PHENOMENOLOGICAL APPROXIMATION OF NUCLEON SPECTRA IN INELASTIC N-N INTERACTIONS AT HIGH ENERGIES

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The phenomenological expressions for differential single-particle inclusive cross sections of proton and nucleon production in inelastic p-p and p-n collisions describing the experimental data known at all values of kinematic variables at energies from 5 to several thousand GeV are obtained. Inelasticity coefficients, average multiplicities, kinetic energies and transversal momenta of protons and neutrons are calculated and compared with the experimental data.

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In analysis of hadron-nuclei interaction mechanisms, when we deal with radiation shielding calculations, in estimation of secondary particles beam intensity and in some other problems, it is necessary to have analytical expressions for differential cross sections of proton and neutron emission in inelastic p-p and p-n interactions, suitable in a wide range of primary particle energies $T \gg 1$ GeV[1].

Such expressions of rather simple type with experimentally sampled coefficients can be obtained in a single-particle inclusive approach using the scaling hypothesis (see Refs. [1–7] for further bibliography). In some cases these expressions may be of sufficiently

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[1] Here and in what follows T is kinetic energy of the projectile particle in the laboratory coordinate system. We use standard notation: S is quadric total energy of colliding particles in C.M.S., $P_{||}$ and $P_{\perp}$ are longitudinal and transversal components of secondary particle momentum in C.M.S., $E = T+m = (p_{||}^2+p_{\perp}^2+m^2)^{1/2}$ is the corresponding energy of the particle, $x = 2p_{||}S^{-1/2}$, $x_{\perp} = 2p_{\perp}S^{-1/2}$.

high accuracy. Unfortunately, such expressions can be used usually only in very narrow areas of kinematic variables. To give a more complete description of experiment one has to use quite a set of approximations, badly coordinated with each other and with a large number of fitting coefficients. In practice it is much more convenient to use phenomenological expressions, which, besides the "scaling" part, contain additional terms chosen with the condition of best agreement with experiment and defined more exactly as the new information is accumulated. Such expressions succeed to cover a wide range of $p_\perp$, x, S values [8][2].

To make matters worse, the data known up to now are not able to define with equal accuracy the numerical coefficients for all types of reactions with differing charges of primary and secondary particles. So far the neutron spectra at p-p collisions are not yet properly investigated; there is lack of data for p-n interactions, especially, for secondary neutrons. In such cases we have to accomodate the expressions obtained for the particles of the other charge sign, inserting, therefore, some errors in absolute cross sections of the particle yield and sometimes essentially changing the character of differential distribution $d^3\sigma(x, p_\perp, S)/d^3p$, especially for large x and $p_\perp$.

Nevertheless, at present, due to lack of experimental data, it is the only way to describe such cases. Especially, as for the considered problems the accuracy of phenomenological expressions $d^3\sigma/d^3p$ is quite sufficient, since for the leading particles where the large momenta are important the distribution is approximated with sufficient accuracy; as for the other particles they rarely have large x and $p_\perp$. In addition, if particle collisions are calculated by the Monte-Carlo method, the errors in the absolute values of cross sections $d^3\sigma/d^3p$ are also unessential since the multiplicity, energies and emission angles of secondary particles can be defined by relative (for example normalized to unity) differential distributions (see Ref. [9]).

It must be kept in mind that due to unsufficient accuracy of experimental cross sections $d^3\sigma/d^3p$, which are the basis for selecting coefficients in approximating formulae, it is usual that the expression good for the differential distribution often has poor agreement with the average energy, transversal momentum and other calculated integral characteristics with the experimental data. At the same time, accuracy of the approximating expressions essentially increases if these expressions are determined provided a good agreement of calculated integral characteristics with measurement results.

Approximations presented here are obtained with regard to this requirement.

We consider the region of energies from $T \simeq 5$ GeV to $T \simeq$ several thousand GeV. To determine the secondary nucleons characteristics at lower energies one can use the well elaborated Monte-Carlo approaches and approximating expressions based on resonance models (see Ref. [9] for further bibliography). At energies exceeding several thousand GeV, where only separate and inaccurate cosmic data are available, our expressions with experimentally defined coefficients become rather rough. However, due to lack of data even such approximations are of great interest now.

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[2] In general, the type of these terms follows from the investigations of analytical properties of the interactions amplitudes but the situation is rather indefinite.

