DIFFRACTIVE AND NON-DIFFRACTIVE PROCESSES IN SCATTERING OF HIGH-ENERGY HADRONS FROM NUCLEAR TARGETS

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We reanalyze the probabilistic description of inelastic hadron-nucleus and nucleus-nucleus collisions with diffractive channels present. We give several formulae which may be useful in analyzing data on multiparticle production in high-energy nuclear scattering.

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1. Introduction

It was shown some time ago [1] that the classical probability calculus can be successfully applied to the description of inelastic hadron-nucleus collisions at high energies. Indeed, in the absence of diffractive processes the formula for inelastic hadron-nucleus cross-section obtained in this way generates, via optical theorem, the correct Glauber formula [2] for the elastic hadron-nucleus amplitude. Also the quasi-elastic cross-section comes out right [1].

In the present paper we extend this application of the probability calculus by taking into account the presence of the diffractive channels (which were neglected in Ref. [1]) in hadron-nucleus and nucleus-nucleus scattering. These diffractive channels are represented in the language of Good and Walker [3, 4] through the eigenmodes which are the eigenstates of the absorption operator and hence propagate through nuclear matter without any diffractive excitations. The methods of Ref. [1] are then applied to these eigenmodes and the formulae for different coherent and incoherent cross-sections can be written down. In particular we calculate the expressions for (a) elastic, (b) diffractive coherent, (c) quasi-

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-elastic, (d) diffractive-incoherent (quasi-diffractive) and (e) non-diffractive hadron-nucleus and nucleus-nucleus cross-sections. The physical meaning of the obtained formulae is discussed and the Glauber model-type approximations are obtained.

In the next two Sections it is shown how the argument of Ref. [1] can be applied to the absorption eigenstates. In Section 4 the formulae for different hadron-nucleus cross-sections are obtained. The Glauber model-type approximations [2] are derived and discussed in Section 5. In Section 6 our analysis is extended to nucleus-nucleus interactions and the Glauber model-type approximations for such interactions are discussed in Section 7. The last Section contains summary and conclusions.

2. *Propagation of the absorption eigenstates in nuclear matter*

Let us consider first hadron-nucleon scattering. Following Good and Walker [3] we expand the states of incident hadrons into the eigenstates of absorption operator:

$$|h; h_T\rangle = \sum_{\alpha,\beta} |\alpha; \beta\rangle \langle\alpha; \beta|h; h_T\rangle, \tag{2.1}$$

where h and h_T denote beam and target hadrons.

The eigenstates $|\alpha; \beta\rangle$ are, by definition, either absorbed or unchanged during the collision. The interaction of the states $|\alpha\rangle$ and $|\beta\rangle$ can thus be described as follows:

Let $\sigma_{abs}(\alpha; \beta; \boldsymbol{b})$ be the probability of absorption of the state $|\alpha; \beta\rangle$ in the collision at impact parameter $\boldsymbol{b}$. The total absorption cross-section is

$$\sigma_{abs}(\alpha; \beta) = \int d^2b\sigma_{abs}(\alpha, \beta; \boldsymbol{b}). \tag{2.2}$$

Assuming that the elastic amplitude $\eta(\alpha; \beta; \boldsymbol{b})$ is purely imaginary, we obtain from the unitarity condition [5]

$$\eta(\alpha; \beta; \boldsymbol{b}) = \tfrac{1}{2}\,\sigma_{tot}(\alpha; \beta, \boldsymbol{b}) = 1-\sqrt{1-\sigma_{abs}(\alpha; \beta; \boldsymbol{b})} \tag{2.3}$$

where $\sigma_{tot}(\alpha; \beta; \boldsymbol{b})$ is the probability of any interaction at impact parameter $\boldsymbol{b}$, with the total cross-section given by

$$\sigma_{tot}(\alpha; \beta) = \int d^2b\sigma_{tot}(\alpha; \beta; \boldsymbol{b}). \tag{2.4}$$

Consider now interaction of hadron h with the nuclear target of atomic number B. Again, to obtain a simple description, we have to expand the initial state into the eigenstates of total absorption. For incident hadron h and for all nucleons of the target h_T^i ($i = 1 \dots B$) this can be done using Eq. (2.1). Furthermore, we observe that the state characterized by *fixed positions of all nucleons* in the target nucleus diagonalizes also nuclear degrees of freedom with respect to absorption.

Thus the initial state can be expanded as follows

$$|h; B\rangle = \sum_{\alpha,\beta_1 \dots \beta_B} \int d^3r_1 \dots d^3r_B|\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B\rangle$$

$$\langle\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B|h; B\rangle, \tag{2.5}$$

where $\boldsymbol{r}_1 \dots \boldsymbol{r}_B$ are the positions of the nucleons in the target nucleus. Since the state $|\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B\rangle$ diagonalizes the absorption operator, we can apply to it the argument from the beginning of this section and write the probability of absorption of the incident state $|\alpha\rangle$ on the target state $|\beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B\rangle$ in the form:

$$\sigma^B_{\text{abs}}(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) = 1 - \prod_{i=1}^{B} \{1 - \sigma_{\text{abs}}(\alpha, \beta_i; \boldsymbol{b} - \boldsymbol{s}_i)\}, \tag{2.6}$$

where $\boldsymbol{b}$ is the impact parameter and $\boldsymbol{s}_i$ are the transverse positions of the nucleons in the target nucleus. To justify the formula (2.6) we observe that $w_i = 1 - \sigma_{\text{abs}}(\alpha; \beta_i; \boldsymbol{b} - \boldsymbol{s}_i)$ is the probability that no absorption takes place at the i-th nucleon in the target. If the nucleons of the target absorb the incident state $|\alpha\rangle$ independently of each other, then $\prod_{i=1}^{B} w_i$ is the probability that no absorption takes place on any of the nucleons. Thus $1 - \prod_{i=1}^{B} w_i$ is the probability of absorption, as stated in Eq. (2.6).

The amplitude for elastic scattering of the states $|\alpha\rangle$ and $|\beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B\rangle$ can now be computed by substituting Eq. (2.6) into the condition of unitarity

$$\eta^B(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) = 1 - \sqrt{1 - \sigma^B_{\text{abs}}(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})}$$

$$= 1 - \prod_{i=1}^{B} \sqrt{1 - \sigma_{\text{abs}}(\alpha; \beta_i; \boldsymbol{b} - \boldsymbol{s}_i)}. \tag{2.7}$$

Using Eq. (2.3), this formula can be written as

$$\eta^B(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) = 1 - \prod_{i=1}^{B} \{1 - \eta(\alpha; \beta_i; \boldsymbol{b} - \boldsymbol{s}_i)\}. \tag{2.8}$$

3. *Diffractive and non-diffractive interactions in hadron-hadron scattering*

To obtain formulae for various hadronic cross-sections from Eqs. (2.1)÷(2.4) of the previous Section we have to invert first the formula (2.1). One obtains

$$|\alpha; \beta\rangle = \sum_{k,l} |h^{(k)}; h_{\text{T}}^{(l)}\rangle \langle h^{(k)}; h_{\text{T}}^{(l)}|\alpha, \beta\rangle, \tag{3.1}$$

where the indices k, l run over all diffractive excitations of the hadrons h and h_{T}.

