THE INFRARED-ULTRAVIOLET CONNECTION

Dedicated to Jacques Prentki on the occasion of his sixtieth birthday.

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Physics below 300 GeV is termed infrared, and physics above 1 TeV is called ultraviolet. Some aspects of the relation between these two regions are discussed. It is argued that the symmetries of the infrared must be symmetries in the ultraviolet. Furthermore, naturalness within the context of the standard model is considered. It is concluded that there is either a threshold in the TeV region, or alternatively a certain mass formula holds. This formula, when true, might be indicative for an underlying supersymmetry.

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1. Introduction

Perhaps the most tantalizing question in particle physics is this: what is the structure, the spectrum and what are the forces at very high energies? High energy is here anything above 1000 GeV, i.e. large with respect to the Fermi mass. This is the mass given by the Fermi coupling constant of weak interactions, being about 350 GeV.

At this moment we do have a reasonably satisfactory theory below the Fermi mass, namely the standard model of quarks and leptons [1]. This model contains e.m., weak and strong interactions, all of them described by a gauge theory, with the symmetries U_1, SU_2 and SU_3.

On the other end of the scale we have the Planck mass, related to the coupling constant of gravitation, and of order 10^{19} GeV. There the situation is much less satisfactory, and all we can say is that perhaps supergravity holds a promise [2].

In between these two domains is a great deal of ignorance. Optimistically, Georgi et al. [3] have cooked up the SU_5 Grand Unified Theory, and the idea is that there is nothing all the way from the Fermi mass up to the unification point at 10^{17} GeV. Many

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physicists find this hard to swallow, and the most objective reason for this unwillingness is the unnaturalness of the SU_5 scheme.

Some time ago we (and also Linde) [4] have pointed out the unnaturalness of the cosmological constant in relation to spontaneous symmetry breaking. To this author this was reason to investigate the possibility that the Higgs be removed, i.e. made very heavy [5]. This problem was also addressed by the Yale group, [6] and a classification of the effects arising in this limit has been obtained. The upshot of this investigation is that the Higgs sector, in this limit, becomes the non-linear σ-model [7]. This model is non-renormalizable, and cut-off dependent effects can be expected. However, it was discovered that at low energy there is a screening effect, and the cut-off dependent terms become virtually unobservable. Whether or not the non-linear σ-model is a viable theory in four dimensions is not known; but it is very well conceivable that at least in some domain (say from 1 to 10 TeV) the theory behaves effectively like this model.

Another attack on the Higgs sector dates to the early days of renormalizable gauge theories, and amounts to the idea that the Higgs is composite, a bound state of two fermions [8]. Even further, strong interactions similar to quantum chromodynamics were suggested [9]. The crucial remark however was made by Susskind, reviving an old remark by Wilson, [10] who observed that scalar particles, in particular the Higgs, are unnatural. Following up on this argument the Stanford group and also Eichten and Lane and others have with great vigor investigated the possibility of composite Higgs, with new strong interactions called technicolor. Indeed, very interesting ideas, such as tumbling, have been forwarded [11].

The above approaches may be seen as an approach from the infrared, from below. There are also a number of papers that approach the problem from above, from the ultraviolet. Ellis et al. [12] have started from a supersymmetric theory, and on the basis of a "theorem" those parts that would survive in the infrared limit were identified. Even if there are many questionable points in the paper it is nevertheless very exciting that something resembling the known structure came out. In the process the discussion of anomalies becomes relevant, and we will come back to that in a moment.

Another refreshingly new approach was advocated by Foerster, Nielsen and Ninomiya [13], Maiani et al. and Iliopoulos et al. [14]. These authors propose that at high energy the world is sort of random, but that in the infrared only the renormalizable gauge symmetric theories as observed would survive.

Relevant to most of these ideas is the work of 't Hooft concerning anomalies [15]. In this work the guiding idea is that neither the infrared nor the ultraviolet theory should contain anomalies. In 't Hooft's work compositness of the known fermions is shown to be hardly compatible with this idea. To this author the idea of composite fermions has greatly lost attraction since. Yet the situation is really not so simple. One could well ask the following question: imagine that there are anomalies below 1 TeV (which there are not, as far as we know). A model for that would be the case that both top and bottom quark are heavier than 1 TeV, while the τ-lepton and its neutrino remain where they are now. Would such a theory display bad effects? One would not think so, bacause the kind of the non-renormalizable effects associated with anomalies are rather hidden, and quite buried in perturbation theory. The anomalies themselves are not even sensitive to the actual mass values. As a matter of fact, cut-off dependent effects due to anomalies appear only on the 3-loop level. Yet it is not possible to let the mass of fermions become large, since in this case there is no decoupling [16], not even at the one-loop level [17]. However, introduction of further heavy particles (scalars) can perhaps cure this problem, and it may be possible that under certain conditions anomalies are relatively harmless. A discussion on this point seems to be relevant, and it applies to the papers of Ellis et al. [12] (who also use absence of anomalies as a guiding principle), as well as those of the Stanford group (for instance, in a tumbling scheme, it need not be required that there is absence of anomalies at all stages). This particular investigation will be presented elsewhere.

The above discussion provides the background for the arguments of this paper. A number of points, relevant to some or most of the above approaches will be discussed. In particular, we will try to elaborate the "theorem", showing that it is really anything but a theorem, but rather an intuitive understanding, based on somewhat conjectural and incomplete arguments. Indeed, that describes reasonably well the contents of this paper.

In Sections 2, 3 and 4 we will discuss the "theorem". The idea is that the particles and forces as known today form the deep infrared part of some unknown theory. The "theorem" is then that such a system must necessarily be renormalizable. If these infrared forces are not renormalizable we would guess that particle masses would move up into the ultra-violet, and also non-perturbative effects would be manifest. In Section 3 we attempt to systematize the non-perturbative effects arising in various non-renormalizable theories. In Section 4 we adopt the point of view that the observed particle masses obtain as a perturbation onto a massless theory. This poses strong constraints on the infrared theory and in that view the "theorem" obtains some respectability.

In Section 5 we discuss the question of naturalness in the context of a gauge theory. This discussion leads to a certain definition of naturalness for the mass of the Higgs scalar and that is worked out in Section 6. In Section 7 the various options that nature may have chosen are listed. The option implying that a rather small Higgs mass may be natural if some mass relation holds is termed semi-natural. This mass-relation, implying a certain cancellation between bosonic and fermionic effects, would in this view be due to an underlying supersymmetry. While quite speculative, the relation has the virtue of being verifiable in the not too distant future.

