THE Λ-PARTICLE BINDING IN NUCLEAR MATTER IN THE FERMI-HYPERNETTED-CHAIN APPROXIMATION AND WITH THE REACTION MATRIX METHOD*

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The binding energy of the Λ particle in nuclear matter, B_Λ, is calculated for a number of central hard core ΛN potentials and for the OMY6 NN potential. Firstly, the Jastrow method in the Fermi-hypernetted-chain (FHNC) approximation is applied. Secondly, the Brueckner reaction method is used. The FHNC results for B_Λ are much bigger than the reaction-matrix matrix results. Possible sources of this discrepancy are discussed.

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1. Introduction

The investigation of the binding energy of a Λ particle in nuclear matter (NM), B_Λ, is of considerable interest as it enables us to gain valuable information of the ΛN interaction, $v_{\Lambda N}$. Furthermore, the Λ + NM system, i.e., NM with a Λ "impurity", is an interesting testing ground for nuclear many-body theories.

In the theoretical analysis of B_Λ, two methods have been applied: the Brueckner reaction matrix method, and the Jastrow correlated function method (for a review, see [1]).

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Most of the existing calculations of B_Λ have used the reaction matrix method. Their results may be summarized as follows:

(*i*) Purely central ΛN potentials, fitted to Λp scattering and to Λ binding in $A = 3, 4$ hypernuclei, with hard cores of radius $c_\Lambda \gtrsim 0.4$ fm, and with suppression in odd-angular-momentum states lead to overbinding. The value of B_Λ, calculated with these potentials, is about 10 MeV larger than the semiempirical value $B_\Lambda(\mathrm{SE}) \lesssim 30$ MeV [2].

(*ii*) Because of the suppression of ΛΣ conversion in NM, realistic ΛN interactions with coupling to the ΣN channel lead to values of B_Λ which agree with $B_\Lambda(\mathrm{SE})$ [3].

The Jastrow type calculations of B_Λ have been restricted to central, possibly spin dependent, ΛN interactions. (Suppression in odd-angular-momentum states requires the presence of a space exchange operator in $v_{\Lambda N}$, and has been treated in [4] (see also [5], [6]) only approximately.)

So far, several central potentials $v_{\Lambda N}$ have been used in calculating B_Λ, both with the reaction matrix and Jastrow method. Values of B_Λ obtained with the Jastrow method turn out to be always much bigger than those obtained with the reaction matrix method.

The purpose of our paper is to analyse the discrepancy between the values of B_Λ obtained by the Jastrow and Brueckner methods with semirealistic ΛN interactions. By semirealistic interactions we mean central, spin dependent ΛN potentials, fitted to Λp scattering and to Λ binding in $A = 3,4$ hypernuclei, with hard cores $c_\Lambda \geqslant 0.4$ fm, and without suppression in odd-angular-momentum states. These interactions are not expected to reproduce the semi-empirical value of B_Λ. Hence, the present paper is of a methodological significance.

Most of the existing Jastrow type calculations of B_Λ apply a low order cluster (LOC) expansion method (see, [4, 5, 7, 8]). The LOC approximation proves reasonable only in the case when it is possible to select the class of the trial functions so that all higher order clusters would yield negligible contributions to the energy. This is not possible for an arbitrary density (see, e.g. [9]). When the matter density is sufficiently large, certain subseries of the full cluster expansion consist of cluster terms, or diagrams, which give increasing contributions to the energy with increasing number of points. In this situation, the LOC approximation loses its validity and it is necessary to sum all the diagrams of these subseries. This may be accomplished by solving appropriate integral (chain) equations, as was shown by Fantoni and Rosati [10] who developed the powerful Fermi hypernetted chain (FHNC) method of summation of cluster integrals.

This is the reason for which the chain summation method has been recently applied in calculating B_Λ [6, 11, 12]. However, these calculations do not seem to be satisfactory due to simplifications of various type. What we mean here is either approximating the expression for B_Λ [6, 12], or not applying a full optimization procedure [11, 12], or applying the chain method in its simplest version — the chain (CH) approximation [11].

One of the main accomplishments of the present paper is a chain type calculation of B_Λ, which appears to be more adequate than the former ones. Thus the full expression for B_Λ is calculated in the FHNC approximation by means of an optimization procedure. We apply the FHNC method for the impurity problem, described in detail in [13], hereafter

referred to as DP. Our calculations are performed for a number of semirealistic ΛN potentials.

The resulting FHNC values of B_Λ do not differ significantly from those obtained previously in the LOC approximation, and are much bigger than the values of B_Λ obtained with the same ΛN and NN interactions by applying the reaction matrix method. Consequently, the serious discrepancy between the Jastrow and Brueckner methods of calculating B_Λ remains an open problem.

The paper is organized as follows. In Section 2, the Jastrow method of calculating B_Λ is outlined, and the expression for B_Λ in terms of radial distribution functions is derived. In Section 3, we apply the FHNC method of DP to calculate B_Λ. The ΛN potentials, and the NN potential as well as the form of the ΛN and NN correlation functions are specified. In Section 4, we compare our results for B_Λ with the results of other Jastrow type calculations. In Section 5, we outline the reaction matrix method of calculating B_Λ and present the resulting values of B_Λ obtained with the ΛN and NN potentials used in our FHNC calculations. In Section 6, we discuss possible sources of the discrepancy between the results obtained for B_Λ with the Brueckner and Jastrow methods.

2. *The Jastrow method of calculating* B_Λ

B_Λ is defined by

$$B_\Lambda = E_{\mathrm{NM}} - E_{\Lambda+\mathrm{NM}}, \tag{2.1}$$

where E_{NM} and $E_{\Lambda+\mathrm{NM}}$ are the ground state energies of NM and of the Λ+NM system.