Using Eqs. (2.1) and (3.1) we can write the amplitude for the scattering $h^{(0)} + h_{\text{T}}^{(0)} \to h^{(k)} + h_{\text{T}}^{(l)}$ in the form

$$\langle h^{(k)}; h_{\text{T}}^{(l)}|\eta(\boldsymbol{b})\,|h^{(0)}; h_{\text{T}}^{(0)}\rangle$$

$$= \sum_{\alpha,\beta} \langle h^{(k)}; h_{\text{T}}^{(l)}|\alpha; \beta\rangle \eta(\alpha; \beta; \boldsymbol{b})\, \langle \alpha; \beta|h^{(0)}; h_{\text{T}}^{(0)}\rangle. \tag{3.2}$$

The special case of elastic scattering reads [4][1]

$$\eta_{el}(b) \equiv \langle h^{(0)}; h_T^{(0)} | \eta(\boldsymbol{b}) | h^{(0)}; h_T^{(0)} \rangle = \sum_{\alpha,\beta} P(\alpha, \beta)\eta(\alpha; \beta; \boldsymbol{b}), \tag{3.3}$$

where

$$P(\alpha; \beta) = |\langle h^{(0)}; h_T^{(0)} | \alpha, \beta \rangle|^2 \tag{3.4}$$

is the probability of finding the state $|\alpha; \beta\rangle$ in the initial state $|h^{(0)}; h_T^{(0)}\rangle$. The elastic scattering probability is given by

$$\sigma_{el}(\boldsymbol{b}) = |\eta_{el}(\boldsymbol{b})|^2. \tag{3.5}$$

It follows from the optical theorem that the total cross-section is given by

$$\sigma_{tot}(\boldsymbol{b}) = 2\eta_{el}(\boldsymbol{b}) \quad \text{and} \quad \sigma_{tot} = \int d^2 b \sigma_{tot}(\boldsymbol{b}). \tag{3.6}$$

The total probability of diffractive collisions (including elastic scattering) is [4, 6]

$$\sigma_d(\boldsymbol{b}) = \sum_{k,l} |\langle h^{(k)}; h_T^{(l)} | \eta(\boldsymbol{b}) | h^{(0)}; h_T^{(0)} \rangle|^2 = \sum_{\alpha\beta} P(\alpha; \beta) \, |\eta(\alpha; \beta; \boldsymbol{b})|^2. \tag{3.7}$$

Consequently, for non-diffractive collisions we obtain

$$\begin{aligned} \sigma_{nd}(\boldsymbol{b}) &= \sigma_{tot}(\boldsymbol{b}) - \sigma_d(\boldsymbol{b}) \\ &= \sum_{\alpha,\beta} P(\alpha; \beta) \{2\eta(\alpha; \beta; \boldsymbol{b}) - [\eta(\alpha; \beta; \boldsymbol{b})]^2\} = \sum_{\alpha,\beta} P(\alpha; \beta)\sigma_{abs}(\alpha; \beta; \boldsymbol{b}), \end{aligned} \tag{3.8}$$

i.e. the average over the absorption probability of the absorption eigenstates. Similarly, for inelastic cross-section we have

$$\begin{aligned} \sigma_{inel}(\boldsymbol{b}) &= \sigma_{tot}(\boldsymbol{b}) - \sigma_{el}(\boldsymbol{b}) \\ &= \sum_{\alpha,\beta} P(\alpha, \beta) 2\eta(\alpha; \beta, \boldsymbol{b}) - \{\sum_{\alpha,\beta} P(\alpha; \beta)\eta(\alpha; \beta; \boldsymbol{b})\}^2. \end{aligned} \tag{3.9}$$

Finally, we may ask for the probability for the single diffraction (i.e. diffractive dissociation of one of the incident hadrons, say h). The result is

$$\begin{aligned} \sigma_{sd}(\boldsymbol{b}) &= \sum_{k \neq 0} |\langle h^{(k)}; h_T^{(0)} | \eta(\boldsymbol{b}) | h^{(0)}; h_T^{(0)} \rangle|^2 \\ &= \sum_{\alpha} P(\alpha) \{\eta(\alpha; \boldsymbol{b})\}^2 - \sigma_{el}(\boldsymbol{b}) \end{aligned} \tag{3.10}$$

where $P(\alpha) = |\langle \alpha | h^{(0)} \rangle|^2$ is the probability of finding the eigenstate $|\alpha\rangle$ in the state $|h^{(0)}\rangle$ and

$$\eta(\alpha; \boldsymbol{b}) = \sum_{\beta} P_T(\beta)\eta(\alpha; \beta; \boldsymbol{b}) \tag{3.11}$$

is the amplitude for elastic scattering of the eigenstate on the target $|h_T^{(0)}\rangle$, $P_T(\beta) = |\langle \beta | h_T^{(0)} \rangle|^2$ being the probability of finding the eigenstate $|\beta\rangle$ in the state $|h_T^{(0)}\rangle$.

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[1] The formulae for all the cross sections given in this Section were first employed for a special case of diffractive and non-diffractive *nuclear* interactions (see e.g. Ref. [6]), where, as we argued in Section 2, the eigenstates of absorption are specified by fixed positions in space of all nucleons in the target nucleus.

We shall now discuss briefly the momentum transfer dependence of the obtained cross-sections. From the amplitude $\eta(\boldsymbol{b})$ in the $\boldsymbol{b}$ space one obtains the amplitude $\eta(\boldsymbol{\Delta})$ as function of momentum transfer $\boldsymbol{\Delta}$ by performing the Fourier transform

$$\eta(\boldsymbol{\Delta}) = \frac{1}{2\pi}\int d^2 b e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\eta(\boldsymbol{b}) \tag{3.12}$$

consequently,

$$\eta_{\rm el}(\boldsymbol{\Delta}) = \sum_{\alpha,\beta} P(\alpha;\beta)\eta(\alpha;\beta;\boldsymbol{\Delta}), \tag{3.13}$$

where

$$\eta(\alpha;\beta;\boldsymbol{\Delta}) = \frac{1}{2\pi}\int d^2 b e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\eta(\alpha;\beta;\boldsymbol{b}). \tag{3.14}$$

The elastic differential cross-section is given by

$$\frac{d\sigma_{\rm el}}{d^2\Delta} = |\eta_{\rm el}(\boldsymbol{\Delta})|^2 = \frac{1}{(2\pi)^2}\int d^2 b d^2 b' e^{i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')}\eta_{\rm el}(\boldsymbol{b})\eta_{\rm el}(\boldsymbol{b}'). \tag{3.15}$$

