[\varrho\nu]}g_{\mu\tau}g^{\varrho\alpha}) \\ &\quad + (\Gamma_{\mu}g^{\alpha}_{\nu} + \Gamma_{\nu}g^{\alpha}_{\mu} - \Gamma_{\varrho}g^{\alpha\varrho}g_{\mu\nu}) \\ &\quad + (Q_{\varrho\nu\mu} + Q_{\varrho\mu\nu} - Q_{\mu\nu\varrho}).\end{aligned} \tag{28}$$

So we are concerned with a (L_4/g) space (see e.g. [16]). The affine connection involves the Christoffel symbols (first line), torsion terms (second line), scale gauge field terms or a semi-metric part (third line) and $\Gamma^{\{ij\}}$ gauge field terms or a non-metric part (last line), which are defined by

$$Q_{\mu\nu\varrho} = \Gamma_{\varrho}{}^{\{ij\}} h_{i\mu} h_{j\nu}. \tag{29}$$

Sources of gravitation are the momentum and the hypermomentum currents. Source identities and the physical concept of the hypermomentum are investigated by Hehl et al. [13, 14, 16]. Independently of the gauge kinematics the metric-affined theory appears as the gauge theory of the GL(4, R) group.

By suppressing the non-metric part and the proper hypermomentum source we are led directly to the U_4 theory. Other restrictions (e.g. omitting the shear potentials $\Gamma_{\mu}{}^{\{ij\}}$) recover the Agnese and Calvini [9] formulation of a conformal gauge theory. The resulting hierarchy is discussed by these authors.

The equations of fields are derived in the appendix. They show a proper hypermomentum dynamics. In our ansatz the Poincaré gauge is related to the Einstein gravitation piece, whereas in the Hehl et al. [14] and Ne'eman and Sijacki [15] ansatz the Poincaré gauge is related almost to a confinement potential via a second coupling constant (it corresponds to our "a" in Eq. (22)).

An inspection of the weak-field limit of our equations of fields does not show a confinement behaviour. The divergences of the shear-like gauge potentials possess a spheric wave solution. Thus, with respect to the EPS work (Ehlers, Pirani and Schild [29]) it is hard to see how the non-metricity remains confined[1]. Further investigation of the non-metric effects and the related coupling constant must be performed.

It would be highly satisfactory to find an ansatz in which only the non-metricity is related to the confinement and for which the U_4 limit is straightforward.

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APPENDIX

The equations of fields follow from the full system of matter fields, their coupling to gravitation and the free gravitational fields (22) which is expressed by

$$\mathscr{L} = \mathscr{L}(\text{Eq. (22)}) + \mathscr{L}(\text{matter}) + \mathscr{L}(\sigma), \tag{A1}$$

where we do not specify the σ-Lagrangian. Using the well-known source identities we get

$$\sigma^2 (F_{i\alpha}{}^{ke} h^{\mu}{}_{[k} h^{\alpha}{}_{e]} - \tfrac{1}{2} h^{\mu}{}_{i} F_{\alpha\beta}{}^{ke} h^{\alpha}{}_{k} h^{\beta}{}_{e}) + 2a (F_{i\alpha}{}^{(ke)} F^{\mu\alpha}{}_{(ke)} - \tfrac{1}{4} F_{\mu\nu}{}^{(ke)} F^{\mu\nu}{}_{(ke)} h^{\mu}{}_{i}) = T^{\mu}{}_{i}, \tag{A2}$$

[1] Other Lagrangians considered by the author (B. Kämpfer, abstract to the GR 9 conference, Jena 1980) do not show confining effects, too.

$$\tfrac{1}{2} D_\mu(\sigma^2 \sqrt{-g}\, h^\mu{}_{[i}h_{j]}{}^\nu) = \sqrt{-g}\, T^\nu_{[ij]}, \tag{A3}$$

$$a D_\mu(\sqrt{-g}\, F^{\mu\nu}{}_{(ij)}) = \sqrt{-g}\, T^\nu{}_{(ij)}, \tag{A4}$$

$$\sigma F_{\mu\nu}{}^{ij} h^\mu{}_i h^\nu{}_i = -\frac{1}{2\sqrt{-g}} \frac{\delta}{\delta\sigma} (\mathscr{L}(\sigma) + \mathscr{L}(\text{matter}) \sqrt{-g}\,. \tag{A5}$$

The sources are the momentum

$$T^\mu{}_i = -\frac{1}{2\sqrt{-g}} \frac{\delta}{\delta h_\mu{}^i} \sqrt{-g}\, \mathscr{L}(\text{matter}) \tag{A6}$$

and the hypermomentum

$$T^\mu{}_{ij} = -\frac{1}{2\sqrt{-g}} \frac{\delta}{\delta \Gamma_\mu{}^{ij}} \sqrt{-g}\, \mathscr{L}(\text{matter}). \tag{A7}$$

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