Let us denote by H, Ψ_0, and $\boldsymbol{H}$, $\boldsymbol{\Psi}_0$ the hamiltonians and the ground state wave functions of NM and of the Λ+NM system. We have

$$E_{\mathrm{NM}} = \langle\Psi_0|H|\Psi_0\rangle/\langle\Psi_0|\Psi_0\rangle \leqslant \langle\Psi|H|\Psi\rangle/\langle\Psi|\Psi\rangle, \tag{2.2}$$

$$E_{\Lambda+\mathrm{NM}} = \langle\boldsymbol{\Psi}_0|\boldsymbol{H}|\boldsymbol{\Psi}_0\rangle/\langle\boldsymbol{\Psi}_0|\boldsymbol{\Psi}_0\rangle \leqslant \langle\boldsymbol{\Psi}|\boldsymbol{H}|\boldsymbol{\Psi}\rangle/\langle\boldsymbol{\Psi}|\boldsymbol{\Psi}\rangle, \tag{2.3}$$

where the last parts of the two equations, with $\Psi(\boldsymbol{\Psi})$ being approximate trial functions of the NM(Λ+NM) ground state, express the variational principle. For B_Λ we have an approximate expression

$$B_\Lambda \simeq \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} - \frac{\langle\boldsymbol{\Psi}|\boldsymbol{H}|\boldsymbol{\Psi}\rangle}{\langle\boldsymbol{\Psi}|\boldsymbol{\Psi}\rangle} \tag{2.4}$$

which becomes exact for $\Psi \equiv \Psi_0$ and $\boldsymbol{\Psi} \equiv \boldsymbol{\Psi}_0$.

If we vary $\boldsymbol{\Psi}$ for a fixed shape of Ψ, expression (2.4) for B_Λ attains its maximum for $\boldsymbol{\Psi} \equiv \boldsymbol{\Psi}_0$. On the other hand, if we vary Ψ for a fixed shape of $\boldsymbol{\Psi}$, expression (2.4) for B_Λ attains its minimum for $\Psi = \Psi_0$. For arbitrary shapes of the trial functions Ψ and $\boldsymbol{\Psi}$, expression (2.4) may be both bigger or smaller than B_Λ, in contradistinction to expressions (2.2) and (2.3) which have the properties of upper bounds on E_{NM} and $E_{\Lambda+\mathrm{NM}}$.

In the Jastrow method, we make the following Ansatz for Ψ and Ψ:

$$\Psi(1, \ldots, A) = \prod_{j<k} f_{\mathrm{NN}}(r_{jk})\Phi(1, \ldots, A), \tag{2.5}$$

$$\Psi(\Lambda, 1, \ldots, A) = \prod_{i} f_{\mathrm{N}\Lambda}(r_{i\Lambda})\varphi_{\Lambda}(\Lambda)\Psi(1, \ldots, A), \tag{2.6}$$

where the arguments of the functions indicate the full sets of space, and spin-isospin coordinates of the respective particle. The Slater determinant Φ is an antisymmetrized product of single particle wave functions (products of spin-isospin functions and plane wave functions normalized in the periodicity box of volume Ω). The function $\varphi_{\Lambda}(\Lambda)$ is the Λ particle spin function multiplied by $\Omega^{-1/2}$, i.e., by the normalized plane wave function with zero momentum (in the ground state of non-interacting $\Lambda+\mathrm{NM}$ system, the Λ particle has zero momentum).

For the NY (Y = N, Λ) correlation functions, we assume a simple, state independent, parametric form. First, we fix the values of the free parameters in f_{NN} by minimizing the expectation value of the NM hamiltonian, Eq. (2.2). (In practice we use the results of other authors who performed this minimalization.) Then we fix the free parameters in $f_{\mathrm{N}\Lambda}$ by maximizing expression (2.4) for B_{Λ}. This maximum value of (2.4) is our Jastrow method result for B_{Λ}.

In our procedure, we use the same optimal NN correlation function f_{NN} in pure NM and in the $\Lambda+\mathrm{NM}$ system. This is justified for the following reason. The difference in f_{NN} in NM and in the $\Lambda+\mathrm{NM}$, δf_{NN}, is of order $1/A$. This difference introduces a change, δE_{NM}, in E_{NM}. Now, E_{NM} is proportional to A, but it attains a minimum for the optimal f_{NN}, $\delta E_{\mathrm{NM}}/\delta f_{\mathrm{NN}} = 0$. Consequently δE_{NM} is proportional to $A \times A^{-2} = A^{-1}$, and the whole difference δf_{NN} may be disregarded. (This point is discussed in [5]; see also the comments after Eq. (4.26) of [14], and Appendix D of [15].)

The expectation value of H may be expressed in terms of the one- and two-body distribution functions of the $\Lambda+\mathrm{NM}$ system:

$$\varrho g_{\mathrm{N}}(\boldsymbol{x}) = \langle \Psi | \sum_{i} \delta(\boldsymbol{r}_i - \boldsymbol{x}) | \Psi \rangle / \langle \Psi | \Psi \rangle, \tag{2.7}$$

$$\varrho^2 g_{\mathrm{NN}}(\boldsymbol{x}_1, \boldsymbol{x}_2) = \varrho^2 f_{\mathrm{NN}}^2(x_{12}) \mathscr{G}_{\mathrm{NN}}(x_{12})$$

$$= \langle \Psi | \sum_{i \neq j} \delta(\boldsymbol{r}_i - \boldsymbol{x}_1)\delta(\boldsymbol{r}_j - \boldsymbol{x}_2) | \Psi \rangle / \langle \Psi | \Psi \rangle, \tag{2.8}$$

$$\Omega^{-1} \varrho g_{\mathrm{N}\Lambda}(\boldsymbol{x}_1, \boldsymbol{x}_{\Lambda}) = \Omega^{-1} \varrho f_{\mathrm{N}\Lambda}^2(x_{1\Lambda}) \mathscr{G}_{\mathrm{N}\Lambda}(x_{1\Lambda})$$

$$= \langle \Psi | \sum_{i} \delta(\boldsymbol{r}_i - \boldsymbol{x}_1)\delta(\boldsymbol{r}_{\Lambda} - \boldsymbol{x}_{\Lambda}) | \Psi \rangle / \langle \Psi | \Psi \rangle, \tag{2.9}$$

where $\varrho = A/\Omega$ is the density of nucleons ($\varrho = 2k_{\mathrm{F}}^3/3\pi^2$, where k_{F} is the Fermi momentum of NM in units of $\hbar$). Similarly, the expectation value of H may be expressed in terms of the one- and two-body distribution function of NM, g_{N} and g_{NN}, defined by Eqs (2.7) and (2.8) with Ψ replaced by Ψ. For the difference in the two expectation values, Eq.