2. *Infrared renormalizability*

In speculating about physics above 1 TeV one often introduces models [18] that give rise to low mass bound states. Usually the spectrum of those states as suggested by the model is considerably richer than the known "elementary" particles; moreover, the couplings between those particles are not evidently those observed. For instance, if the electron is a bound state it is quite difficult to understand the dynamics that would guarantee the observed, very particular, magnetic moment.

On the other hand, a substantial anomalous moment for the electron would generate, through radiative corrections, all kinds of effects of a non-perturbative nature. One would observe at low energy departures of a point like structure. In other words, the non-local structure of this bound state would extend outwards, through radiative corrections. The heavy constituents, extremely tightly bound to produce this very low mass electron, would seemingly momentarily escape the very small region of confinement. Also, for an anomalous magnetic moment of the same order as the normal magnetic moment one would get a self mass contribution to the electron mass of order Λ^2/m_e, where Λ is the inverse size of the bound state. Thus the electron mass could hardly be expected to be small with respect to Λ. To obtain a stable situation the dynamics must therefore carefully balance things out such that to very good approximation there is no anomalous magnetic moment.

Now nobody has suggested any dynamical system that would do such tight rope balancing. Surely, one would expect that in such a system there is a symmetry that guarantees the near vanishing of the small masses. Typically, chiral symmetry would be a candidate, but to this date nobody has found ways to implement this. Of course, that is the real problem: to find a model that produces very low mass states. But the outcome seems inevitable: the low mass states and their interactions must behave like a renormalizable theory. It is clear from the above discussion that it is very difficult, if not impossible, to construct a proof of this conjecture. Furthermore, some theories are more non-renormalizable than others. There are essentially two subjects that deserve discussion in this context. First, we will try to get some insight in the non-perturbative effects arising from non-renormalizability. Second, we will discuss the possibility that the small mass situation arises from a small perturbation onto a system that generates zero mass states due to some symmetry. This latter case is often thought of in connection with the presently observed fermion spectrum: somehow small perturbations of chiral invariance are thought to be responsible for the fermion masses.

3. *Non-renormalizable theories*

In discussing non-renormalizable theories we will suppose the existence of some threshold Λ above which the theory changes, and becomes in fact renormalizable. This may happen in very different ways: for instance the non-renormalizable four-fermion theory of weak interactions becomes the usual standard model for energies of the order of the vector boson mass. Alternatively, electrons, muons, etc. may show structure above 100 GeV, and break up in constituents that have renormalizable interactions. The question now is what Λ dependent effects could be expected at low energy, characterized by some energy scale $\lambda \ll \Lambda$. To be precise, we suppose that at energies λ the theory is well described perturbatively by a Lagrangian involving a small coupling constant g, but Λ, λ and g such that $g\Lambda/\lambda \gtrsim 1$.

In working out the perturbation theory for such a Lagrangian we will encounter series in the variable $\xi = g^2\Lambda^2/\lambda^2$. To facilitate the discussion we introduce the notation $S^m(\xi)$ for a series starting with ξ^m:

$$S^m(\xi) = a_m\xi^m + a_{m+1}\xi^{m+1} + \ldots$$

In general, we will have no detailed information on the coefficients a, other than that they are of order 1. Thus, for $\xi \gtrsim 1$ we have no idea of the magnitude of these divergent series. For the purposes of the discussion in this section we arbitrarily assume that $S^1(\xi)$ is of order 1. This assumption corresponds to the idea that the sum of all non-perturbative effects is of the same order as the cut-off independent contribution of the previous order in perturbation theory. Amputated series $S^m(\xi)$, $m > 1$ are supposedly badly behaving, we assume behavior as ξ^{m-1}. While being consistent with the former assumption it is nevertheless another arbitrary assumption.

In a non-renormalizable theory any measurable quantity will correspond to a series in the coupling constant that at sufficiently high order diverges as $\Lambda \to \infty$. In other words, we have series of the form

$$g^l\left(a_0+a_2g^2+a_4g^4+\ \dots\ a_ig^i\frac{\Lambda^2}{\lambda^2}+a_{i+2}g^{i+2}\frac{\Lambda^4}{\lambda^4}+\ \dots\right)$$

$$= g^l\{a_0+a_2g^2+a_4g^4+\ \dots\ a_{i-2}g^{i-2}+g^{i-2}S^1(\xi)\}.$$

The important question is at which order the cut-off dependence occurs for which amplitude. Related to this is the question of the unitarity limit.

Most non-renormalizable theories give rise in lowest order to some amplitude that for sufficiently high energy E_u exceeds the bounds required by unitarity. Then evidently the radiative corrections must be at least of the same order of magnitude as the lowest order amplitude, i.e. there is evident non-perturbative behavior. An exception to this rule are gauge theories with anomalies, such as the Weinberg model unsupplemented with quarks coupled according to the GIM scheme. The discussion of anomalies is, as usual, quite complicated, and will not be given here.

However, even if there is a unitarity limit for some amplitude, it may well be that E_u is larger than λ, i.e. outside the supposed domain of validity of perturbation theory. This may still be consistent with perturbative behavior in the domain λ. Such is the case of the current-current theory of weak interaction, where the unitarity limit in fermion-fermion scattering is of the order of 300 GeV. As an alternative example: if the electron had an anomalous moment equal to the normal moment, then in that case for electron-positron scattering the unitarity limit would be at about $m_e/\sqrt{\alpha} \simeq 10$ MeV.

However, even if E_u is well above λ there may be other non-perturbative effects. We will proceed by way of an example, namely the four fermion theory of weak interactions. Specifically we will take only the electron and muon part of the Lagrangian

$$\mathscr{L}_1 = Gj_\alpha j_\alpha^+, \qquad G = \frac{g^2}{m_p^2}, \qquad g^2 \sim 10^{-5}, \qquad m_p = \text{proton mass}$$

$$j_\alpha = \{\bar{\nu}_e\gamma^\alpha(1+\gamma^5)e\}+\{\bar{\nu}_\mu\gamma^\alpha(1+\gamma^5)\mu\}.$$

This theory is non-renormalizable, and there is a cut-off Λ above which another theory takes over. The parameter λ is of order of m_p, and is typical for the range of energy in which the theory has been tested.

Consider now ν_e e scattering. In lowest order we have a tree graph, giving rise to a contribution of order g^2. In the next order we have 3 diagrams (Fig. 1).