One checks the correct normalization of this formula by observing that it reproduces the condition $\sigma_{\rm el} = \int d^2\Delta \frac{d\sigma_{\rm el}}{d^2\Delta} = \int d^2 b|\eta_{\rm el}(\boldsymbol{b})|^2$. A special case of the Eq. (3.14) is of particular interest. Taking $\boldsymbol{\Delta} = 0$ we have

$$\left.\frac{d\sigma_{\rm el}}{d^2\Delta}\right|_{\Delta=0} = \frac{1}{(2\pi)^2}\int d^2 b d^2 b' \eta_{\rm el}(\boldsymbol{b})\eta_{\rm el}(\boldsymbol{b}') = \frac{1}{(2\pi)^2}\frac{\sigma_{\rm tot}^2}{4}, \tag{3.16}$$

where in the last equality we used the Eq. (3.6). Similarly, the amplitude for diffractive dissociation of the beam at momentum transfer $\boldsymbol{\Delta}$ is given by

$$\langle h^{(k)}; h_{\rm T}^{(0)}|\eta(\boldsymbol{\Delta})\,|h^{(0)}; h_{\rm T}^{(0)}\rangle = \frac{1}{2\pi}\int d^2 b e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\langle h^{(k)}; h_{\rm T}^{(0)}|\eta(\boldsymbol{b})\,|h^{(0)}; h_{\rm T}^{(0)}\rangle. \tag{3.17}$$

The differential cross-section for single diffraction of the beam hadron (including elastic scattering) at momentum transfer $\boldsymbol{\Delta}$ is thus

$$\frac{d\sigma_{\rm el}}{d^2\Delta} + \frac{d\sigma_{\rm sd}}{d^2\Delta} \equiv \sum_k |\langle h^{(k)}; h_{\rm T}^{(0)}|\eta(\boldsymbol{\Delta})\,|h^{(0)}; h_{\rm T}^{(0)}\rangle|^2$$

$$= \frac{1}{(2\pi)^2}\int d^2 b d^2 b' e^{i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')}\sum_\alpha P(\alpha)\eta(\alpha;\boldsymbol{b})\eta(\alpha;\boldsymbol{b}'), \tag{3.18}$$

where $\eta(\alpha;\boldsymbol{b})$ is given by Eq. (3.11). For $\boldsymbol{\Delta} = 0$ this formula takes a simple form

$$\left(\frac{d\sigma_{\rm el}}{d^2\Delta} + \frac{d\sigma_{\rm sd}}{d^2\Delta}\right)_{\Delta=0} = \frac{1}{(2\pi)^2}\sum_\alpha P(\alpha)\,(\eta^\alpha)^2 \equiv \frac{1}{(2\pi)^2}\overline{\eta^2}, \tag{3.19}$$

where

$$\eta^\alpha = \int d^2 b\eta(\alpha;\boldsymbol{b}). \tag{3.20}$$

4. Diffractive and non-diffractive interactions in hadron-nucleus collisions

Using the results from Section 2, in particular the Eq. (2.8), we can write the amplitude for hadron-nucleus scattering in the form

$$\langle h^{(k)}, B^*|\eta(\boldsymbol{b})\,|h^{(0)}, B\rangle = \sum_{\substack{\alpha\\ \beta_1 \dots \beta_B}} \int d^3r_1 \dots d^3r_B \langle h^{(k)}; B^*|\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B\rangle$$

$$\times \eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) \langle \alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B|h^{(0)}; B\rangle, \tag{4.1}$$

where B^* denotes the final state of the target nucleus and $\eta(\alpha; \beta_1 \dots \beta_B;\ \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})$ is given by Eq. (2.8). The nuclear excitations can be of two kinds: the diffractive excitations of individual nucleons (corresponding to parameters $\beta_1 \dots \beta_B$) and excitations of the nuclear structure including a possible break up (parameters $\boldsymbol{r}_1 \dots \boldsymbol{r}_B$).

From Eq. (4.1) one can derive formulae for probabilities and cross-sections of various physical processes.

Elastic amplitude is given by

$$\eta^B_{\text{el}}(\boldsymbol{b}) = \langle h^{(0)}; B|\eta(\boldsymbol{b})\,|h^{(0)}; B\rangle$$

$$= \sum_{\substack{\alpha\\ \beta_1 \dots \beta_B}} \int d^3r_1 \dots d^3r_B P(\alpha; \beta_1 \dots \beta_B)\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) \tag{4.2}$$

where $P(\alpha; \beta_1 \dots \beta_B)$ is the probability of finding the state $|\alpha; \beta_1 \dots \beta_B\rangle$ in the physical state $|h^{(0)}; B\rangle$ and $\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)$ is the nuclear density normalized to unity.

The total cross-section is thus

$$\sigma^B_{\text{tot}} = 2\int d^2b\,\eta^B_{\text{el}}(\boldsymbol{b}). \tag{4.3}$$

The total cross-section consists of several contributions corresponding to different physical processes. We list them below together with the formulae for the corresponding probabilities.

(*i*) *Elastic scattering*, when both beam and target remain in the ground state

$$\sigma^B_{\text{el}}(\boldsymbol{b}) = \{\eta^B_{\text{el}}(\boldsymbol{b})\}^2 \tag{4.4}$$

where $\eta^B_{\text{el}}(\boldsymbol{b})$ is given by Eq. (4.2).

(*ii*) *Quasi-elastic scattering* in which nucleons undergo elastic scattering but target does not remain in the ground state

$$\sigma^B_{\text{q.el}}(\boldsymbol{b}) = \sum_{B^*} |\langle h^{(0)}; B^*|\eta(\boldsymbol{b})\,|h^{(0)}; B\rangle|^2 - \sigma^B_{\text{el}}(\boldsymbol{b})$$

$$= \int d^3r_1 \dots d^3r_B\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)\Big\{\sum_{\substack{\alpha\\ \beta_1 \dots \beta_B}} P(\alpha)P(\beta_1 \dots \beta_B)\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})\Big\}^2$$

$$-\{\eta^B_{\text{el}}(\boldsymbol{b})\}^2, \tag{4.5}$$

where we accepted

$$P(\alpha; \beta_1 \dots \beta_B) = P(\alpha)P(\beta_1 \dots \beta_B).$$

(*iii*) *Diffractive production* including all processes in which the incident hadron and/or any nucleon of the target nucleus become diffractively excited, but no non-diffractive interaction takes place. The probability of such collisions is

$$\sigma^B_{\text{diff}}(\boldsymbol{b}) = \sum_k \sum_{B^*} |\langle h^{(k)}; B^*|\eta(\boldsymbol{b})\, |h^{(0)}; B\rangle|^2 - (\sigma^B_{\text{el}}(\boldsymbol{b}) + \sigma^B_{\text{q.el}}(\boldsymbol{b}))$$

$$= \sum_\alpha \sum_{\beta_1 \dots \beta_B} \int d^3r_1 \dots d^3r_B P(\alpha)P(\beta_1 \dots \beta_B)\varrho_B(r_1 \dots r_B)\, \{\eta(\alpha; \beta_1 \dots \beta_B; r_1 \dots r_B; \boldsymbol{b})\}^2$$
$$-(\sigma^B_{\text{el}}(\boldsymbol{b}) + \sigma^B_{\text{q.el}}(\boldsymbol{b})). \tag{4.6}$$

The processes of diffractive production represented in formula (4.6) include both coherent and incoherent interactions. *Coherent diffractive production* in which the incident hadron is diffractively excited and the target nucleus remains in the ground state deserve special attention. The probability for such coherent diffractive production is

$$\sigma^B_{\text{coh}}(\boldsymbol{b}) = \sum_k |\langle h^{(k)}; B|\eta(\boldsymbol{b})\, |h^{(0)}; B\rangle|^2 - \sigma^B_{\text{el}}(\boldsymbol{b})$$

$$= \sum_\alpha P(\alpha) \{ \sum_{\beta_1 \dots \beta_B} \int d^3r_1 \dots d^3r_B P(\beta_1 \dots \beta_B)\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})\}^2 - \sigma^B_{\text{el}}(\boldsymbol{b}). \tag{4.7}$$

It is characterized by very steep momentum transfer dependence, controlled by the radius of the target nucleus.