(2.4), we get

$$-B_\Lambda = \tilde{V}_{N\Lambda} + \Delta\tilde{V}_{NN} + \tfrac{1}{4}(\hbar^2/2M_N)\Delta\tau, \tag{2.10}$$

where

$$\tilde{V}_{N\Lambda} = \varrho \int dr f_{N\Lambda}^2(r)\mathscr{G}_{N\Lambda}(r)\tilde{v}_{N\Lambda}(r), \tag{2.11}$$

$$\Delta\tilde{V}_{NN} = \tfrac{1}{2}\varrho^2\Omega \int dr f_{NN}^2(r)\hat{\mathscr{G}}_{NN}(r)\tilde{v}_{NN}(r), \tag{2.12}$$

$$\Delta\tau = \varrho \int dr \Delta^{\Phi}[g_N(r) - g_N(r)], \tag{2.13}$$

where $\hat{\mathscr{G}}_{NN} = \mathscr{G}_{NN} - \mathscr{G}_{NN}$, and the effective two-body potentials are:

$$\tilde{v}_{NN}(r) = v_{NN}(r) + \tfrac{1}{2}(\hbar^2/M_N)\left[(\nabla f_{NN}(r)/f_{NN})^2 - \Delta f_{NN}(r)/f_{NN}\right], \tag{2.14}$$

$$\tilde{v}_{N\Lambda}(r) = v_{N\Lambda}(r) + \tfrac{1}{4}\hbar^2[(M_N + M_\Lambda)/M_N M_\Lambda]\left[(\nabla f_{N\Lambda}(r)/f_{N\Lambda})^2 - \Delta f_{N\Lambda}(r)/f_{N\Lambda}\right], \tag{2.15}$$

where v_{NY} is the NY potential. (To simplify the formulae, we assume here that v_{NY} is of pure Wigner type.)

Notice that for the Jastrow trial functions, Eqs. (2.5), (2.6), and for the Jackson–Feenberg form of the kinetic energy applied here, only one- and two-body distribution functions appear in our formulae. By Δ^{Φ}, we denote in (2.13) the Laplace operator acting on r which appears in the $|\Phi|^2$ component of the one-body distribution functions.

In calculating the radial distribution functions, we apply the FHNC method described in detail in DP. In the FHNC method, we approximate the complete cluster expansion of the distribution function by hypernetted chains of cluster integrals. The summation of these hypernetted chains is accomplished by solving (numerically) the chain equations derived in DP.

3. *Calculational procedure and FHNC results for* B_Λ

In our calculations, we have used the OMY [16] NN interaction,

$$v_{NN} = \begin{cases} \infty, & r \leqslant c_N, \\ \tfrac{1}{2}(1+P_r)\left\{\tfrac{1}{2}(1+P_\sigma)v_{NN}^t(r) + \tfrac{1}{2}(1-P_\sigma)v_{NN}^s(r)\right\}, & r > c_N, \end{cases} \tag{3.1}$$

where P_r and P_σ are the space and spin exchange operators, and

$$v_{NN}^{t,s}(r) = -V_{NN}^{t,s}\exp\left[-\kappa_{t,s}(r - c_N)\right]. \tag{3.2}$$

The parameters of this potential are:

$$c_N = 0.6\ \text{fm}, \quad \kappa_t = 3.6765\ \text{fm}^{-1}, \quad \kappa_s = 2.6272\ \text{fm}^{-1},$$

$$V_{NN}^t = 947.02\ \text{MeV}, \quad V_{NN}^s = 397.31\ \text{MeV}. \tag{3.3}$$

We chose the OMY potential, because it has been used in almost all Jastrow type calculations of B_Λ [4–8, 11].

For the NΛ interactions we use the potentials HNX, ENX, E'NX of Herndon and Tang [17], and the potential DW of Downs and Ware [18]. Our notation for the potentials is the same as in [19] (hereafter referred to as DH), e.g. HNX is the potential H of [17] with no exchange ($x = 0$). All these potentials have the following form:

$$v_{N\Lambda} = \begin{cases} \infty, & r \leqslant c_\Lambda, \\ \frac{1}{2}(1+P_\sigma)v^t_{N\Lambda}(r)+\frac{1}{2}(1-P_\sigma)v^s_{N\Lambda}(r), & r \geqslant c_\Lambda, \end{cases} \tag{3.4}$$

where

$$v^{t,s}_{N\Lambda}(r) = -V^{t,s}_{N\Lambda} \exp\left[-\lambda(r-c_\Lambda)\right]. \tag{3.5}$$

The parameters of these potentials are given in Table I, which also contains the intrinsic range b, and the triplet and singlet well-depth parameters s_t, s_s. All these potentials have been used in several previous calculations of B_Λ.

TABLE I

Parameters of $v_{N\Lambda}$

$v_{N\Lambda}$	c_Λ (fm)	λ (fm^{-1})	b (fm)	$V^t_{N\Lambda}$ (MeV)	$V^s_{N\Lambda}$ (MeV)	s_t	s_s
HNX[a]	0.6	3.935	2.1	676.9	713.1	0.792	0.834
ENX[a]	0.45	3.219	2.0	402.1	451.7	0.703	0.790
E'NX[a]	0.45	3.219	2.0	378.7	459.5	0.662	0.803
DW[b]	0.4	3.219	1.9	330.9	330.9	0.578	0.578

[a] Ref. [17]. [b] Ref. [18].

For the correlation functions, we assume the following form:

$$f_{NY}(r) = \begin{cases} 0, & r \leqslant c_Y, \\ \{1-\exp\left[-\alpha_Y(r-c_Y)\right]\}\{1+\beta_Y \exp\left[-\gamma_Y(r-c_Y)\right]\}, & r > c_Y. \end{cases} \tag{3.6}$$

This form of the NN correlation function was used by the Pisa group [20] in the NM calculation, in which the expectation value of the NM hamiltonian, in the FHNC approximation, with the OMY interaction was minimized with respect to the parameters of the NN correlation function, Eq. (3.6), under the subsidiary "average Pauli condition",

$$\varrho \int d\boldsymbol{r}[1-f_{NN}(r)]g_F(r) = 0, \tag{3.7}$$

where $g_F(r)$ is the radial distribution function of noninteracting NM. The parameters of this correlation function (which we denote by f^P_{NN}), for $k_F = 1.366\,\text{fm}^{-1}$, are given in Table II which also contains the value, I_N(FHNC), of the normalization integral

$$I_N = \varrho \int d\boldsymbol{r}[1-g_{NN}(r)], \tag{3.8}$$

for g_{NN} calculated in the FHNC approximation. The fact that the value of I_N is very close to one indicates a good accuracy of the FHNC approximation with the f^P_{NN} correlation function ($I_N = 1$ for an exact g_{NN}).