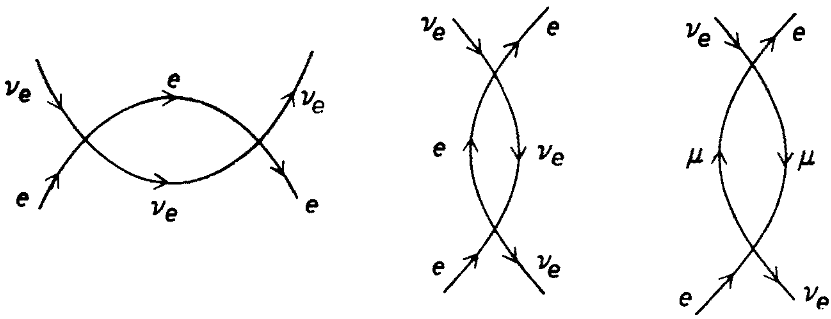

Fig. 1

These diagrams are quadratically divergent, and they give rise to a contribution of order $G^2 \Lambda^2$, i.e. $g^4 \Lambda^2/\lambda^2$. In the next order (a typical graph is shown in Fig. 2) we find contribution of order $g^6 \Lambda^4/\lambda^4$, and so on.

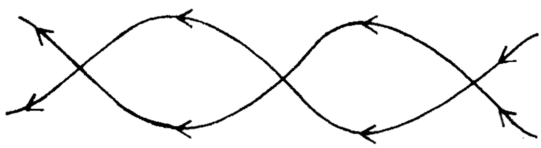

Fig. 2

Thus for the ν_e e amplitude at low energy we find a series of the form:

$$g^2\{a_0 + a_1\xi + a_2\xi^2 + \ldots\}, \qquad \xi = g^2\Lambda^2/\lambda^2$$

In this a_0 is of order 1, and the remaining series also sums up to something of order 1[1]. Thus we would expect a large deviation from naive perturbation theory, except that it is not measurable at low energy. The reason is that through renormalization of g this effect can be transformed away.

Another amplitude suffering large corrections is that for μ-decay. The situation is precisely as that for ν_e e scattering. However, the series in ξ is a different one, and the renormalization of g on the basis of ν_e e scattering will not neutralize that series. Thus now the correction becomes observable, and we can rule out values of Λ larger than λ/g ($\simeq$ 350 GeV), because experiments supposedly agree with some precision with the lowest order theory (the tree graphs).

What if accidentally the series in μ-decay would sum up to something very nearly

[1] Incidentally, taking only graphs of the form as in Fig. 2, repeated bubble insertion, then the series can be summed. The result is of the form $(1-\xi)^{-1}$, and shows occurence of resonances, both physical and unphysical, if ξ exceeds 1.

equal to that in ν_e e scattering? Well, by our criteria there would be no way to observe Λ dependence. The reason is that divergencies in all other amplitudes are much weaker and become less visible. Consider for instance the process $\mu \to \nu_\mu \bar{\nu}_e e\bar{e}e$. In lowest order that is zero. Next we have one loop graphs, see for example Fig. 3. This graph gives a con-

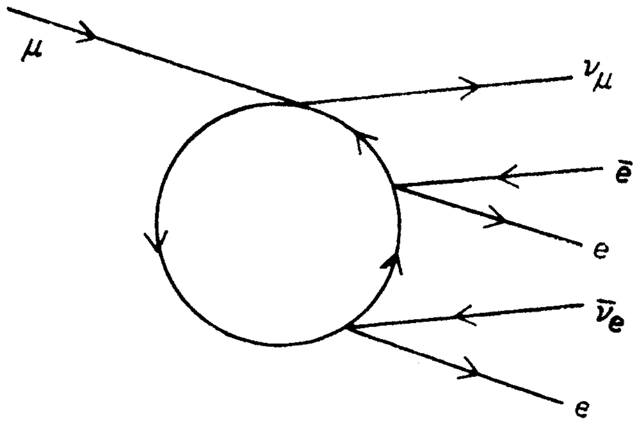

Fig. 3

tribution of the form $g^6 \ln \Lambda/\lambda$, which is still very small compared to lowest order μ-decay. If this graph had been divergent as Λ^4 then by out criteria one could have ruled it out. But now the Λ dependence effects come in as a series $g^6(\xi+\xi^2+\ldots)$, which is of order g^6.

The above shows how fragile the situation really is. The four-fermi theory, supplemented by quark currents, and with neutral and charm currents such as suggested by the standard model needs only a few accidents to escape conflict with experiment. We refer here to the data on ν_μ e scattering, μ-decay and β-decay.

An even more remarkable case is provided by the standard model however without Higgs mechanism. There the parameter λ is the vector boson mass, and g is roughly like the e.m. coupling constant e. It turns out that there is simply no observable correction of order Λ^2 at the one loop level, the worst case is $\ln \Lambda^2/\lambda^2$. By our criteria therefore nothing can be said on the range of validity of this theory. It could well be the infrared part of some theory with mass scale in the 10 TeV range and higher. This state of affairs has been referred to as the screening theorem [5, 6].

The situation becomes much clearer in the case of particles of spin 1 and higher. Such particles have propagators that behave as $(k^2)^i$, $i \geqslant 0$, where k is the four-momentum flowing through that propagator. Then amplitudes with 5 (or more) external legs are still large and observable by our criteria. For instance, consider the standard model without the quartic vector boson coupling, but with 2 vector boson, 2 photon coupling in order to retain electromagnetic gauge invariance. The 6 vector boson amplitude behaves then as $g^6\Lambda^4/\lambda^4$, which by our criteria is of order g^2, i.e. as large as the 4 vector boson amplitude. Even worse, the 3 vector boson amplitude behaves as $g^3\Lambda^4$, which seems intolerable for a perturbative theory[2]. Note however that no vector boson has yet been seen, let alone that perturbation theory has been tested.

[2] Note that g can be fixed by the coupling of the vector bosons to the leptons.

Thus in order for a theory containing particles of spin 1 and higher to be viable as an infrared perturbative theory we must require that there is a symmetry such that the worst divergencies cancel. For the case of vector bosons one can follow the arguments of Llewellyn-Smith, and Cornwall et al. [19] to deduce uniquely that there must be a local gauge invariance. Of course, we would stop at the introduction of the Higgs mechanism, since that is not needed to cancel the really bad divergencies.

In case of a theory containing particles of spin 3/2 and higher one may perhaps construct an argument along those same lines, and so arrive at a local supersymmetric theory. Thus we would argue that in the infrared particles of spin 3/2 and higher are acceptable only if there is a local supersymmetry. Barring supersymmetry, such theories are really unacceptable as perturbation theories, by our criteria, because propagators behave as $(k^2)^i$, $i > 0$, and generally one loop diagrams become progressively more divergent as the number of external legs increases.

We close this section by noting that we have not discussed the trivial case where the low mass particles interact solely with particles in the Λ region. Such particles would behave as non-interacting particles, provided that no effective self-interaction in the λ region is generated.

4. *Small mass approximation*

In this section we focus attention on the possibility that the infrared masses obtain as small perturbation on a zero mass situation. The crucial question is here the smoothness of the transition from zero mass to finite mass.