Other diffractive processes, in which the target nucleus is excited or breaks up (incoherent interactions) shall be called quasi-diffractive. The corresponding probability is

$$\sigma^B_{\text{qd}}(\boldsymbol{b}) = \sigma^B_{\text{diff}}(\boldsymbol{b}) - \sigma^B_{\text{coh}}(\boldsymbol{b}). \tag{4.8}$$

The quasi-diffractive collisions can be again split into two categories: (a) those in which the beam hadron remains unexcited (but at least one of the nucleons in the target nucleus becomes diffractively excited) with the probability

$$\sigma^B_{\text{qd}}(\boldsymbol{b})_{\text{target}} = \sum_{\beta_1 \dots \beta_B} \int d^3r_1 \dots d^3r_B P(\beta_1 \dots \beta_B)\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)$$
$$\times \{\sum_\alpha P(\alpha)\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})\}^2 - (\sigma^B_{\text{el}}(\boldsymbol{b}) + \sigma^B_{\text{q.el}}(\boldsymbol{b})), \tag{4.9}$$

and (b) those in which the beam hadron is diffractively excited with the probability

$$\sigma^B_{\text{qd}}(\boldsymbol{b})_{\text{beam}} = \sigma^B_{\text{qd}}(\boldsymbol{b}) - \sigma^B_{\text{qd}}(\boldsymbol{b})_{\text{target}} \tag{4.10}$$

(*iv*) *Non-diffractive processes*, which represent all the remaining cross-section

$$\sigma^B_{\text{nd}}(\boldsymbol{b}) = \sigma_{\text{tot}} - \sum_{k,B^*} |\langle h^{(k)}; B^*|\eta(\boldsymbol{b})\, |h^{(0)}; B\rangle|^2$$

$$= \sum_\alpha \sum_{\beta_1 \dots \beta_B} \int d^3r_1 \dots d^3r_B P(\alpha)P(\beta_1 \dots \beta_B)\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)$$
$$\times \{2\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}) - [\eta(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})]^2\}$$
$$= \sum_\alpha \sum_{\beta_1 \dots \beta_B} \int d^2r_1 \dots d^3r_B P(\alpha)P(\beta_1 \dots \beta_B)\varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)\sigma_{\text{abs}}(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b}), \tag{4.11}$$

where $\sigma_{\text{abs}}(\alpha; \beta_1 \dots \beta_B; \boldsymbol{r}_1 \dots \boldsymbol{r}_B; \boldsymbol{b})$ is given by Eq. (2.6).

5. *Glauber model-type approximations*

The general formulae of Section 4 are not directly applicable to practical calculations. For that one needs information on probabilities $P(\alpha)$ and $P(\beta_1 \ldots \beta_B)$ and on absorption of different eigenmodes. But this information is not available from hadron-nucleon scattering where, as was seen in Section 3, only some averages can be measured. Consequently, it is of interest to obtain the formulae which, although approximate, would allow one to calculate the nuclear cross-sections from the measured hadron-nucleon cross-sections. The well-known example of such an approximation is the Glauber formula for elastic amplitude. It reads

$$^{G}\eta_{el}^{B}(\boldsymbol{b}) = \int d^3r_1 \ldots d^3r_B \varrho_B(\boldsymbol{r}_1 \ldots \boldsymbol{r}_B) \{1 - \prod_{i=1}^{B} [1 - \eta_{el}(\boldsymbol{b} - \boldsymbol{s}_i)]\} \tag{5.1}$$

where $\eta_{el}(\boldsymbol{b})$ is the elementary elastic hadron-nucleon amplitude at impact parameter $\boldsymbol{b}$.

A comparison of the general formula (4.2) and the Glauber formula (5.1) shows that the Glauber formula arises from (4.2) if the fluctuations in the elementary amplitude are neglected, so that the actual value of the elementary amplitude can be replaced by its average value $\eta_{el}(\boldsymbol{b})$. It is known that the Glauber formula reproduces quite well the elastic hadron-nucleus amplitudes. One may hope therefore that analogous approximations shall also work for other cross-sections listed in Section 4.

In the following we shall also use two simplifying assumptions which allow us to write down compact formulae and do not seem to introduce serious errors (particularly for large nuclei): we shall assume that both $P(\beta_1 \ldots \beta_B)$ and $\varrho_B(\boldsymbol{r}_1 \ldots \boldsymbol{r}_B)$ factorize:

$$P(\beta_1 \ldots \beta_B) = \prod_{i=1}^{B} P_T(\beta_i), \tag{5.2}$$

and

$$\varrho_B(\boldsymbol{r}_1 \ldots \boldsymbol{r}_B) = \prod_{i=1}^{B} \varrho_B(\boldsymbol{r}_i). \tag{5.3}$$

The approximate formulas following from (5.2) and (5.3) shall be called "optical" formulae.

Using Eq. (5.3) we obtain from (5.1)

$$^{G}\eta_{el}^{B}(\boldsymbol{b}) = 1 - \{1 - \tfrac{1}{2}\sigma_{tot} D(\boldsymbol{b})\}^B, \tag{5.4}$$

where

$$D(\boldsymbol{b}) = \frac{2}{\sigma_{tot}} \int d^3r \varrho_B(\boldsymbol{r}) \eta_{el}(\boldsymbol{b} - \boldsymbol{s}) \approx \int_{-\infty}^{+\infty} dz \varrho_B(\boldsymbol{b}, z), \tag{5.5}$$

and σ_{tot} is given by Eq. (3.6).

Consequently, in the "optical" approximation the probability of elastic interaction at impact parameter $\boldsymbol{b}$ can be written as

$$^{G}\sigma_{el}^{B}(\boldsymbol{b}) = |^{G}\eta_{el}^{B}(\boldsymbol{b})|^2 = 1 - 2\{1 - \tfrac{1}{2}\sigma_{tot} D(\boldsymbol{b})\}^B + \{1 - (\sigma_{tot} D(\boldsymbol{b}) - \tfrac{1}{4}\sigma_{tot}^2 D^2(\boldsymbol{b})\}^B. \tag{5.6}$$

Let us now proceed to other cross-sections discussed in Section 4.