TABLE II

Parameters of f_{NN} and values of I_N (FHNC)

	$\alpha_N(\text{fm}^{-1})$	β_N	$\gamma_N(\text{fm}^{-1})$	I_N(FHNC)
f_{NN}^{P}	2.5[a]	0.884[a]	2.0[a]	1.06
f_{NN}^{MC}	2.3[b]	1.394[b]	2.3[b]	0.87

[a] Ref. [20]. [b] Ref. [4].

To test the sensitivity of the calculated value of B_Λ to the shape of f_{NN}, we have also used the NN correlation function f_{NN}^{MC} of Mueller and Clark [4] who minimized (at $k_F = 1.366\ \text{fm}^{-1}$) the expectation value of the NM hamiltonian (with v_{NN}(OMY)) in the lowest (two-body) cluster approximation, with the subsidiary normalization condition

$$\varrho \int dr[f_{NN}^2(r) - 1]g_F(r) = 0, \tag{3.9}$$

which expresses the lowest (two-body) cluster approximation to the requirement, $I_N = 1$. The parameters of f_{NN}^{MC}, as well as the corresponding value of I_N(FHNC), are given in Table II.

The NN and NΛ interactions, and the NN and NΛ correlation functions described above, were used in calculating B_Λ according to Eq. (2.10). Since our interactions depend on spin and parity, we applied the properly modified form of expressions (2.11) and (2.12), given in Appendix C of DP. The main computational effort of this work consisted in determining $\mathcal{G}_{N\Lambda}$ and $\hat{\mathcal{G}}_{NN}$. The functions $\mathcal{G}_{N\Lambda}$ and $\hat{\mathcal{G}}_{NN}$ were determined by solving chain Eqs. (4.6) and (4.14) od DP, respectively. Both systems of chain equations, (4.6) and (4.14) of DP, require the knowledge of the radial distribution function in pure NM, $g_{NN} = f_{NN}^2 \mathcal{G}_{NN}$, which was determined by solving chain equations (A.5–8) of DP.

Each of the three systems of chain equations was solved numerically by an iterative procedure. In the chain equations for the chain functions (denoted in DP by N_{mn}) in a given iteration, the link functions (denoted in DP by X_{mn}) resulting from the previous iteration were used. Consequently, the chain equations in a given iteration were systems of linear integral equations which were solved in terms of Fourier transforms.

The $\Delta\tau$ part of B_Λ, Eq. (2.10), turned out to be very small (of the order of our numerical accuracy) and therefore have been neglected. Notice, that the corresponding τ term gives only a very small contribution to NM binding. Now, the $\Delta\tau$ term in (2.10) represents the change in τ caused by the presence of Λ in NM, and one expects it to be very small.

The determination of B_Λ consists in finding the maximum value of expression (2.10) for B_Λ with respect to the parameters in $f_{N\Lambda}$. With $f_{N\Lambda}$ given by Eq. (3.6), it would require a search for a maximum in three-dimensional parameter space. The size of the computation of B_Λ for a given $f_{N\Lambda}$ prevented us from such an extensive search, and led us to consider a restricted two-parameter version of expression (3.6) for $f_{N\Lambda}$ with $\alpha_\Lambda = \gamma_\Lambda$. To justify our procedure, let us mention that the one-parameter $f_{N\Lambda}$ function (a special case ($\beta_\Lambda = 0$) of our two-param-

eter function) was found by Clark and Mueller [5] to work almost as well as the more elaborate $f_{N\Lambda}$ function [4] in their Iwamoto-Yamada cluster expansion method of calculating B_Λ. Moreover, we know that an optimal three-parameter NΛ correlation function of Eq. (3.6) would obviously lead to an even bigger value of B_Λ than our two-parameter function.

All the calculations have been performed for $k_F = 1.366\ \mathrm{fm}^{-1}$ with the NN correlation function f^{P}_{NN}, and for comparison, also with f^{MC}_{NN}. Our results are shown in Table III.

TABLE III

FHNC results for B_Λ obtained for $k_F = 1.366\ \mathrm{fm}^{-1}$ with the two-parameter $f_{N\Lambda}$ with the indicated optimal values of $\alpha_\Lambda = \gamma_\Lambda$, and β_Λ

$v_{N\Lambda}$	f_{NN}	α_Λ (fm^{-1})	β_Λ	$-\tilde{V}_{N\Lambda}$ (MeV)	$-\Delta\tilde{V}_{NN}$ (MeV	B_Λ (MeV)
HNX	P	6.0	−0.05	80.1	−3.1	77.0
	MC	6.5	−0.10	80.0	−3.9	76.1
ENX	P	6.3	−0.25	69.8	−1.5	68.3
	MC	6.3	−0.20	69.7	−1.9	67.7
E'NX	P	6.0	−0.20	64.3	−1.5	62.7
	MC	6.0	−0.20	64.2	−2.0	62.2
DW	P	5.5	0.05	44.7	−1.1	43.6
	MC	5.5	0.05	44.7	−1.4	43.3

We see that values of B_Λ obtained with f^{P}_{NN} are systematically larger than those obtained with f^{MC}_{NN}. We consider f^{P}_{NN} to be a better correlation function since it is optimized in the FHNC scheme and gives the value of I_N closer to one than f^{MC}_{NN} does. And by changing f^{MC}_{NN} into f^{P}_{NN}, we expect to improve the wave function Ψ, Eq. (2.5), of NM, and also the wave function $\boldsymbol{\Psi}$, Eq. (2.6), of the Λ+NM system. These simultaneous improvements lead to an increase in B_Λ, Eq. (2.4), which is practically equal to the decrease in $-\Delta\tilde{V}_{NN}$.