As is well known all zero mass particles of spin 1/2 or higher possess two (or one if we do not include parity conjugate states) degrees of freedom. Massive particles of spin 1 or higher possess more than 2 degrees of freedom, and in order for the transition to finite mass to be smooth something has to be done for particles with spin larger than 1/2. Furthermore, massless particles of spin 1 or higher need to be invariant to some local symmetry requirement, or else unphysical degrees of freedom arise (alternatively Lorentz invariance breaks down, or negative energy states are needed, etc.). In quantum-electrodynamics for instance, gauge invariance renders the negative norm states harmless in the context of the Gupta-Bleuler scheme [20]. All of this is well-known, and needs no further discussion here.

Thus the massless theory obeys some local symmetry. In passing to the massive case extra degrees of freedom are needed. Thus extra particles are needed that become ghosts in the transition. Spontaneously broken gauge theories are of course the perfect example of this case. Thus it is at least conceivable that such gauge theories obtain as a perturbation theory on a zero mass situation. One must however deal with the scalar particles, for which we know of no way to guarantee zero mass other than by a supersymmetry. This is a separate problem, to be discussed later.

It may also happen that the extra particles decouple completely. This happens in the Higgs model [21] and it is for this reason that the limit of massive to massless photons is continuous. The extra degree of freedom decouples. However, this is the only case where this happens. Thus without the introduction of extra particles there are discontinuities for non-abelian spin 1 systems, for spin 2 (gravitation) [22], for spin 3/2 [23], and higher [24]. From the point of view of perturbation theory in the masses the only acceptable objects (apart from spin 0 and spin 1/2 particles) are spontaneously broken gauge theories, or supersymmetries.

Let us now investigate this in some more detail. In the unbroken gauge theory one has invariance with respect to transformations involving scalar gauge functions $\Lambda(x)$. For instance, in an SU_2 model, there are three vector bosons and four scalars, and the infinitesimal transformation laws are

$$W_\mu^a \to W_\mu^a + g\varepsilon_{abc}\Lambda^b W_\mu^c - \partial_\mu \Lambda^a,$$

$$\phi^a \to \phi^a + \tfrac{1}{2}\, g\varepsilon_{abc}\Lambda^b\phi^c - \tfrac{1}{2}\, gH\Lambda^a,$$

$$H \to H + \tfrac{1}{2}\, g\Lambda^a\phi^a.$$

If the Higgs field H develops a vaccuum expectation value $\langle H \rangle = 2M/g$ then the transformation law for the ϕ^a acquires the constant part $-M\Lambda^a$. This part, not containing the coupling constant g, shows that ϕ can be transformed away, and thus are now ghosts. Of course, these degrees of freedom reappear in the vector fields.

Similarly, in case of a spontaneously broken supersymmetry one has a spin 1/2 particle having a term of the form $S\varepsilon$ in its transformation law

$$\psi \to \psi + gS\varepsilon.$$

In this ε is the anticommuting infinitesimal super gauge function, and S is some scalar field. If this field S develops a vacuum expectation value the ψ becomes a ghost, and conceivably a spin 3/2 particle may become massive. Indeed, this is the way that it can be done [25].

If we now want a spin 2 particle to become massive, then there must be a spin 1 particle that turns into a ghost. For reasons of Lorentz invariance the only particles that can be allowed to have a vacuum expectation value are scalar particles[3]. That means that we require a spin 1 particle with a term of the form $S\Lambda_\mu$ in its trnsformation law, where the Λ_μ are now the gauge functions as occurring in the transformation laws of the spin 2 particle (the graviton), and where S is a scalar field. The point is now that there exist no such transformations forming a group[4], and this holds also for particles with spin higher than 2 [5].

[3] It is anybody's guess why nature would want to keep the vacuum Lorentz invariant. See also Ref. [30].

[4] In the case at hand, we know of course precisely what can and must be, since this is just the theory of gravitation interacting with vectors and scalars.

[5] J. Scherk, private communication, 1972. The significance of this fact is only now becoming clear to me. The argument was not discussed in detail, but by considering the possible transformation laws of the scalar and the higher spin particle the result becomes quickly transparent. At that time this investigation arose from the question whether renormalizable theories of particles with spin larger than 1 could be constructed, and supersymmetries were not considered.

The conclusion of this section is now clear: from a mass-perturbative point of view there may be scalars, spin 1/2 particles, or spontaneously broken gauge theories or supersymmetric gauge theories. Furthermore, these theories must be anomaly free, because massless theories are meaningless if they are not anomaly free (they would contain unphysical degrees of freedom).

5. Naturalness and regularization

The arguments of Section 3 deal with non-renormalizable theories, and rely on the fact that in such theories observable effects are cut-off dependent. In renormalizable theories the cut-off dependence is not observable, and can be absorbed in the parameters of the theory, e.g. coupling constants and masses.

Nevertheless it is possible to say something by introducing the criterium of naturalness. This criterium is that radiative corrections are supposed to be of the same order (or much smaller) than the actually observed values. And this then is taken to apply also for coupling constants and masses. Symmetries may be important here too; radiative corrections may be made small if threre is a symmetry guaranteeing this smallness. This has recently been discussed in detail by 't Hooft [15], and we refer to this work for further details. An example of this kind of naturalness is the well-known vector boson mass relation $M/M_0 = \cos\theta_w$, which relation follows to good approximation of the Higgs sector has the symmetries of the σ-model.

One of the most interesting applications concerns the Higgs mass in the standard model. The radiative corrections to this mass are quadratically divergent, and the mass is thus expected to be of order $g^2\Lambda^2$, where Λ is a cut-off. In order to keep this mass (and also the vacuum expectation value of the Higgs, giving rise to the vector boson mass) in the 100 GeV range it follows that Λ should be in the 1000 GeV range. This observation is then the underlying rationale for the technicolor constructs [10, 11].

This reasoning contains two gaps that we would like to discuss. The first gap concerns the actual cut-off dependence, which requires a specification of the cut-off mechanism. The second gap is that there may be an underlying supersymmetry that leads to cancellations of these quadratic divergencies.

Let us now first discuss the question of cut-off dependence. The most popular scheme in connection with gauge theories is the dimensional regularization method. This method is unphysical, in the sense that for $n \neq 4$ the theory is unphysical. For comparison, consider the (non-renormalizable) non-linear σ-model. This is the limit of the linear σ-model where some mass becomes infinite. But the linear σ-model is physically acceptable, and can be seen as a physically realizable cut-off method for the non-linear σ-model.

Now we have really no idea what cut-off method compares to the case of a composite Higgs particle. For relatively low energies one sees a point particle. For energies above the cut-off one sees the constituents. Compare for instance an atom. At low energies an atom is a point particle, and we observe a Lennard-Jones potential. At higher energies we see an electron and a proton, and Coulomb forces. It would be very difficult to describe the behavior in the transition region. And yet, in the case of gauge theories, this is of crucial importance.