Neglecting the fluctuations of absorption in hadron-nucleus collisions we obtain from Eq. (4.5) and (2.8)

$$ {}^{G}\sigma^{B}_{\mathrm{q.el}}(\boldsymbol{b}) = \int d^3r_1 \dots d^3r_B \varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B) \{1 - \prod_{i=1}^{B} [1-\eta_{\mathrm{el}}(\boldsymbol{b}-\boldsymbol{s}_i)]\}^2 - {}^{G}\sigma^{B}_{\mathrm{el}}(\boldsymbol{b}), \tag{5.7} $$

which in the "optical" approximation becomes

$$ {}^{G}\sigma^{B}_{\mathrm{q.el}}(\boldsymbol{b}) = 1-2\{1-\tfrac{1}{2}\,\sigma_{\mathrm{tot}}D(\boldsymbol{b})\}^{B}+\{1-(\sigma_{\mathrm{tot}}-\sigma_{\mathrm{el}})D(\boldsymbol{b})\}^{B}-{}^{G}\sigma^{B}_{\mathrm{el}}(\boldsymbol{b}) $$

$$ = [1-\sigma_{\mathrm{inel}}D(\boldsymbol{b})]^{B}-[1-(\sigma_{\mathrm{tot}}D(\boldsymbol{b})-\tfrac{1}{4}\,\sigma^{2}_{\mathrm{tot}}D^{2}(\boldsymbol{b}))]^{B}. \tag{5.8} $$

This formula has a simple physical meaning. Indeed, the probability of quasi-elastic collisions is the sum of probabilities of any number of elastic collisions which lead to break up of the target nucleus. This probability can be calculated as a difference between the probability of any type of collision (breaking the nucleus) equal to [1]

$$ 1-\{1-(\sigma_{\mathrm{tot}}D(\boldsymbol{b})-\tfrac{1}{4}\,\sigma^{2}_{\mathrm{tot}}D^{2}(\boldsymbol{b}))\}^{B} \tag{5.9} $$

and the probability of inelastic collision equal to

$$ 1-\{1-\sigma_{\mathrm{inel}}D(\boldsymbol{b})\}^{B}. \tag{5.10} $$

Consider now the *coherent diffractive production*. According to Eq. (4.7) and using Eqs. (5.2) and (5.3) we obtain in the "optical" approximation

$$ \sigma^{B}_{\mathrm{coh}}(\boldsymbol{b}) = \sum_{\alpha} P(\alpha)\,\{1-2[1-\eta^{\alpha}D(\boldsymbol{b})]^{B}+[1-2\eta^{\alpha}D(\boldsymbol{b})+(\eta^{\alpha})^{2}D^{2}(\boldsymbol{b})]^{B}\}-\sigma^{B}_{\mathrm{el}}(\boldsymbol{b}), \tag{5.11} $$

where $\eta^{\alpha} = \int d^2b\eta(\alpha;\, \boldsymbol{b})$ and $\eta(\alpha;\, \boldsymbol{b})$ is given by Eq. (3.11). Using Eq. (5.6) we thus have

$$ {}^{G}\sigma^{B}_{\mathrm{coh}}(\boldsymbol{b}) = \{1-[\sigma_{\mathrm{tot}}D(\boldsymbol{b})-\overline{\eta^{2}}D^{2}(\boldsymbol{b})]\}^{B}-\{1-[\sigma_{\mathrm{tot}}D(\boldsymbol{b})-\tfrac{1}{4}\,\sigma^{2}_{\mathrm{tot}}D^{2}(\boldsymbol{b})]\}^{B}. \tag{5.12} $$

As shown in Section 3, the coefficients $\frac{1}{4}\sigma^2_{\mathrm{tot}}$ and $\overline{\eta^2} \equiv \sum_{\alpha} P(\alpha)\,(\eta^{\alpha})^2$ can be expressed by hadron-nucleon elastic and single diffractive cross-sections at zero momentum transfer (cf. Eqs. (3.16) and (3.19)). Consequently, we obtain

$$ {}^{G}\sigma^{B}_{\mathrm{coh}}(\boldsymbol{b}) = \left\{1-\left[\sigma_{\mathrm{tot}}D(\boldsymbol{b})-(2\pi)^{2}\left(\frac{d\sigma_{\mathrm{el}}}{d^{2}\Delta}+\frac{d\sigma_{\mathrm{d}}}{d^{2}\Delta}\right)_{\Delta=0}D^{2}(\boldsymbol{b})\right]\right\}^{B} $$

$$ -\left\{1-\left[\sigma_{\mathrm{tot}}D(\boldsymbol{b})-(2\pi)^{2}\left.\frac{d\sigma_{\mathrm{el}}}{d^{2}\Delta}\right|_{\Delta=0}D^{2}(\boldsymbol{b})\right]\right\}^{B}. \tag{5.13} $$

Using the Eq. (4.6), the cross-section for *diffractive production* in Glauber approximation can be written as

$$ {}^{G}\sigma^{B}_{\mathrm{diff}}(\boldsymbol{b}) = \int d^3r_1 \dots d^3r_B \varrho_B(\boldsymbol{r}_1 \dots \boldsymbol{r}_B)\,\{1-2\prod_{i=1}^{B}[1-\eta_{\mathrm{el}}(\boldsymbol{b}-\boldsymbol{s}_i)] $$

$$ +\prod_{i=1}^{B}[1-\sigma_{\mathrm{tot}}(\boldsymbol{b}-\boldsymbol{s}_i)+\sigma_{\mathrm{d}}(\boldsymbol{b}-\boldsymbol{s}_i)]\}-({}^{G}\sigma^{B}_{\mathrm{el}}(\boldsymbol{b})+{}^{G}\sigma^{B}_{\mathrm{q.el}}(\boldsymbol{b})), \tag{5.14} $$

where we have used Eqs. (3.5) and (3.6). Substituting into this formula Eq. (5.7) we obtain

$$ {}^{G}\sigma^{B}_{\text{diff}}(b) = \int d^3r_1 \ldots d^3r_B \varrho_B(r_1 \ldots r_B) \{\prod_{i=1}^{B} [1-\sigma_{\text{nd}}(b-s_i)] - \prod_{i=1}^{B} [1-\sigma_{\text{inel}}(b-s_i)]\}. \tag{5.15} $$

In the "optical" approximation we thus have

$$ {}^{G}\sigma^{B}_{\text{diff}}(b) = [1-\sigma_{\text{nd}}D(b)]^B - [1-\sigma_{\text{inel}}D(b)]^B. \tag{5.16} $$

The physical meaning of the formula (5.16) is analogous to that of Eq. (5.8). ${}^{G}\sigma^{B}_{\text{diff}}(b)$ can be interpreted as a difference of the probability for any number of inelastic collisions (both diffractive and non-diffractive), equal to

$$ 1-[1-\sigma_{\text{inel}}D(b)]^B \tag{5.17} $$

and the probability for any number of non-diffractive collisions, equal to $1-[1-\sigma_{\text{nd}}D(b)]^B$. The result of this difference is given by Eq. (5.16).

It is also of interest to calculate the probability of quasi-diffractive collisions in which the beam hadron does not dissociate. Starting from Eq. (4.9) and using the arguments similar to those employed before one obtains

$$ {}^{G}\sigma^{B}_{\text{diff}}(b)_{\text{target}} = \{1-(\sigma_{\text{inel}}-\sigma^{(\text{T})}_{\text{sd}})D(b)\}^B - \{1-\sigma_{\text{inel}}D(b)\}^B, \tag{5.18} $$

where $\sigma^{(\text{T})}_{\text{sd}}$ is the cross-section for single diffractive dissociation of the target proton in hadron-proton collision.