The differential distribution of "conserved" particles in N-N collisions is[3]

$$E\frac{d^3\sigma(x, p_\perp, S)}{d^3p}\,[\mathrm{mb}\cdot\mathrm{GeV}^{-2}\cdot c^3]$$

$$=\begin{cases} a(1+bxS^{c-x_\perp S^{0.27}})\dfrac{(1-x)^{fp_\perp{}^2}}{(p_\perp^2+\mu)^d}\exp(gx)+uS^v(1-x)\exp(-5p_\perp^2) \\ 0\leqslant |x| < 0.7, \\ \dfrac{A|t|(1-x)^{1-\alpha t}}{(0.0195-t)^2}\exp(Rt)+\displaystyle\sum_{i=1}^{4} G_iS^{-\alpha_i}(1-x)^{\beta_i-\gamma_i t}\exp(R_it) \\ 0.7\leqslant |x| < 1, \end{cases} \tag{1}$$

$$t = -p_\perp^2/x-(1-x)(M_c^2/x-M_a^2). \tag{2}$$

Here M_a and M_c are the particle masses in inclusive reaction a+b → c+ ... (in the present case $M_a = M_c$ is a mass of a nucleon). The values of parameters are shown in Tables I and II. In all expressions the absolute value of x must be used. In the case of p-p and n-n interactions particle distributions in the region of positive and negative x are symmetrical with respect to the point $x = 0$. Such a symmetry is absent in p-n interactions where the conserved particles are assumed to be proton at $x > 0$ and neutron at $x < 0$ (in n-p interactions the x sign is inversed). All energy variables are in GeV units.

The number of coefficients in the expression (1) could be reduced by considering several smaller intervals $\Delta S = (S_1, S_2)$; however, in this case for the whole energy region the number of constants is not reduced but increased.

TABLE I

Coefficients for conserved and unconserved nucleon spectra at $|x| < 0.7$

Reaction \ Coef.	a	b	c	d	f	g	μ	u	v
pp → p+ ...	100	9.4	−0.06	6.43	0.21	1.5	1.81	230	−1
$x < 0$	6	3.6	−0.01	4.9	1.0	3.0	1.36	230	−1
pn → p+ ...									
$x > 0$	95	8.5	−0.06	6.43	0.45	1.4	1.84	210	−1
pp → n+ ...	13	0.35	0.17	5.5	1.2	−0.4	1.25	210	−1

[3] We call the particle a "conserved" in reaction a+b → a+ ... if after the interaction this particle is emitted into the same hemisphere it came from. In this case the distribution of the conserved particle b in reaction a+b → b+ ... is given by formulae (1), (2) by the change $|x| \to |-x|$ (i.e. azimuthal emission angles of the particles a and b are related by $\theta_b = \pi-\theta_a$). In p-p and n-n collisions particles a and b are completely interchangable.

If the conserved particle is emitted at a small ($\theta \sim 0$, a) or at a big ($\theta \sim \pi$, b) angle, it usually takes the large part of energy of colliding particle. This particle, a or b, is called the "leading particle".

TABLE II

Coefficients for conserved and unconserved nucleon spectra at $|x| \geqslant 0.7$

Identical for all reactions				pp → p+ ...		pn → p+ ...		pp → n+ ...	
		A		19		23 (23)[1]		3.8	
		R		4.3		5.6 (0.08)		0.23	
		a		0.6		1.2 (1.3)		1.62	
i	α_i	β_i	γ_i	G_i	R_i	G_i	R_i	G_i	R_i
1	0	0	1.5	3.3	−0.38	5.7 (3.8)	0.07 (4.8)		
2	0	−0.5	0.85	9	6.5	1	10	0.007	0.45
3	0.5	−0.5	1.5	57	3.6	24	1.4	5.1	4.1
4	0	−1	0.2	0.2	1.5	0.07	1.3		

[1] For the region $x < 0$.

The coefficients listed in tables are obtained taking into account all known experimental values of $d^3\sigma/d^3p$ and integral quantities: the average multiplicity of secondary particles, their transverse momenta, and inelasticity coefficient of interactions. Figs. 1–3 illustrate the typical agreement between the calculated and experimental spectra.