The basic question at issue is the algebra of the cut-off dependence[6]. Consider the standard model. In the unitary gauge (and this is always the starting point if one wants to discuss physics of divergencies) the one loop diagrams with internal vector bosons are very divergent. For instance, the 6 boson amplitude has at the one loop level a divergence as Λ^{10}. But of course such divergencies cancel out, provided they are treated according to the rules of the symmetry. Only if the divergencies obey an algebra in harmony with the symmetry a renormalizable theory results. And the dimensional regularization scheme provides for such an algebra.

This question becomes very significant if one suspects that there is an actual physical cut-off at high energies. Then the transition from point particles to constituents must very precisely go such that the algebra of the cut-off dependence is respected. Thus if the vector bosons were composite objects, then the way they dissolve in their constituents is very strongly constrained, or else for example for the 6 boson amplitude, some of the divergences, ranging from Λ^{10} to Λ^2 will not cancel out precisely, with disastrous results. In fact, this seems to us utterly fantastic. We therefore believe that it is absolutely necessary that the symmetry of the theory remains valid throughout the cut-off range. In other words, the gauge symmetries of the infrared must be the symmetries in the ultra-violet. We consider this a constraint onto the philosophy of Foerster et al. [13]. In the actual case they consider the symmetry is broken by a mass term only, which leaves the cancellation of most infinities unaffected. Symmetry breaking, by for instance a very small quartic vector boson interaction, is much more violent.

Thus, if we consider divergencies it seems best to use dimensional regularization, since this guarantees the correct algebra. But in this scheme integrals such as

$$\int d_n q (q^2)^i$$

are zero, and a naive person could conclude that there are no quadratic divergencies, the behavior in the transition range to be such that the various contributions cancel. However, that need not be true. Furthermore the sceptical reader may point out that the algebra is somewhat less important with respect to the Higgs mass. In fact, in the unitary gauge the Higgs mass receives at the one loop level contributions at most proportional to Λ^4.

A suitable criterion, within the framework of dimensional regularization, is the occurrence of poles in the complex dimensional plane for n less than four. Thus naive quadratic divergencies at the one loop level correspond to poles for $n = 2$. We therefore inquire after the existence of poles for $n = 2$ in the standard model.

6. *Quadratic divergencies*

In the standard model, and within the dimensional regularization scheme poles for $n = 2$ occur in vector boson and Higgs self energy diagrams, and in tadpole diagrams. They also occur in connection with the cosmological constant, but we will not discuss

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[6] I recall very early conversations with 't Hooft in which he mentioned this concept, although unrelated to the points that we discuss here.

that explicitly. We work in the 't Hooft-Feynman gauge throughout, and we must take care to obtain gauge invariant results. This may be done following the procedure as outlined in an earlier paper [26].

Following methods[7] introduced before [27] we express the contributions of the various diagrams in terms of a few basic functions

$$A(m) = \int d_n q \, \frac{1}{q^2+m^2},$$

$$B_0;\, B_\mu;\, B_{\mu\nu} = \int d_n q \, \frac{1;\, q_\mu;\, q_\mu q_\nu}{(q^2+m_1^2)((q+k)^2+m_2^2)},$$

$$B_0 = B_0(k, m_1, m_2),$$

$$B_\mu = B_1(k, m_1, m_2)k_\mu,$$

$$B_{\mu\nu} = B_{21}(k, m_1, m_2)k_\mu k_\nu + B_{22}(k, m_1, m_2)\delta_{\mu\nu}.$$

Of these functions only A and B_{22} are quadratically divergent, i.e. contain poles for $n = 2$. Most of the result of these calculations for $n = 4$ has been given before; in Appendix A the results are given keeping n explicitly as a variable. We work within the standard model, and of the various parameters both the mass of the top quark and the Higgs are yet unknown. Summing up these contributions, and keeping only the A and B_{22} terms we have the following result, in terms of an effective Lagrangian

$$\begin{aligned}\mathscr{L}(\text{eff., 1 loop}) = &-M^2 W_\mu^+ W_\mu^- Z_W - \tfrac{1}{2}\frac{M^2}{c^2} W_\mu^0 W_\mu^0 Z_W^0 \\ &-M^2\phi^+\phi^- Z_\phi^+ - \tfrac{1}{2}\frac{M^2}{c^2}\phi^0\phi^0 Z_\phi^0 - \tfrac{1}{2} m_H^2 H^2 Z_H \\ &-2MHZ_T,\end{aligned} \tag{6.1}$$

$$Z_W^+ = -\frac{g^2}{M^2}\{3-2n+4(aa'+bb')\}(A-2B_{22})/(2\pi)^4 i,$$

$$Z_W^0 = -\frac{g^2c^4}{M^2}\left\{2+\frac{2}{c^2}-\frac{1}{c^4}-2n+4(aa'+bb')\right\}(A-2B_{22})/(2\pi)^4 i,$$

$$Z_\phi^+ = -\frac{g^2}{M^2}\left\{(1-n)\left(\tfrac{1}{2}+\frac{1}{4c^2}\right)-3\alpha-4(a^2+b^2)\right\}A/(2\pi)^4 i,$$

$$Z_\phi^0 = c^2 Z_\phi^+; \qquad Z_H = \frac{g}{m_H^2} Z_T,$$

$$Z_T = g\left\{(1-n)\left(\tfrac{1}{2}+\frac{1}{4c^2}\right)-3\alpha+\sum_f \frac{m_f 2}{M^2}\right\}A/(2\pi)^4 i.$$

[7] For the purposes of this paper one could also do the calculations in the unbroken theory.

We have not bothered to work out in detail the total fermion contribution to Z_W^+, Z_W^0, Z_ϕ^+ and Z_ϕ^0. Note that $\alpha = m_H{}^2/4M^2$. In the original Lagrangian the relevant terms are

$$-\tfrac{1}{2} m_H^2 H^2 - \alpha g M H^3 - \alpha g M H(\phi^{02} + 2\phi^+\phi^-) - gMHW_\mu^+ W_\mu^-$$

$$-\tfrac{1}{2}\frac{M}{c^2} gHW_\mu^0 W_\mu^0 + \tfrac{1}{2} HW_\mu^+ \partial_\mu \phi^- + \tfrac{1}{2} gHW_\mu^- \partial_\mu \phi^+ + \frac{g}{2c} HW_\mu^0 \partial_\mu \phi^0$$

$$-M^2 W_\mu^+ W_\mu^- - \tfrac{1}{2}\frac{M^2}{c^2} W_\mu^0 W_\mu^0 - M^2\phi^+\phi^- - \tfrac{1}{2}\frac{M^2}{c^2}\phi^0\phi^0. \tag{6.2}$$

Remember that there is no $W_\mu \partial_\mu \phi$ term due to our choice of gauge. In other words, the $W_\mu \partial_\mu \phi$ term of the gauge invariant Lagrangian cancels against that of the gauge fixing term.