Finally, using Eqs. (4.10), (5.16) and (5.18) we obtain

$$ {}^{G}\sigma^{B}_{\text{diff}}(b)_{\text{beam}} = \{1-\sigma_{\text{nd}}D(b)\}^B - \{1-(\sigma_{\text{inel}}-\sigma^{(\text{T})}_{\text{sd}})D(b)\}^B. \tag{5.19} $$

The last formula we like to quote is that expressing the non-diffractive cross-section. Using Eqs. (4.11), (2.6) and (3.8) we have

$$ {}^{G}\sigma^{B}_{\text{nd}}(b) = 1 - \int d^3r_1 \ldots d^3r_B \varrho_B(r_1 \ldots r_B) \prod_{i=1}^{B} [1-\sigma_{\text{nd}}(b-s_i)]. \tag{5.20} $$

In the "optical" approximation we obtain

$$ {}^{G}\sigma^{B}_{\text{nd}}(b) = 1-[1-\sigma_{\text{nd}}D(b)]^B, \tag{5.21} $$

which confirms interpretation of $1-[1-\sigma_{\text{nd}}D(b)]^B$ as the probability for any number of non-diffractive collisions.

6. Diffractive and non-diffractive interactions in nucleus-nucleus collisions

The generalization of the expressions for various hadron-nucleus cross-sections to the nucleus-nucleus case is fairly straightforward but leads to formulae of considerable complexity. Without going into any details, we quote the definitions and results.

Now we have the following eigenstates of diffraction of the two colliding nuclei with the atomic numbers A and B

$$ |\alpha_1 \ldots \alpha_A; x_1 \ldots x_A; \beta_1 \ldots \beta_B; y_1 \ldots y_B\rangle, \tag{6.1} $$

where $\alpha_i(\beta_j)$ label the eigenstates of diffraction of the nucleon i — in the nucleus A (j — in the nucleus B), and $\boldsymbol{x}_i$, $\boldsymbol{y}_j$ are the spatial positions of the nucleons in the nuclei A, B[2]. The analogon of the "eigenprofile" (2.8) is now

$$\eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A, \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b}) = 1 - \prod_{i=1}^{A} \prod_{j=1}^{B} \{1-\eta(\alpha_i; \beta_j; \boldsymbol{b}-\boldsymbol{x}_i^{\perp}+\boldsymbol{y}_j^{\perp})\}. \tag{6.2}$$

The general transition amplitude from the ground states of the colliding nuclei A, B to a diffractively excited state (i.e. a state where both the spatial structure of the nuclei and the internal structure of the nucleons is changed) is a straightforward generalization of (4.1)

$$\begin{aligned}&\langle A^*B^*; h_1^* \ldots h_A^*; g_1^* \ldots g_B^* | \eta(\boldsymbol{b}) | AB; h_1 \ldots h_A; g_1 \ldots g_B\rangle \\ &= \sum_{\substack{\alpha_1 \ldots \alpha_A \\ \beta_1 \ldots \beta_B}} \int d^3x_1 \ldots d^3x_A d^3y_1 \ldots d^3y_B \langle A^*B^*; h_1^* \ldots h_A^*; g_1^* \ldots g_B^* | \alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \\ &\quad \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B\rangle \eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b}) \\ &\quad \times \langle \alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B | AB; h_1 \ldots h_A; g_1 \ldots g_B\rangle,\end{aligned} \tag{6.3}$$

where $A, B; h_1 \ldots h_A; g_1 \ldots g_B$ and $A^*, B^*; h_1^* \ldots h_A^*; g_1^* \ldots g_B^*$ label the ground and the excited states of the nuclei and of the nucleons in them.

We are interested only in the cross-sections in which a sum over all possible excitations is performed and therefore we need only the probabilities $P_A(\alpha_1 \ldots \alpha_A)$, $P_B(\beta_1 \ldots \beta_B)$ of finding the states $|\alpha_1 \ldots \alpha_A\rangle$ in the physical ground states of A nucleons in the nucleus A and of the states $|\beta_1 \ldots \beta_B\rangle$ in the physical ground states of B nucleons in the nucleus B, respectively, and also the ground state densities $\varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)$ and $\varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B)$.

Eq. (6.3) implies the following expression for the elastic amplitude

$$\begin{aligned}\eta_{\text{el}}^{AB}(\boldsymbol{b}) &= \sum_{\substack{\alpha_1 \ldots \alpha_A \\ \beta_1 \ldots \beta_B}} \int d^3x_1 \ldots d^3x_A d^3y_1 \ldots d^3y_B \varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A) \varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B) \\ &\times P_A(\alpha_1 \ldots \alpha_A) P_B(\beta_1 \ldots \beta_B) \eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b}),\end{aligned} \tag{6.4}$$

and for the total cross-section

$$\sigma_{\text{tot}}^{AB} = \int d^2b \sigma_{\text{tot}}^{AB}(\boldsymbol{b}) = 2 \int d^2b \eta_{\text{el}}^{AB}(\boldsymbol{b}). \tag{6.5}$$

Without going into any details we list the components of the total cross-section in the same order as in Section 4.

(*i*) *Elastic scattering.* Both nuclei remain in the ground state

$$\sigma_{\text{el}}^{AB}(\boldsymbol{b}) = |\eta_{\text{el}}^{AB}(\boldsymbol{b})|^2, \tag{6.6}$$

with $\eta_{\text{el}}^{AB}(\boldsymbol{b})$ given by Eq. (6.4).

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[2] $x_i^{\perp}, y_j^{\perp}$ are the projections of spatial positions of nucleons (in A and B, respectively) on the impact parameter ($\boldsymbol{b}$) plane.

(*ii*) *Quasi-elastic scattering*. At least one of the nuclei gets excited, but the nucleons do not

$$\sigma^{AB}_{\text{q.el}}(\boldsymbol{b}) = \int d^3x_1 \ldots d^3x_A d^3y_1 \ldots d^3y_B \varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)\varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B)$$

$$\times \Big| \sum_{\substack{\alpha_1 \cdots \alpha_A \\ \beta_1 \cdots \beta_B}} P_A(\alpha_1 \ldots \alpha_A)P_B(\beta_1 \ldots \beta_B)\eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b})\Big|^2 - \sigma^{AB}_{\text{el}}(\boldsymbol{b}). \tag{6.7}$$

The quasi-elastic processes summed up in Eq. (6.7) can be further split into two classes, according to the degree of coherence: (a) proper quasi-elastic reactions in which both nuclei become excited (or break up); (b) semi-elastic reactions in which one of the colliding nuclei remains in the ground state whereas the other one is excited. These two classes differ in their dependence on momentum transfer: for semi-elastic reactions the momentum transfer dependence is controlled by the radius of the unexcited nucleus and is, consequently, rather steep — particularly for heavy nuclei. For proper quasi-elastic reactions the momentum transfer dependence is controlled by the nucleon-nucleon elastic scattering cross--section. We shall not discuss these relations in further detail here. Let us just quote as an example the probability for the semi-elastic process which leaves the nucleus A in the ground state

$$\int d^3y_1 \ldots d^3y_B \varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B)\Big| \sum_{\substack{\alpha_1 \cdots \alpha_A \\ \beta_1 \cdots \beta_B}} \int d^3x_1 \ldots d^3x_A \varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)P_A(\alpha_1 \ldots \alpha_A)P_B(\beta_1 \ldots \beta_B)$$

$$\times \eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B)\Big|^2 - \sigma^{AB}_{\text{el}}(\boldsymbol{b}). \tag{6.8}$$

The cross-section for proper quasi-elastic reactions can be obtained by subtracting two semi-elastic cross-sections from the Eq. (6.7).