4. Comparison with other Jastrow type calculations of B_Λ

We start with the LOC calculations of B_Λ. In these calculations, only few leading terms in the cluster expansion of radial distribution functions (2.7–9) are taken into account. For comparison with our FHNC results, we choose the work by Mueller and Clark [4], [5], and by Ali, Grypeos, and Kargas [8].

Mueller and Clark [4] apply the Iwamoto–Yamada [21] cluster expansion, and for $f_{N\Lambda}$ assume the form:

$$f_{N\Lambda}(r) = \begin{cases} 0, & r \leqslant c_\Lambda, \\ \{1-(c_\Lambda/r)\exp[-\tilde{\alpha}_\Lambda(r-c_\Lambda)]\}\{1+\tilde{\beta}_\Lambda\exp[-\tilde{\gamma}_\Lambda(r-c_\Lambda)]\}, & r > c_\Lambda, \end{cases} \tag{4.1}$$

with the subsidiary normalization condition (compare (3.9)):

$$\varrho \int dr[f_{N\Lambda}^2(r)-1] = 0. \tag{4.2}$$

For v_{NN}(OMY) and with the NN correlation function f_{NN}^{MC} (see Table II), they determine the parameters $\tilde{\alpha}_\Lambda, \tilde{\beta}_\Lambda, \tilde{\gamma}_\Lambda$ by maximizing B_Λ in the Iwamoto-Yamada approximation. The optimal values of $\tilde{\alpha}_\Lambda = \gamma, \tilde{\beta}_\Lambda = \mu, \tilde{\gamma}_\Lambda = \nu$ are given in Table III of [4]. (Throughout the present paper, we use results of their procedure (*i*).) Their results for B_Λ, denoted by B_Λ^{IY}, are given in Table IV. Actually, Mueller and Clark consider NΛ potentials H, E, E' of [14], with suppression in odd angular-momentum states. (They treat the suppression

TABLE IV

Results of Jastrow type calculations of B_Λ in MeV at $k_F = 1.366\ \text{fm}^{-1}$ for OMY NN interaction (except for [12])

Ref.	HNX		ENX	E'NX	DW
[4]	B_Λ^0	83	77	70	46
	B_Λ^{IY}	74	71	65	42
	$\mathscr{B}_\Lambda^{IY}$	75	71	65	42
[8]	B_Λ^0	89	80	74	48
	$\mathscr{B}_\Lambda^{IY}$	75	71	65	42
11]	B_Λ^{CH}	71	70	63	40
	$\mathscr{B}_\Lambda^{CH}$	77	72	65	43
[6]	$\mathscr{B}_\Lambda^{FHNC}$	80	75	68	45
[12]	B_Λ^{FHNC}	78	73	65	44
present calcul.	B_Λ^0	74	70	64	43
	B_Λ^{FHNC}	77	68	63	44

in the approximation, in which the NΛ space exchange operator does not act on the correlation function.) The values of B_Λ^{IY} in Table IV have been obtained by rejecting the suppression effect (with the help of Table IV of [4]).

Westhaus and Clark [22] (see also [5]) derived an expression for B_Λ alternative to (2.10), by assuming that Ψ in (2.6) is the exact wave function of NM ground state. With this assumption, definition (2.4) of B_Λ leads to:

$$\mathscr{B}_\Lambda = -\varrho \int dr f_{N\Lambda}^2(r)\mathscr{G}_{N\Lambda}(r)w_{N\Lambda}(r), \tag{4.3}$$

where

$$w_{N\Lambda}(r) = \tilde{v}_{N\Lambda}(r)+(\hbar^2/4M_N)\,[(\nabla f_{N\Lambda}(r)/f_{N\Lambda})^2+\Delta f_{N\Lambda}(r)/f_{N\Lambda}]. \tag{4.4}$$

To derive Eq. (4.3), no assumption concerning the specific form of Ψ is necessary. However, if we want to use Eq. (4.3), we have to calculate $\mathscr{G}_{N\Lambda}$. And to do it, we have to know Ψ.

Here, we make Ansatz (2.5) for Ψ, and calculate $\mathscr{G}_{N\Lambda}$ by applying cluster expansion. Now, Ψ given by (2.5) is not the exact ground state wave function of NM, and consequently expression (4.3) is an approximate one. We use the notation $\mathscr{B}_\Lambda$ for expression (4.3), to visualize the difference between (4.3) and our original expression (2.10) for B_Λ. Notice that $\mathscr{B}_\Lambda$, in contradistinction to B_Λ, does not depend explicitly on v_{NN}.

Results for $\mathscr{B}_\Lambda$, $\mathscr{B}_\Lambda^{IY}$, obtained with the Iwamoto-Yamada approximation to $\mathscr{G}_{N\Lambda}$, and with the ΛN and NN correlation functions of [4], are shown in Table IV.

The paper by Ali, Grypeos, and Kargas [8] is based on the cluster expansion scheme of Aviles [23], applied to the Λ binding problem by Downs and Grypeos [7]. In the low order approximation (three points), used in [8], the cluster expansion scheme of Aviles applied to the B_Λ problem coincides with the Iwamoto–Yamada approximation. For this reason, we denote the results of [8] with the superscript IY. The ΛN correlation function is determined in [8] by maximizing B_Λ in the lowest order of the cluster expansion (in this order $B_\Lambda = \mathscr{B}_\Lambda$), with the subsidiary healing condition,

$$\kappa_{N\Lambda}^{J} = \varrho \int dr (f_{N\Lambda} - 1)^2 = \text{const.} \tag{4.5}$$

The Lagrange multiplier, which appears in the Euler equation for $f_{N\Lambda}$, is determined by maximizing the sum of the lowest and next order term in the cluster expansion of $\mathscr{B}_\Lambda$. The results of [8], denoted by $\mathscr{B}_\Lambda^{IY}$, obtained with the NN correlation function f_{NN}^{MC} (see Table II) are shown in Table IV. The use of f_{NN}^{MC} implies that the OMY NN interaction is assumed.