Now, the original Lagrangian yields a contribution that precisely equals that of the effective Lagrangian if we make the substitutions

$$H \to H + 2\frac{M}{m_H^2} Z_T,$$

$$m_H^2 \to m_H^2 - 3gZ_T + m_H^2 Z_H, \tag{6.3}$$

$$M^2 \to M^2 - 2g\frac{M^2}{m_H^2} Z_T + M^2 Z_W^+,$$

$$\frac{M^2}{c^2} \to \frac{M^2}{c^2} - 2g\frac{M^2}{c^2 m_H^2} Z_T + \frac{M^2}{c^2} Z_W^0. \tag{6.4}$$

Note that the Z_ϕ^0 and Z_ϕ^+ cancel against the terms generated by the shift of the Higgs field H in the terms $H\phi^+\phi^-$ and $H\phi^0\phi^0$.

We draw attention to the fact that this procedure leads to gauge invariant replacements, as shown in Ref. [26]. The shift in the Higgs field, giving rise to Z_T terms in the mass renormalizations, is essential to this result.

Now the poles for $n = 2$. The n-dependence is contained in the functions A and B_{22} apart from the explicitly shown dependence. Since in two dimensions

$$\int d_n q \frac{q_\nu q_\mu}{(q^2+m^2)^2} = \tfrac{1}{2}\delta_{\mu\nu} \int d_n q \frac{1}{q^2+m^2} = \tfrac{1}{2}\delta_{\mu\nu} A \quad (n = 2 \text{ poles}), \tag{6.5}$$

we see that the coefficient of $1/(n-2)$ in B_{22} is half that of A. Then only Z_H and Z_T contain divergencies

$$Z_T, Z_H \propto \left\{(1-n)\left(\tfrac{1}{2} + \frac{1}{4c^2}\right) - \frac{3m_H^2}{4M^2} + \sum_f \frac{m_f^2}{M^2}\right\} A, \tag{6.6}$$

where the sum is over all fermions in the theory (insofar as they are coupled to the Higgs like the usual quarks and leptons). We see that both the vector boson mass and the Higgs mass get quadratically divergent contributions, but that is all. The ratio of m_H^2 to M^2 is however finite.

Customarily one would write $A = i\pi^2 \Lambda^2$, with Λ the cut-off of the theory. Then Z_T is negative, unless the fermion contribution becomes large due to higher mass fermions, as yet unknown.

7. *Naturalness and semi-naturalness*

What conclusions can be drawn from the results of the previous section? Since we are speaking about essentially unobservable effects it is really extremely difficult. We have no knowledge on the values of the vector boson and Higgs mass before the radiative corrections are taken into account, and that effectively precludes any hard conclusions. Let us try to systematize things, and let us for a start write $A = i\pi^2 \Lambda^2$, and try to deduce something about Λ. We may now distinguish three cases:

(*i*) The fermion contribution is relatively small, i.e. such that $Z_T < 0$.

(*ii*) The fermion contribution cancels the other contributions so that $Z_T = 0$.

(*iii*) The fermion contribution dominates.

In the latter case Z_T becomes positive, and no meaningful statement can be made. It seems that this case (case *iii*) must be considered unphysical. This leaves us with two alternatives.

Let us now introduce the parameter $\kappa = m_H^2/M^2$. In terms of this parameter we have

$$m_H^2 \to m_H^2 - 2gZ_T, \qquad M^2 \to M^2 - \frac{2g}{\kappa} Z_T,$$

$$Z_T \equiv g\left\{(1-n)\left(\tfrac{1}{2} + \frac{1}{4c^2}\right) - \tfrac{3}{4}\kappa + \sum_f \frac{m_f^2}{M^2}\right\}\frac{\Lambda^2}{16\pi^2},$$

where we have substituted $A = i\pi^2 \Lambda^2$. As we will argue below we must take $n = 4$ in the equation for Z_T.

Next we assume that the radiative correction to M^2 is of the same order of magnitude as the experimentally observed value (i.e. $M \sim 75$ GeV). Thus

$$-\frac{2g}{\kappa} Z_T = (75 \text{ GeV})^2.$$

Solving this for Λ we find

$$\Lambda \equiv (75 \text{ GeV})\frac{4\pi}{g}\sqrt{\frac{\kappa}{3 + 3/2c^2 + 3\kappa/2 - \sum m_f^2/M^2}}. \tag{7.1}$$

This may be worked out somewhat further if we substitute the numerical values $s^2 = 0.22$ and $g^2/4\pi = 1/(137 s^2) \cong 1/30$. This gives $4\pi/g = 19.5$. We then get

$$\Lambda = (1460 \text{ GeV})\sqrt{\frac{\kappa}{4.9 + 3\kappa/2 - \sum m_f^2/M^2}}. \tag{7.2}$$

If we assume that the top quark mass is substantially below the vector boson mass, and if we assume that there are no further fermion generations, then the fermion contribution m_f^2/M^2 can be ignored. In that case, for small κ also Λ is small, and in fact, since Λ cannot be too small this poses a limit on κ. If $\kappa = 1$ then we find 580 GeV for Λ. The expected Higgs mass is in that case equal to the vector boson mass. In the limit of large κ we find $\Lambda = 1200$ GeV, and a very large Higgs mass.

These cases correspond physically to situations where a strongly interacting system emerges above Λ. This may be like technicolor, or, in particular for the large Higgs mass cases, like the non-linear σ-model.

Let us now consider case (*ii*). Thus suppose that the fermionic contribution cancels precisely the bosonic contribution. Then Λ may be infinite, or rather there are no quadratic divergencies at the one loop level. As a matter of fact, there are no further poles at $n = 2$ for higher order in perturbation theory. At the m-loop level, a quadratic divergence corresponds to a pole at $n = 4-2/m$, for instance at $n = 3$ for a two loop quadratic divergence. Thus to eliminate all quadratic divergencies infinitely many conditions must be met. One may ask if this is conceivable. The answer is yes, because there exist supersymmetric theories where bosonic and fermion masses are related, and since fermion masses are only logarithmically divergent, so would boson masses. The second question is whether this may happen in the standard model. There the answer cannot be given; while at the one-loop level only a restricted set of particles participates, on the two-loop level all kinds of particles up to the Planck mass enter in the discussion. For example, very heavy bosons associated with a horizontal symmetry would enter in the calculation[8].