As we can see, agreement is very good for $x > 0.5$ at all energies $T > 5$ GeV (as a rule, the theoretical curves are placed inside the corridor of experimental errors); the agreement

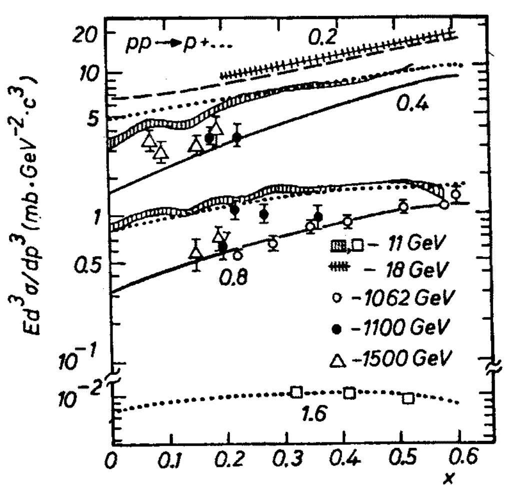

Fig. 1. Differential distribution of secondary protons in inelastic p-p collisions in the region $x \leqslant 0.6$. Dotted curves are results of calculations for $T = 11$ GeV, dashed — for $T = 18$ GeV and solid — for $T = 1062$ GeV. The corresponding values of $p_\perp$ (GeV/c) are indicated near the curves. Experimental points are taken from Ref. [10]

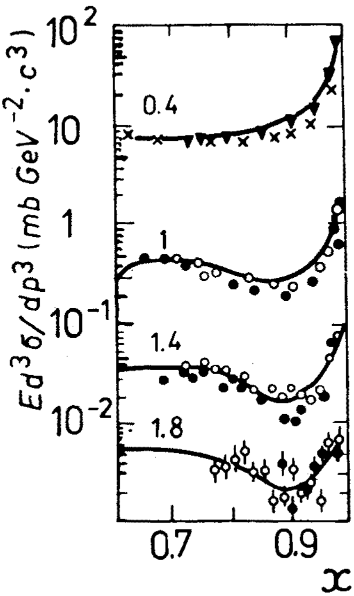

Fig. 2. Differential distribution of secondary protons in inelastic p-p collisions in the region $x \geqslant 0.6$. All curves are calculated for $T = 1062$ GeV. The corresponding values of $p_\perp$ (GeV/c) are indicated near the curves. The experimental points for $T = 385$, 659, 1062 and 1482 GeV are shown by the marks ×, ▼, ●, ○, respectively

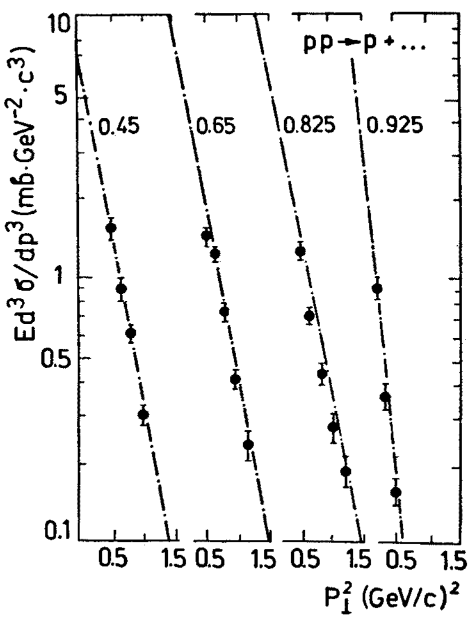

Fig. 3. Transverse momentum distribution of protons in inelastic p-p collisions at $T = 1062$ GeV. Curves are the results of calculations and points are experimental data [13]. The corresponding values of x are nearby

is somewhat worse for smaller x. Few and rather scattered experimental points are known yet in the region $x \lesssim 0.5$, therefore, one can define the coefficients in the expression (1) for this region by means of known integral quantities only. It seems difficult to define a more accurate approximation here.

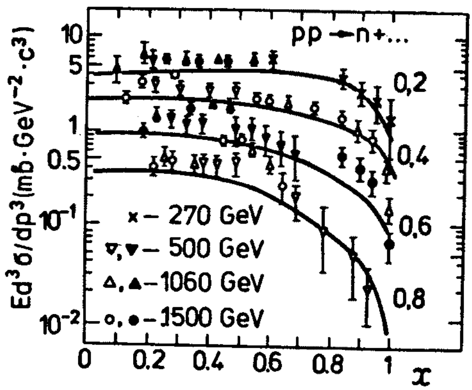

Fig. 4. Differential distribution of neutrons in p-p interactions. Curves are the results of calculations for $T = 1062$ GeV. The corresponding values of $p_\perp$ are nearby. The experimental points are taken from Ref. [14]