Notice that for a given $N\Lambda$ interaction, all the LOC results, B_Λ^{IY} and $\mathscr{B}_\Lambda^{IY}$, shown in Table IV are identical, except for a small (1 MeV) discrepancy between B_Λ^{IY} and $\mathscr{B}_\Lambda^{IY}$ in the case of the HNX interaction.

Now let us turn to the recent calculations of B_Λ which apply the chain summation technique.

The simplest chain summation is accomplished in the CH approximation. In this approximation, the summation is restricted to single chains built of the simplest links, without parallel connection of chains (see DP). The CH results for B_Λ and $\mathscr{B}_\Lambda$, B_Λ^{CH} and $\mathscr{B}_\Lambda^{CH}$, obtained in [11] and [15] for v_{NN}(OMY) with the Mueller-Clark form of $f_{N\Lambda}$, Eq. (4.1), and with f_{NN}^{MC} (see Table II), are shown in Table IV. The reliability of these results appears to be less certain because they have been obtained without any optimization procedure, and with the very simple CH approximation.

An FHNC calculation of $\mathscr{B}_\Lambda$ has been performed by Barbato [6]. Her results, $\mathscr{B}_\Lambda^{FHNC}$, for $N\Lambda$ potentials without suppression in odd angular momentum states are shown in Table IV. Among her results, we have selected those obtained with the Mueller-Clark form of $f_{N\Lambda}$, Eq. (4.1), and with f_{NN}^{MC} (see Table II) corresponding to v_{NN}(OMY). Although Barbato performs a complete FHNC calculation, she uses the approximate expression (4.3), i.e., she calculates only $\mathscr{B}_\Lambda$ but not B_Λ.

In a recent paper, Usmani [12] derives the FHNC equations for the $\tilde{V}_{N\Lambda}$ and $\Delta\tilde{V}_{NN}$ parts of B_Λ, Eq. (2.10), which coincide with the corresponding equations of DP (he uses chain equations for $\mathscr{G}_{N\Lambda}$ and $\hat{\mathscr{G}}_{NN}$ in the form denoted in DP as Eqs. (4.6) and (4.14)). The $\Delta\tau$ part of B_Λ is not considered in [12]. In his numerical calculation of B_Λ, Usmani

uses the f_{NN}^{MC} form of the NN correlation function (with parameters given in Table II), and $f_{N\Lambda}$ in the form given in Eq. (4.1). The values of the parameters $\tilde{\alpha} = \gamma$, $\tilde{\beta}_\Lambda = \mu$, $\tilde{\gamma}_\Lambda = \nu$, quoted by Usmani, are the optimal parameters of [4] for the NΛ potential E, and their use for other NΛ potentials would be questionable. In [4], the NN correlation function f_{NN}^{MC} was adjusted to v_{NN}(OMY). In [12], however, f_{NN}^{MC} is used inconsistently, together with the pure Wigner type potential of Tang and Herndon [24], which has a hard core radius $c_\Lambda = 0.45$ fm. Furthermore, the values of B_Λ at two different densities ϱ are calculated in [12] with unchanged correlation functions, whereas the optimal correlation functions change their shape with changing density. Nevertheless, the B_Λ^{FHNC} results obtained by Usmani for B_Λ at $k_F = 1.366\,\mathrm{fm}^{-1}$, shown in Table IV, approximately agree with the results of other calculations.

Table IV also contains results of our own FHNC calculations (with f_{NN}^{P}) described in Section 4.

It is instructive to consider the lowest order (two body) part of B_Λ,

$$B_\Lambda^0 = \varrho \int dr f_{N\Lambda}^2(r)\tilde{v}_{N\Lambda}(r). \tag{4.6}$$

Values of B_Λ^0 for all optimal correlation functions under consideration are given in Table IV (notice that $\mathscr{B}_\Lambda^0 = B_\Lambda^0$). We see that B_Λ is determined predominantly by the lowest order cluster term B_Λ^0. Next order cluster terms, present in the LOC results, constitute only a correction to B_Λ^0, of an order of magnitude of 10%. Higher order terms present in the CH and FHNC results are much smaller. Thus, for the class of correlation functions considered, and at the equilibrium density of NM ($k_F = 1.366\,\mathrm{fm}^{-1}$), the LOC approximation turns to be adequate.

5. B_Λ *calculated with the reaction matrix method*

The starting point in the reaction matrix method of calculating B_Λ (see [1] for a review) is Eq. (2.1) in which both E_{NM} and $E_{NM+\Lambda}$ are expressed in terms of the Brueckner reaction matrices K_{NN} and $K_{N\Lambda}$. Following the systematic approach in terms of the number of hole lines, worked out for pure NM (see [25] for a recent review), we group the contributions to B_Λ (energy diagrams) according to the number of interacting particles (number of hole lines). In the low-order Brueckner method (LOB), only diagrams with two hole lines are considered usually with the "standard choice" of pure kinetic single particle (s.p.) energies in the reaction matrix equation. In this "standard" LOB the self-consistent expression for $B_\Lambda = -V_\Lambda$ (V_Λ = s.p. potential of zero momentum Λ) is:

$$-B_\Lambda = V_\Lambda = 4 \sum_{\boldsymbol{k}_N}^{<k_F} (\boldsymbol{k}_N\boldsymbol{k}_\Lambda = 0|K_{N\Lambda}|\boldsymbol{k}_N\boldsymbol{k}_\Lambda = 0), \tag{5.1}$$

with the reaction matrix equation (for $k_N < k_F$):

$$K_{N\Lambda}|\boldsymbol{k}_N 0) = v_{N\Lambda}|\boldsymbol{k}_N 0) + \sum_{\boldsymbol{p}_N}^{>k_F} \sum_{\boldsymbol{p}_\Lambda} \frac{v_{N\Lambda}|\boldsymbol{p}_N\boldsymbol{p}_\Lambda)\,(\boldsymbol{p}_N\boldsymbol{p}_\Lambda|K_{N\Lambda}|\boldsymbol{k}_N 0)}{e_N(k_N) + V_\Lambda - \varepsilon_N(p_N) - \varepsilon_\Lambda(p_\Lambda)}, \tag{5.2}$$

where ε_Y is the kinetic energy of Y, and e_N the nucleon s.p. energy ($e_N = \varepsilon_N + V_N$, where V_N is the nucleon s.p. potential). To simplify our formulae, we assume here that $v_{N\Lambda}$ is central, of the pure Wigner type.