(*iii*) *Diffractive production.*

$$\sigma^{AB}_{\text{diff}}(\boldsymbol{b}) = \sum_{A^*B^*} \sum_{\substack{h_1^* \cdots h_A^* \\ g_1^* \cdots g_B^*}} \Big| \sum_{\substack{\alpha_1 \cdots \alpha_A \\ \beta_1 \cdots \beta_B}} \langle A^*B^*; h_1^* \ldots h_A^*; g_1^* \ldots g_B^* | \alpha_1 \ldots \alpha_A; \boldsymbol{x}_1$$

$$\ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B, \boldsymbol{y}_1 \ldots \boldsymbol{y}_B\rangle \eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b})$$

$$\times \langle \alpha_1 \ldots \alpha_A \;\; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B | AB; h_1 \ldots h_A; g_1 \ldots g_B\rangle\Big|^2$$

$$-(\sigma^{AB}_{\text{el}}(\boldsymbol{b}) + \sigma^{AB}_{\text{q.el}}(\boldsymbol{b})) = \int d^3x_1 \ldots d^3x_A d^3y_1 \ldots d^3y_B \varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)\varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B)$$

$$\times \Big\{ \sum_{\substack{\alpha_1 \cdots \alpha_A \\ \beta_1 \cdots \beta_B}} P_A(\alpha_1 \ldots \alpha_A)P_B(\beta_1 \ldots \beta_B)\,[\eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b})]^2$$

$$-\Big[\sum_{\substack{\alpha_1 \cdots \alpha_A \\ \beta_1 \cdots \beta_B}} P_A(\alpha_1 \ldots \alpha_A)P_B(\beta_1 \ldots \beta_B)\eta(\alpha_1 \ldots \alpha_A; \boldsymbol{x}_1 \ldots \boldsymbol{x}_A; \beta_1 \ldots \beta_B; \boldsymbol{y}_1 \ldots \boldsymbol{y}_B; \boldsymbol{b})\Big]^2\Big\}. \tag{6.9}$$

The processes of diffractive production can also be divided into several classes, according to the exhibited degree of coherence:

(a) quasi-diffractive processes when both nuclei are excited (including breakup),

(b) semi-coherent diffractive processes when one of the nuclei remains in the ground state, whereas the other is excited or breaks up,

(c) coherent diffractive processes when both nuclei remain in the ground state. These processes are presumably very rare.

Different classes should exhibit markedly different momentum transfer dependences.

(*iv*) *Non-diffractive processes*, which include all remaining contributions to the cross--section.

$$\sigma_{\text{nd}}^{AB}(\boldsymbol{b}) = \sigma_{\text{tot}}^{AB}(\boldsymbol{b}) - \sum_{A^*B^*} \sum_{\substack{h_1^* \dots h_A^* \\ g_1^* \dots g_B^*}} \Big| \sum_{\substack{\alpha_1 \dots \alpha_A \\ \beta_1 \dots \beta_B}} \langle A^*B^*; h_1^* \dots h_A^*; g_1^* \dots g_B^* | \alpha_1 \dots \alpha_A; \beta_1$$
$$\dots \beta_B; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \boldsymbol{y}_1 \dots \boldsymbol{y}_B \rangle \eta(\alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B; \boldsymbol{b})$$
$$\times \langle \alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B | AB; h_1 \dots h_A; g_1 \dots g_B \rangle \Big|^2$$
$$= 2 \sum_{\substack{\alpha_1 \dots \alpha_A \\ \beta_1 \dots \beta_B}} \int d^3x_1 \dots d^3x_A d^3y_1 \dots d^3y_B \varrho_A(\boldsymbol{x}_1 \dots \boldsymbol{x}_A) \varrho_B(\boldsymbol{y}_1 \dots \boldsymbol{y}_B) P_A(\alpha_1 \dots \alpha_A) P_B(\beta_1 \dots \beta_B)$$
$$\times \eta(\alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B; \boldsymbol{b}) - \sum_{\substack{\alpha_1 \dots \alpha_A \\ \beta_1 \dots \beta_B}} P_A(\alpha_1 \dots \alpha_A) P_B(\beta_1 \dots \beta_B)$$
$$\times \int d^3x_1 \dots d^3x_A d^3y_1 \dots d^3y_B \varrho_A(\boldsymbol{x}_1 \dots \boldsymbol{x}_A) \varrho_B(\boldsymbol{y}_1 \dots \boldsymbol{y}_B)$$
$$\times [\eta(\alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B; \boldsymbol{b})]^2$$
$$= \sum_{\substack{\alpha_1 \dots \alpha_A \\ \beta_1 \dots \beta_B}} P_A(\alpha_1 \dots \alpha_A) P_B(\beta_1 \dots \beta_B) \int d^3x_1 \dots d^3x_A d^3y_1 \dots d^3y_B \varrho_A(\boldsymbol{x}_1 \dots \boldsymbol{x}_A)$$
$$\times \varrho_B(\boldsymbol{y}_1 \dots \boldsymbol{y}_B) \sigma_{\text{abs}}^{AB}(\alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B), \tag{6.10}$$

where, in analogy to (2.6),

$$\sigma_{\text{abs}}^{AB}(\alpha_1 \dots \alpha_A; \boldsymbol{x}_1 \dots \boldsymbol{x}_A; \beta_1 \dots \beta_B; \boldsymbol{y}_1 \dots \boldsymbol{y}_B)$$
$$= 1 - \prod_{i=1}^{A} \prod_{j=1}^{B} [1 - \sigma_{\text{abs}}(\alpha_i; \beta_j; \boldsymbol{b} - \boldsymbol{x}_i^{\perp} + \boldsymbol{y}_j^{\perp})]. \tag{6.11}$$

7. *Glauber model type approximation in nucleus-nucleus collisions*

The motivation and approach are the same as in Section 5. By replacing in the formulae of the previous Section the fluctuating elementary amplitudes by their average values we obtain several formulae which express different nucleus-nucleus cross-sections in terms of measurable nucleon-nucleon cross-sections and nuclear densities. These formulae can thus be used for practical calculations of the nucleus-nucleus interactions. They may be considered to be generalizations of the formula for the elastic nucleus-nucleus amplitude [7]

$$\eta_{\text{el}}^{AB}(\boldsymbol{b}) = \int d^3x_1 \dots d^3x_A d^3y_1 \dots d^3y_B \varrho_A(\boldsymbol{x}_1 \dots \boldsymbol{x}_A) \varrho_B(\boldsymbol{y}_1 \dots \boldsymbol{y}_B)$$
$$\times [1 - \prod_{i=1}^{A} \prod_{j=1}^{B} \{1 - \eta_{\text{el}}(\boldsymbol{b} - \boldsymbol{x}_i^{\perp} + \boldsymbol{y}_j^{\perp})\}]. \tag{7.1}$$