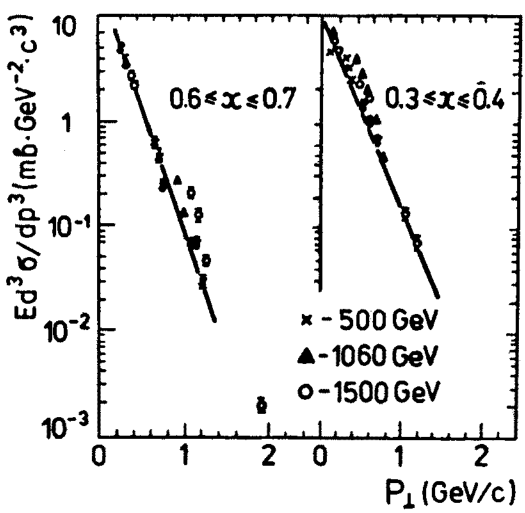

Fig. 5. Transverse momentum distribution of neutrons in p-p interactions. Curves are calculated for $T = 1062$ GeV. The intervals of x corresponding to the experimental points are shown near the curves [14]. (Calculations have been done for average values of these intervals.)

Expressions (1) and (2) can also be used for description of unconserved nucleon spectra in the reactions with overcharge of colliding particles: neutrons in p-p interactions, protons at $x < 0$ and neutrons at $x > 0$ in p-n interactions (and corresponding particles in charge-symmetrical n-n and n-p interactions). Expressions differ from those for the conserved particles only in values of coefficients (see Tables I and II).

In Figs. 4 and 5, where characteristic examples of the accuracy of the considered approximation are shown again, one can see that in the case of p-p interaction the calculated cross sections describe the experiment with the accuracy which does not go out of the limits of average experimental errors. The case of p-n collisions is more difficult due to lack

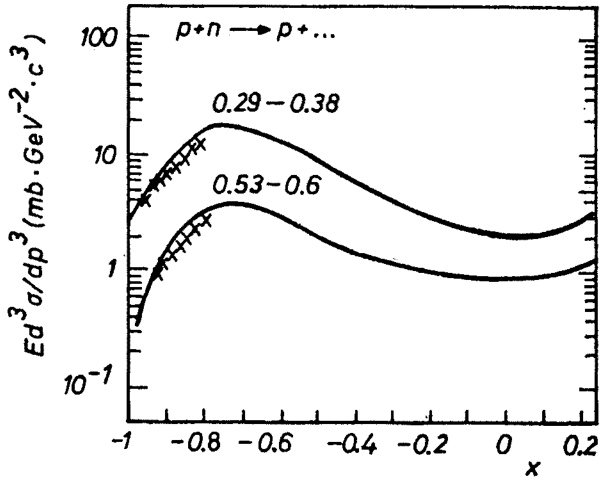

Fig. 6. Protons distribution in inelastic p-n interaction at $T = 56.7$ GeV. Curves are calculated for the indicated values of $p_\perp$(GeV/c). Experimental points are taken from Ref. [15]

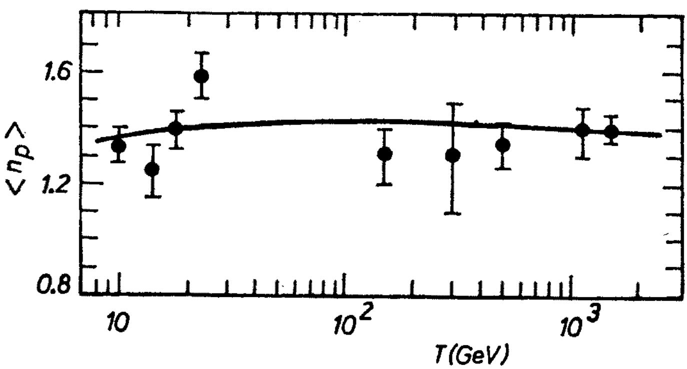

Fig. 7. Average number of secondary protons in inelastic p-p collision. The curve is calculated by the formula (3), the points correspond to experimental data [16–26]

of experimental data. Tables I and II present values of coefficients giving the best agreement with known experiments; however, it is likely that the results of more detailed measurements will require the refinement of these coefficients.

In the region $T \gtrsim 5$ GeV the calculated multiplicities of protons and neutrons $\langle n_p \rangle \simeq 1.4$, $\langle n_n \rangle \simeq 0.6$, where

$$\langle n_x \rangle = \sigma_{in}^{-1} \int d^3\sigma_x = \sigma_{in}^{-1} \int \left(E \frac{d^3\sigma_x}{d^3p} \right) \frac{d^3p}{E}, \tag{3}$$

σ_{in} is the experimental value of the total two nucleon inelastic cross section. The results of calculations are in good agreement with experimental data.