Calculation of B_Λ, according to Eqs (5.1) and (5.2), have been performed in DH and [26] with practically identical results. Here, we shall rely on the calculation of DH which we are going to modify before comparing the resulting values of B_Λ with the Jastrow method results. (The calculations in DH were done at $k_F = 1.35\,\mathrm{fm}^{-1}$, and the Jastrow calculations reported in the previous sections were done at $k_F = 1.366\,\mathrm{fm}^{-1}$. To correct this difference we multiply all the DH results by $(1.366/1.35)^3$ before discussing them here.)

The self-consistent standard LOB value of B_Λ depends only indirectly on the NN interaction, v_{NN}, through the dispersive effect, i.e., the appearance of the nucleon s.p. energy $e_N(k_N)$ in Eq. (5.2). It was found in DH that B_Λ is not very sensitive to changes in the shape of $e_N(k_N)$, as long as the average value of $e_N(k_N)$ in the Fermi sea, $\bar{e}_N$, remains unchanged. Consequently, we may replace $e_N(k_N)$ in Eq. (5.2) by $\bar{e}_N$ (in DH, this approximation was called the choice (*iii*) of the s.p. nucleon potential). In the LOB theory of NM, the expression for the energy per nucleon in NM, $-\varepsilon_{vol}$, is

$$-\varepsilon_{vol} = \bar{\varepsilon}_N + \bar{V}_N/2 = (\bar{e}_N + \bar{\varepsilon}_N)/2, \tag{5.3}$$

i.e., we have for $\bar{e}_N$:

$$\bar{e}_N = -[\tfrac{3}{5}\,\varepsilon_N(k_F) + 2\varepsilon_{vol}]. \tag{5.4}$$

In the LOB calculation of B_Λ at a given value of k_F, it is sufficient to assume a given value of ε_{vol}. There is no need to specify v_{NN}. The tacit assumption concerning v_{NN} is here that in the LOB theory of NM it leads to the assumed value of ε_{vol}.

TABLE V

LOB values of B_Λ (in (MeV) and of $\kappa_{N\Lambda}$ at $k_F = 1.336\,\mathrm{fm}^{-1}$

$v_{N\Lambda}$	B_Λ(DH)	$\kappa_{N\Lambda}$	B_Λ(OMY)	Uncertainty in B_Λ(OMY)
HNX	56.7	0.23	59.5	46–73
ENX	61.1	0.12	62.8	49–77
E'NX	55.7	0.12	57.3	45–70
DW	36.5	0.09	37.7	29–46

In DH, the assumed value of ε_{vol} was the empirical value of ε_{vol}(EMP) = 15.8 MeV. (The resulting self-consistent values of B_Λ, denoted as B_Λ(DH), are shown in Table V.) On the other hand, in the Jastrow calculations of B_Λ, discussed in the previous sections, the OMY potential v_{NN} was used. This potential, when used in the LOB theory of NM, leads to ε_{vol}(OMY) = 8.2 MeV at $k_F = 1.366\,\mathrm{fm}^{-1}$ [27]. Hence, to make a meaningful comparison between the LOB and Jastrow results for B_Λ, we should use the value of $\varepsilon_{vol} = 8.2$ MeV as the input in the LOB calculations, instead of the value of 15.8 MeV used in DH. The resulting values of B_Λ, denoted as B_Λ(OMY), are shown in Table V.

The values of B_Λ(OMY) have been obtained in the following way. To get the self consistent value of B_Λ, one solves the K matrix equation, Eq. (5.2), for a few values of $V_\Lambda = -\Delta_\Lambda$ (Δ_Λ is the gap in the s.p. Λ spectrum). For each Δ_Λ, one obtains with the help of Eq. (5.2) a value of $B_\Lambda = f(\Delta_\Lambda)$. Self-consistency is achieved when $B_\Lambda = f(B_\Lambda)$. In DH, the curve $B_\Lambda = f(B_\Lambda)$ was approximated by a straight line,

$$B_\Lambda = \kappa_{N\Lambda}[\Delta_\Lambda - B_\Lambda(\mathrm{DH})] + B_\Lambda(\mathrm{DH}), \tag{5.5}$$

with the slope

$$\kappa_{N\Lambda} = \partial B_\Lambda / \partial \Delta_\Lambda. \tag{5.6}$$

The values of the derivatives $\partial B_\Lambda / \partial \Delta_\Lambda$ obtained in DH are shown in Table V. It may be easily shown (see, e.g., [28] for similar considerations in pure NM) that $\kappa_{N\Lambda}$ in Eq. (5.6) is equal to the NΛ wound integral.

Now, let us estimate the inaccuracy of the LOB method of calculating B_Λ. The LOB method is the first step in the hole-line expansion. In pure NM, simple considerations suggest (see [25], [29]) that by introducing into a diagram an additional independent hole line, we change its contribution to E_{NM} by a factor of order κ_{NN} (the NN wound integral) which plays the role of the smallness parameter in the hole-line expansion method. The same considerations applied to diagrams which contribute to B_Λ suggest that by introducing an additional nucleon hole-line, we change its contribution to B_Λ by a factor of the order of κ_{NN} or $\kappa_{N\Lambda}$, depending on the location of the additional hole-line. Consequently, we expect that the order of magnitude of $|B'_\Lambda / B_\Lambda(\mathrm{LOB})|$ is determined by κ_{NN} and $\kappa_{N\Lambda}$. By B'_Λ we denote the first correction to B_Λ(LOB) in the hole-line expansion, i.e., the contribution to B_Λ from the interaction of the Λ particle and two nucleons (diagrams with the Λ particle line and two independent nucleon hole-lines).

For the OMY potential, we have $\kappa_{NN} = 0.22$ at $k_F = 1.366$ fm^{-1} [27]. This value of κ_{NN} is slightly smaller than $\kappa_{N\Lambda}$(HNX), and is bigger than the $\kappa_{N\Lambda}$ values for the other NΛ potentials, shown in Table V. To be on the safe side, we use the estimate $B'_\Lambda \sim \pm \kappa B_\Lambda$(LOB), where κ is the bigger one of the two wound integrals, κ_{NN} and $\kappa_{N\Lambda}$. The resulting estimated upper and lower limits of $B_\Lambda(\mathrm{LOB}) + B'_\Lambda$ are shown in the last column of Table V.