Thus it would be very interesting if the relation

$$\sum \frac{m_f^2}{M^2} = \tfrac{3}{2} + \frac{3}{4c^2} + \tfrac{3}{4}\frac{m_H^2}{M^2} \tag{7.3}$$

would hold[9]. Numerically, if $m_H^2 \ll M^2$ we may solve for the top quark mass (occurring 3 times), with the result $m_t = 69$ GeV. For increasing Higgs mass also m_t increases, and if $m_H = M$ we find $m_t = 77.5$ GeV.

Another possibility is that the top quark is relatively light (say, 20 GeV), and that there is yet another fermion generation.

If this equality holds then this would be very suggestive of an underlying supersymmetry. While we have no idea how this would come about we nevertheless can say that there is a chance that this happens. This in contrast to cases where fermionic and bosonic contributions have the same sign. We therefore would like to call this case "semi-natural": there is a fighting chance that supersymmetry removes all quadratic divergencies. In that case the cut-off Λ could be very high, since all divergencies are now logarithmic. The Ellis et al. [12] attempt is then something in line with this idea, and furthermore consistent with all other observations made in the previous sections.

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[8] Thanks are due to Dr. Yao and Dr. Wilson for a discussion on this point.

[9] The quantities involved are those of the bare Langrangian, and differ by radiative corrections from the experimental ones.

Let us examine this possibility in slightly more detail. The underlying idea is that supersymmetry is responsible for a conspiracy that leads to the equation:

$$\sum \frac{m_f^2}{M^2} = (n-1)\left(\tfrac{1}{2} + \frac{1}{4c^2}\right) + \frac{3m_H^2}{4M^2}\,. \tag{7.4}$$

The summation is over all fermions that are coupled to the Higgs system precisely like the known generations. The parameter n is the dimension of space-time, and in the original concept of dimensional regularization we would have to take it equal to 2 since we are considering the residue of a pole at $n = 2$. However, as the reader may suspect from the above equation, the n-dependence here derives from the number of degrees of freedom of a vector boson in n dimensions. One would suspect that in a supersymmetry scheme this number is to be kept fixed. Indeed, in recent work by Siegel [28] and Capper, Jones and Van Nieuwenhuizen [29], it has been worked out in detail how dimensional regularization must be modified in order to have a scheme suitable also for supersymmetric theories. In that scheme the number of vector degrees of freedom is kept at four. We therefore, following their prescriptions, have taken $n = 4$ in the above equation.

It should be noted that Eq. (7.4) is sensitive to very heavy particles. For instance, the SU_5 heavy X and Y bosons contribute comparably to the SU_2 bosons. However, constraints following from the experimental validity of the equation for the vector boson masses, $M/M_0 = \cos\theta_W$, put severe limits on possible heavy fermions.

8. Conclusions

The interesting question to this author is whether the mass-relation (7.4) holds. In view of the fact that there are bounds on fermion masses (and mass differences) we would expect fermions relevant to this equation to be below 300 GeV in mass. Note that only fermions coupled to the SU_2 breaking Higgs system enter this equation. Thus the next generation of accelerators would presumably give exhaustive information on the validity of this equation. If the equation holds, then the future of supersymmetry seems assured.

Great profit has been derived from lively discussions at SLAC during this summer, in particular with V. Baluni, K. Lane and L. Susskind. Furthermore thanks are due to my colleagues at Michigan University, in particular M. Einhorn and E. Yao.

APPENDIX A

Self-energies and tadpoles

Most of the self-energy contributions in terms of the A and B functions have been listed in Ref. [27]. There the limit $n = 4$ was taken, but here we want to keep the n-dependence since we are interested in the poles for $n = 2$. Most of this dependence resides in the A and B functions, so that the same equations can be used, but there are some exceptions, notably when there is a sum over the vector boson degrees of freedom. For these cases the equations will be rewritten here. Referring to page 157 and following of Ref. [27], the changes are (note that the Higgs is denoted by Z in this reference):

Graph S_1. Define

$$A_1 = -\{(m_1^2+m_2^2-4k^2)B_0(k, m_1, m_2)-A(m_1)-A(m_2)+(4n-6)B_{22}(k, m_1, m_2)\},$$

$$B_1 = -\{-(4n-6)B_{21}(k, m_1, m_2)-(4n-6)B_1(k, m_1, m_2)+(n-6)B_0(k, m_1, m_2)\}.$$

Hopefully the reader is not confused by the use of two different B_1.

Graph S_2

$$A_2(m) = (1-n)A(m) \quad \text{and} \quad A_2^+ = A_2(M)+c^2A_2(M_0)+s^2A_2(0).$$

Furthermore there are the fermion contributions. Assigning the number S_9 to the graph of Fig. A1 we find the contribution:

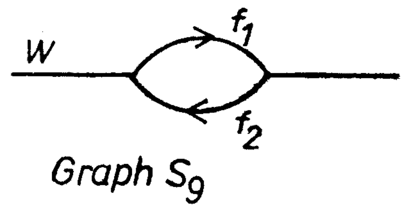

Fig. A1

Graph S_9

$$A_9 = (aa'+bb')\{-8B_{22}(k, m_1, m_2)+2A(m_1)+2A(m\ )$$
$$-2(k^2+m_1^2+m_2^2)B_0(k, m_1, m_2)\}$$
$$+4(aa'-bb')m_1m_2B_0(k, m_1, m_2),$$
$$B_9 = -8(aa'+bb')\{B_{21}(k, m_1, m_2)+B_1(k, m_1, m_2)\}.$$

Right resp. left the vertices $\gamma^\mu(a+b\gamma^5)$ and $\gamma^\mu(a'+b'\gamma^5)$ are supposed. One has for the couplings to e, ν_e, u-quark, d-quark

$$A_\mu(\text{ee})-s\gamma^\mu; \quad A_\mu(\text{uu})\,\tfrac{2}{3}\,s\gamma^\mu; \quad A^\mu(\text{dd})-\tfrac{1}{3}\,s\gamma^\mu,$$

$$W_\mu^+(\text{e}\nu)\,\frac{1}{2\sqrt{2}}\,\gamma^\mu(1+\gamma^5); \quad W^+(\text{ud})\,\frac{1}{2\sqrt{2}}\,\gamma^\mu(1+\gamma^5),$$

$$W^0(\nu\nu)\,\frac{1}{4c}\,\gamma^\mu(1+\gamma^5); \quad W^0(\text{ee})\,\frac{1}{4c}\,\gamma^\mu(4s^2-1-\gamma^5),$$

$$W^0(\text{dd})\,\frac{1}{4c}\,\gamma^\mu(\tfrac{4}{3}\,s^2-1-\gamma^5); \quad W^0(\text{uu})\,\frac{1}{4c}\,\gamma^\mu(1-\tfrac{8}{3}\,s^2+\gamma^5).$$

We ignore Cabibbo mixing.