From statistical considerations based on isotopic invariance it is often confirmed that at large multiplicities of secondary particles $\langle n_p \rangle \simeq \langle n_n \rangle$, regardless of the charges of colliding nucleons. The presented data show that in p-p collisions the average multiplicities of protons and neutrons remain different even at very high energies. We can talk about the equality of $\langle n_p \rangle$ and $\langle n_n \rangle$ in the case of p-n interactions only.

TABLE III

Integral characteristics of secondary protons in inelastic p-p interactions (C.M.S.)

T, [GeV]	$\langle p_{\perp p} \rangle$, [GeV/$c$]	$\langle T_p \rangle$, [GeV]	$\langle K_p \rangle$
10	0.38	0.62	0.34
20	0.39	0.99	0.31
10^2	0.41	3.17	0.30
$5 \cdot 10^2$	0.42	7.02	0.29
10^3	0.44	9.6	0.29
$1.5 \cdot 10^3$	0.44	11.4	0.29
$5 \cdot 10^3$	0.44	21.9	0.29

The average transversal momenta of protons in p-p collisions are shown in Table III, where

$$\langle p_\perp(S) \rangle = \sigma[p_\perp; S]/\sigma[1; S] \tag{4}$$

with functional

$$\sigma[Z; S] = \pi \int_{-1}^{+1} dx \int_0^{p^2_{\perp max}} E \frac{d^3\sigma(x, p_\perp, S)}{d^3p} \frac{Z(x, p_\perp, S) dp_\perp^2}{(x^2 + 4S^{-1}(p_\perp^2 + M_c^2))^{1/2}}, \tag{5}$$

here $p_{\perp max}$ is the maximum value of proton transversal momentum. The calculated quantities are close to experimental ones and are about 0.4 GeV at $T = 5 \div 30$ GeV and increase to 0.44 at $T \sim 10^3$ GeV.

Compared with those of protons, the average transversal momentum of neutrons emitted in p-p collisions are some percents more, though it is hardly provable within the accuracy of the approximations used.

Table III shows the values of the proton average kinetic energy in C.M.S.

$$\langle T_p(S) \rangle = \{\sigma[E; S]/\sigma[1; S]\} - M_p, \tag{6}$$

and the average share of energy taken by proton

$$\langle K_p(S) \rangle = \{\langle T_p(S) \rangle + M_p\} S^{-1/2}. \tag{7}$$

At $T \simeq 10$ GeV the quantity $\langle T_p \rangle$ is close to experimental value 0.6 GeV [27, 28] and then increases more rapidly than S (i.e. $\gtrsim T^{1/2}$). Values of $\langle K_p \rangle$ remain practically constant in the whole energy region $T \gtrsim 20$ GeV. The average kinetic energy of a neutron created in p-p collision $\langle T_n \rangle$ is $1.5 \div 2$ times less than that of proton. As the primary particle energy increases, the value $\langle T_n \rangle$ rises something slower than $\langle T_p \rangle$; so the ratio $\langle T_p \rangle / \langle T_n \rangle = 1.4$ at $T = 10$ GeV and is 2 and 2.5 at $T = 10^2$ and 10^3 GeV.

Taking into account the relative probability of protons and neutrons creation we obtain that the magnitude of the average energy taken by proton in inelastic p-p interaction exceeds the average neutron energy approximately four times. The inelasticity coefficient of p-p interaction, i.e. the part of energy taken by all created particles

$$\begin{aligned}\langle K \rangle &= 1 - \langle n_p \rangle \{\langle T_p \rangle + M_p\} S^{-1/2} - \langle n_n \rangle \{\langle T_n \rangle + M_n\} S^{-1/2} \\ &= 1 - \langle n_p \rangle \{\langle T_p \rangle - \langle T_n \rangle\} S^{-1/2} - 2\{\langle T_n \rangle + M_n\} S^{-1/2} \end{aligned} \tag{8}$$

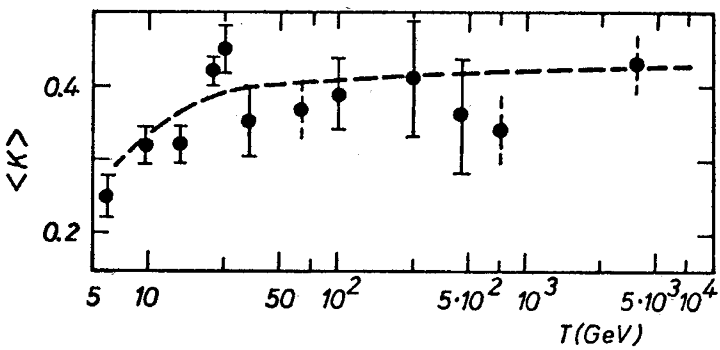

Fig. 8. Energy dependence of inelasticity coefficients for primary proton in proton-nucleon collisions. Dashed line is the result of calculations. Experimental points are taken from Refs. [29, 30] and compilations [31–32].