6. Discussion

The striking feature of the results described in the last two sections is, that the Jastrow results for B_Λ are much bigger than Brueckner results. The biggest discrepancy occurs for the ΛN potential HNX with the biggest hard core radius, and with the biggest strength parameters s. We do not know how to explain this discrepancy, and restrict ourselves to the following comments.

To remove a similar discrepancy in the case of pure NM, it was essential to go in the reaction matrix method beyond the LOB approximation, and consider at least the contribution to E_{NM} of the three-hole-line diagrams (Day [25]). The corresponding first correction to B_Λ(LOB) is the sum of the contributions of all B_Λ diagrams with two nucleon-

-hole lines, denoted by B'_Λ in Section 5. Our estimate of the order of magnitude of B'_Λ leads us to the expected ranges of $B_\Lambda(\mathrm{LOB})+B'_\Lambda$, shown in Table V, whose upper limits agree with the Jastrow results of Table IV (notice that the Jastrow values of B_Λ do not have any strict lower bound character).

The problem, however, is that the existing estimates of B'_Λ suggest that B'_Λ is negative, and a negative B'_Λ would increase the discrepancy between B_Λ (Jastrow) and B_Λ (Brueckner). B'_Λ consists of two parts: the rearrangement energy $B_{\Lambda R}$, and the three-body cluster energy $B_{\Lambda 3}$. For the rearrangement energy, we have the approximate expression derived in [30]

$$B_{\Lambda R} = -\kappa_{NN} B_\Lambda(\mathrm{LOB}), \tag{6.1}$$

which clearly shows that $B_{\Lambda R}$ is negative. To calculate the three-body cluster energy, $B_{\Lambda 3}$, we have to solve the ΛNN Bethe-Faddeev equation, what is a very serious numerical problem. So far, $B_{\Lambda 3}$ has been calculated only in case of simplified separable Puff type [31] potentials $v_{N\Lambda}$ and v_{NN} [32]. The result for $B_{\Lambda 3}$ of [32] is negative, and is supported by an earlier simple estimate [33] based on the method applied by Moszkowski [34] in pure NM. However, in view of the numerical difficulties in determining $B_{\Lambda 3}$, it certainly would be desirable to have more results for $B_{\Lambda 3}$. It should be reminded that also in the case of pure NM, the early estimates of E_{MN3}, the three-hole-line contributions to E_{NM}. could not remove the discrepancy between the Brueckner and Jastrow method results for E_{NM}. The discrepancy disappeared only after the recent extensive calculations by Day (see [25]) produced values of E_{NM3} significantly different from the earlier results.

Obviously, the LOB method of calculating B_Λ is burdened with the problem of the proper choice of the s.p. energies in the K matrix equation. To get an idea about the values of $B_\Lambda = \tilde{B}_\Lambda$, which one would obtain without any selfenergy insertions into hole lines ($e_N = \varepsilon_N$, $V_\Lambda = 0$ in Eq. (5.2)), we have applied the linear extrapolation, Eq. (5.5), and obtained: $\tilde{B}_\Lambda(\mathrm{HNX}) \simeq 88$ MeV, $\tilde{B}_\Lambda(\mathrm{ENX}) \simeq 78$ MeV, $\tilde{B}_\Lambda(E'\mathrm{NX}) \simeq 72$ MeV, and $\tilde{B}_\Lambda(\mathrm{DW}) \simeq 47$ MeV.

One may show that $\tilde{B}_\Lambda$ is equal to the lowest order Jastrow value of B^0_Λ if the NΛ correlation function coincides with the corresponding part of the Bethe-Goldstone NΛ wave function. In pure NM, an analogical theorem concerning E_{NM} was shown by Wong [35]. A look at Table IV shows that the B^0_Λ values of [4] and [8] approximately agree with the above values of $\tilde{B}_\Lambda$. Now, the NΛ correlation functions of [4] and [8] have an overshoot and thus are similar to the Bethe-Goldstone wave function. This is not the case with the optimal $f_{N\Lambda}$ of our calculation and here we do not find such an agreement between B^0_Λ and $\tilde{B}_\Lambda$.

The important problem of determining the optimal correlation functions, in particular the function $f_{N\Lambda}$, has been solved in the present paper in a simplified way. The assumed parametric form of $f_{N\Lambda}$ and f_{NN} might be not sufficiently flexible, and it might be a source of inaccuracy in our resulting FHNC values of B_Λ. In principle, one should determine $f_{N\Lambda}$ by functional maximalization of $B^{\mathrm{FHNC}}_\Lambda$, similarly as it has been done in the pure Fermi system by Lantto and Siemens [36].

The error of our FHNC result caused by neglecting elementary diagrams depends on the shape of the correlation functions. With our correlation functions $f_{N\Lambda}$ and f_{NN}, this

error is not expected to be significant at the density considered ($k_F = 1.366$ fm^{-1}). Namely, in the case of pure NM at this density, elementary diagrams appear to give relatively small contributions for the type of the f_{NN} function used in the present paper [20]. Actually, as we noticed in Section 4, all the higher order cluster terms, beyond those considered in the LOC approximation, are very small for our optimal correlation functions. Similarly, a possible state dependence of the correlation functions is not expected to introduce essential changes into our FHNC results for B_Λ, for our simple two-body interactions. According to estimates by the Pisa group [37], the state dependence of the correlation functions in the case of pure NM should change the energy per nucleon by not more than 2 MeV. But obviously, conclusions based on the analogy[1] with pure NM should be tested in the B_Λ problem.

It appears that the best way of resolving the problem of the discrepancy between the Jastrow and Brueckner method of calculating B_Λ would be to consider a model case of simple central ΛN and NN hard core potentials of the pure Wigner type. In this model case one should calculate B_Λ with both methods. In particular in the Brueckner method, one should calculate accurately the three-hole-line contributions. And in the Jastrow method one should determine carefully the optimal correlations. Furthermore, one should also calculate the radial distribution functions with the Brueckner method, and compare them with the Jastrow distribution functions.

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