Of the tadpole diagrams only that with an internal W line needs to be mentioned.

$$T_3 = -nM\left\{A(M)+\frac{1}{2c^2}\,A(M_0)\right\}.$$

Further there is a contribution due to fermions, Fig. A2. We denote that as graph T_6. The contribution is the same for all fermions in the standard model

$$T_6 = 2\,\frac{m_f^2}{M}\,A(m_f)$$

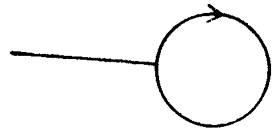

Fig. A2

Graph H_1

$$H_1^+ = nM^2\left\{\frac{s^4}{c^2}\,B_0(k, M_0, M) + s^2 B_0(k, 0, M)\right\}.$$

Graph H_7

$$H_7^+ = n\left\{\left(s^2 - \frac{1}{4c^2}\right)A(M_0) - s^2 A(0) - \tfrac{1}{2}\,A(M)\right\},$$

$$H_7^0 = -n\left\{\frac{1}{4c^2}\,A(M_0) + \tfrac{1}{2}\,A(M)\right\}.$$

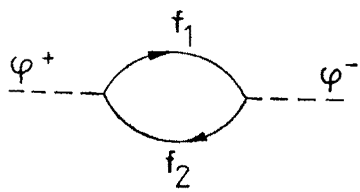

Fig. A3

Graph H_{11}. The fermion insertion, Fig. A3
Right $a+b\gamma^5$, left $a-b\gamma^5$

$$H_{11} = 4\,[-\tfrac{1}{2}\,(a^2+b^2)\,\{A(m_1)+A(m_2)-\tfrac{1}{2}\,(k^2+m_1^2+m_2^2)B_0(k, m_1, m_2)\}$$
$$+(a^2-b^2)m_1 m_2 B_0(k, m_1, m_2)].$$

The coefficients a and b for the various cases are

$$\phi^+(\mathrm{e}\nu) - \frac{m_e}{2\sqrt{2}M}\,(1-\gamma^5); \quad \phi^+(\mathrm{ud})\,\frac{1}{2\sqrt{2}M}\,\{m_d(1-\gamma^5)+m_u(-1-\gamma^5)\},$$

$$\phi^0(\mathrm{ee}) - \frac{m_e}{2M}\,\gamma^5; \quad \phi^0(\mathrm{uu})\,\frac{m_u}{2M}\,\gamma^5; \quad \phi^0(\mathrm{dd}) - \frac{m_d}{2M}\,\gamma^5.$$

The subscript 1 refers to the first fermion, e.g. the electron for the $\phi^+(\mathrm{e}\nu)$ case.
Graph R_5

$$R_5^+ = (1-n)s^2 R_3(M_0, M) + (1-n)s^2 R_3(0, M).$$

Graph R_6 The fermion insertion, Fig. A4.

Right $\gamma^\mu(a+b\gamma^5)$, left $a'+b'\gamma^5$.

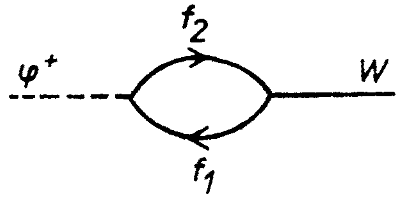

Fig. A4

$$R_6 = -4(aa'-bb')\frac{m_1}{M}\{B_0(k, m_1, m_2)+B_1(k, m_1, m_2)\}$$

$$-4(aa'+bb')\frac{m_2}{M}B_1(k, m_1, m_2).$$

The coefficients a, b are as given with graph S_9, and a', b' as graph H_{11}.

Finally there are the physical Higgs self-energy diagrams. There are 11 cases, see Fig. A5.

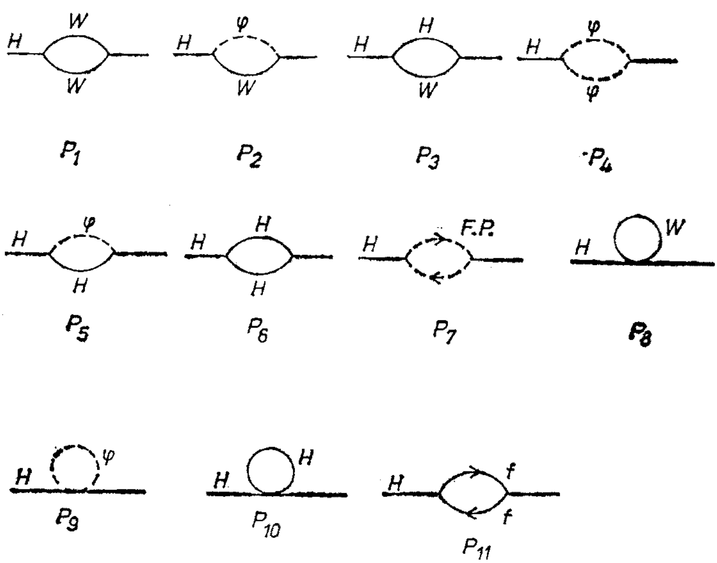

Fig. A5

The various contributions are

$$P_1 = nM^2\left\{B_0(k, M, M)+\frac{1}{2c^4}B_0(k, M_0, M_0)\right\},$$

$$P_2 = \frac{1}{4c^2}\{A(M_0)+(k^2-M_0^2)B_0(k, M_0, M_0)-2k^2B_1(k, M_0, M_0)\}$$

$$+\tfrac{1}{2}\{A(M)+(k^2-M^2)B_0(k, M, M)-2k^2B_1(k, M, M)\},$$

$$P_3 = 0,$$

$$P_4 = 2\alpha^2 M^2 \{B_0(k, M_0, M_0) + 2B_0(k, M, M)\},$$

$$P_5 = 0,$$

$$P_6 = 18\alpha^2 M^2 B_0(k, m_H, m_H),$$

$$P_7 = -\tfrac{1}{2} M^2 \left\{ B_0(k, M, M) + \frac{1}{2c^4} B_0(k, M_0, M_0) \right\},$$

$$P_8 = -\frac{n}{4} \left\{ 2A(M) + \frac{1}{c^2} A(M_0) \right\},$$

$$P_9 = -\alpha \{A(M) + \tfrac{1}{2} A(M_0)\},$$

$$P_{10} = -\tfrac{3}{2} \alpha A(m_H),$$

$$P_{11} = -\frac{m_f^2}{M^2} \{-A(m_f) + 2m_f^2 B_0(k, m_f, m_f) - k^2 B_1(k, m_f, m_f)\}$$

The contribution of all fermions in the standard model is the same, as given by P_{11}.

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