However, as pointed out in Ref. [7], such expressions are rather difficult to handle and lead to calculations of great complexity. Therefore, we consider also further approximation which, although perhaps more questionable, provides much simpler and tractable formulae. In this second step we *neglect the spatial fluctuations of elementary interactions*. This means that the *actual* position of the colliding nucleons is replaced by their *average* positions inside the colliding nuclei. This procedure gives expressions which become, in the limit $A, B \to \infty$ the so called "optical limits". For elastic amplitude (7.1) this "optical approximation" gives

$$\eta_{el}^{AB}(b) = 1-\{1-\int d^2xd^2y\eta_{el}(b-x^\perp+y^\perp)D_A(x^\perp)D_B(y^\perp)\}^{AB}$$
$$= 1-\{1-\eta_{el}D_{AB}(b)\}^{AB}, \qquad (7.2)$$

where

$$D_{AB}(b) = \frac{1}{\eta_{el}}\int d^2xd^2y\eta_{el}(b-x^\perp+y^\perp)D_A(x^\perp)D_B(y^\perp) \approx \int d^2x^\perp D_A(b-x^\perp)D_B(x^\perp), \qquad (7.3)$$

and $D_A(x^\perp)$, $D_B(y^\perp)$ are given by Eq. (5.5), and $\eta_{el} = \int d^2b\eta_{el}(b)$.

To estimate errors introduced through such a procedure is a very complicated task which we shall not undertake in this paper. We believe however that the formulae obtained in this way do give a reasonable first approximation.

We follow steps of Section 5. First, for elastic amplitude, starting from Eq. (6.4) and replacing the fluctuating elementary amplitudes by their averages, we clearly obtain the formulae (7.1) and (7.2). From this formula and from Eqs. (4.3) and (4.4) we obtain the total and elastic cross-sections. In particular, the "optical" formula for elastic cross-section reads

$$\sigma_{ql}^{AB}(b) = 1-2[1-\tfrac{1}{2}\sigma_{tot}D_{AB}(b)]^{AB}+[1-\sigma_{tot}D_{AB}(b)+\tfrac{1}{4}\sigma_{tot}^2D_{AB}^2(b)]^{AB}, \qquad (7.4)$$

where we have used the relation (3.5).

For the *quasi-elastic cross-section*, by introducing the averaging over the densities $P_A(\alpha_1 \dots \alpha_A)$, $P_B(\beta_1 \dots \beta_B)$ under the products of the Eq. (6.7) we get

$$\sigma_{q.el}^{AB}(b) = \int d^3x_1 \dots d^3x_Ad^3y_1 \dots d^3y_B\varrho_A(x_1 \dots x_A)\varrho_B(y_1 \dots y_B)$$
$$\times\{1-2\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\eta_{el}(b-x_i^\perp+y_j^\perp)]+\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\sigma_{inel}(b-x_i^\perp+y_j^\perp)]\}-\sigma_{el}^{AB}(b), \qquad (7.5)$$

where we have used the Eq. (3.9). Consequently, using the Eq. (7.4) we obtain in "optical approximation"

$$\sigma_{q.el}^{AB}(b) = [1-\sigma_{inel}D_{AB}(b)]^{AB}-[1-\sigma_{tot}D_{AB}(b)+\tfrac{1}{4}\sigma_{tot}^2D_{AB}^2(b)]^{AB}. \qquad (7.6)$$

We go over to *cross-section for diffractive production*. Starting from Eq. (6.9) and going through the same steps as in the case of Eqs (6.7) and (7.5) we obtain

$$\sigma_{diff}^{AB}(b) = \int d^3x_1 \dots d^3x_Ad^3y_1 \dots d^3y_B\varrho_A(x_1 \dots x_A)\varrho_B(y_1 \dots y_B)$$
$$\times\{\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\sigma_{nd}(b-x_i^\perp+y_j^\perp)]-\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\sigma_{inel}(b-x_i^\perp+y_j^\perp)]\}, \qquad (7.7)$$

where we have used the relations (3.7) and (3.8). The corresponding "optical" formula reads

$$\sigma_{\text{diff}}^{AB}(\boldsymbol{b}) = [1-\sigma_{\text{nd}}D_{AB}(\boldsymbol{b})]^{AB}-[1-\sigma_{\text{inel}}D_{AB}(\boldsymbol{b})]^{AB}. \tag{7.8}$$

Finally, the approximate formula for probability of non-diffractive collision is obtained immediately from Eq. (6.10) and (3.7)

$$\sigma_{\text{nd}}^{AB}(\boldsymbol{b}) = \int d^3x_1 \ldots d^3x_A d^3y_1 \ldots d^3y_B \varrho_A(\boldsymbol{x}_1 \ldots \boldsymbol{x}_A)\varrho_B(\boldsymbol{y}_1 \ldots \boldsymbol{y}_B)$$
$$\times \{1-\prod_{i=1}^{A}\prod_{j=1}^{B}[1-\sigma_{\text{nd}}(\boldsymbol{b}-x_i^{\perp}+y_j^{\perp})]\}. \tag{7.9}$$

The optical approximation gives

$$\sigma_{\text{nd}}^{AB}(\boldsymbol{b}) = [1-\sigma_{\text{nd}}D_{AB}(\boldsymbol{b})]^{AB}. \tag{7.10}$$

To close this Section let us comment on possible corrections to the Glauber-model type formulas presented in Sections 5 and 7. One possible method of constructing such corrections [8] is by expanding the formulas of Sections 4 and 6 in powers of

$$\eta(\alpha;\beta;\boldsymbol{b})-\eta_{\text{el}}(\boldsymbol{b}),$$

where $\eta_{\text{el}}(\boldsymbol{b})$ is given by Eq. (3.3). The resulting formulae (which are rather involved) in the lowest non-vanishing order can be expressed in terms of measurable hadron-nucleon cross-sections. The higher orders, however, require information on eigenmodes $|\alpha, \beta\rangle$.

9. *Summary and conclusions*

Having in mind recent results on multiparticle production in nuclei [9] which clearly show the effects of diffractive collisions in such processes [10] we reanalyzed the probabilistic description of inelastic hadron-nucleus and nucleus-nucleus collisions, with diffractive channels present.

We obtained formulae for elastic, quasi-elastic, coherent diffractive, incoherent diffractive (quasi-diffractive), and non-diffractive hadron-nucleus and nucleus-nucleus cross-sections. We discussed these formulae in the Glauber model type approximations which express the hadron-nucleus and nucleus-nucleus cross-sections in terms of nuclear densities and hadron-nucleon cross-sections.

In Ref. [10] we have already shown that such probabilistic approach is useful in analyzing experimental data. We believe that it may be successfully applied to the forthcoming hadron-nucleus and nucleus-nucleus data.

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