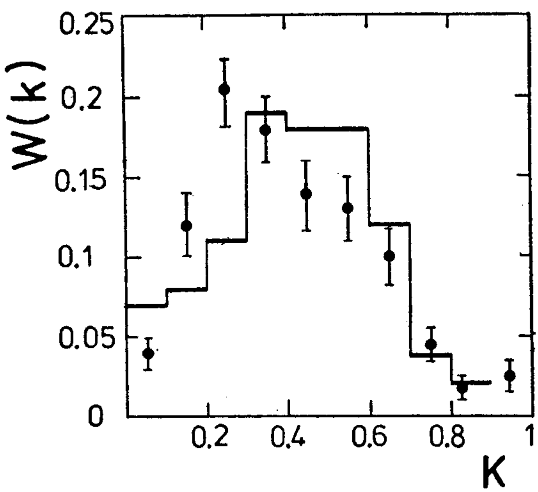

Fig. 9. Distribution of inelasticity coefficient for p-p collisions. The histogram is the result of calculation for $T = 10^2 - 10^3$ GeV. The points correspond to experimental data [33–35] for p-N collisions obtained at $T = 20 - 5 \cdot 10^4$ GeV

is about 42% and is in good agreement with experimental data (see Fig. 8; for p-n interactions a close value is obtained)[4].

The calculated and experimental distributions of inelasticity coefficients $W(K)$ for the energy interval $T \simeq 10^2 \div 10^3$ GeV are shown in Fig. 9[5]. These distributions are close to one another, though it should be remembered that experimental points are obtained at different energies T, containing therefore significant errors, so one can expect that theoretical distribution $W(K)$ is more accurate.

The distribution $W(K)$ and average inelasticity coefficient $\langle K \rangle$ are independent of the system of reference (C.M.S. or lab frame) due to the symmetry of N-N system; at

TABLE IV

The share of energy taken by the leading proton and the recoil proton in p-p interaction in the laboratory coordinate system

T, [GeV]	$\langle K_{lead} \rangle$	$\langle K_{rec} \rangle$
10	0.53	0.15
10^2	0.55	0.05
10^3	0.56	0.02

the same time nucleon coefficients $\langle K_{N_1} \rangle$ and $\langle K_{N_2} \rangle$ which are equal in C.M.S. are essentially different in the laboratory coordinate system (see Table IV). The part of energy taken by the leading nucleon is actually constant at all energies $T \gtrsim 5$ GeV.

In conclusion we can say that within the accuracy of experimental data available at present the inclusive expression (1) describes sufficiently the differential and integral characteristics of secondary nucleons in inelastic N-N collisions and can be used for different theoretical estimations at $T > 5$ GeV, including the calculation of quantities which are not measured in experiments yet.

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[4] As it is usual in experiment all secondary particles except two nucleons are considered to be new. Including the neutron into the number of new particles we have the corresponding inelasticity coefficient $\langle K \rangle = 1 - \langle n_p \rangle \cdot \langle K_p \rangle \simeq 0.55$ at $T = 10$ GeV and $\langle K \rangle \simeq 0.6$ at $T \gtrsim 500$ GeV.

[5] The distribution of the part of energy taken by nucleon is obtained by simple transformation

$$\int \frac{d^3\sigma(x, p_\perp, S)}{dp_\perp^2 dx} dp_\perp^2 dx = \int \frac{d^3\sigma(K_N, p_\perp, S)}{dp_\perp^2 dK_N} dp_\perp^2 dK_N = \int W(K_N, S) dK_N.$$

The part of energy consumed for the production of secondary mesons depends on the energy of both nucleons (see formula (8)). Calculation of this energy distribution and distributions $W(K_{N1})$ and $W(K_{N2})$ in the laboratory system may be done by the Monte-Carlo method (by means of von Neumann rejection procedure), sampling values of x and $p_\perp$ corresponding to formula (1) and calculating the corresponding nucleon energies in lab system and in C.M.S. This procedure turns out to be quite effective.

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