TRANSPARENCY IN THERMODYNAMIC MODELS OF RELATIVISTIC ION COLLISIONS

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We show how to incorporate systematically a transparency into thermodynamic models of relativistic ion collisions. The momentum loss of a nucleon per penetration length is evaluated from the free N-N scattering data. Inclusion of transparency in the firestreak model improves agreement with the inclusive data of Gazzaly et al. from 1.8 GeV/nucl lab energy collisions.

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Models of relativistic ion collisions based on geometry and thermodynamics, like the fireball [1], firestreak [2, 3] and two-fireball [1, 4] models, are widely used in describing the data of particle production. Usually it is assumed that the total available energy of the colliding objects undergoes thermalization. In calculations of Ref. [4] a partial thermalization of the colliding ions, leading to two fireballs, was assumed, in order to reproduce the experimentally observed anisotropy of the proton distribution [5]. In Ref. [3] the discrepancy between the firestreak model predictions and the experimental data of Ref. [6] was attributed to some transparency of the colliding nuclei.

The partial thermalization is also suggested by the following study of the momentum loss in the colliding ions. First, we evaluate the momentum loss of the colliding nucleons which can scatter either elastically or produce the Δ resonance. The average center of mass nucleon z-direction momentum loss per collision Δp_z is given by

$$\sigma \Delta p_z = 2\pi p \int\limits_0^1 \frac{d\sigma_{NN\to NN}}{d\Omega}(1-\cos\theta)d\theta + (p-p')\sigma_{NN\to N\Delta} + 4\pi p \int\limits_0^1 \frac{d\sigma_{NN\to N\Delta}}{d\Omega}(1-\cos\theta)d\theta, \quad (1)$$

where p and p' are the center of mass momenta in the NN and NΔ channels. The quantity $\sigma\Delta p_z$ multiplied by the normal nuclear density $\varrho_0 = 0.145\ \text{fm}^{-3}$ gives us the average mo-

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mentum loss of a nucleon per penetration length. This loss is plotted in Fig. 1a, while in Fig. 1b we give the nucleon-nucleon cross section for reference. From Fig. 1a we can infer that for the lab energy 800 MeV/nucl a nucleon would require about 5 fm of nuclear matter for the complete stopping in the nucleon-nucleon c.m. system. Thus for 800 MeV/nucl C on C (diameter 5.5 fm) or Ne on NaF (diameters around 6.5 fm) collisions we may

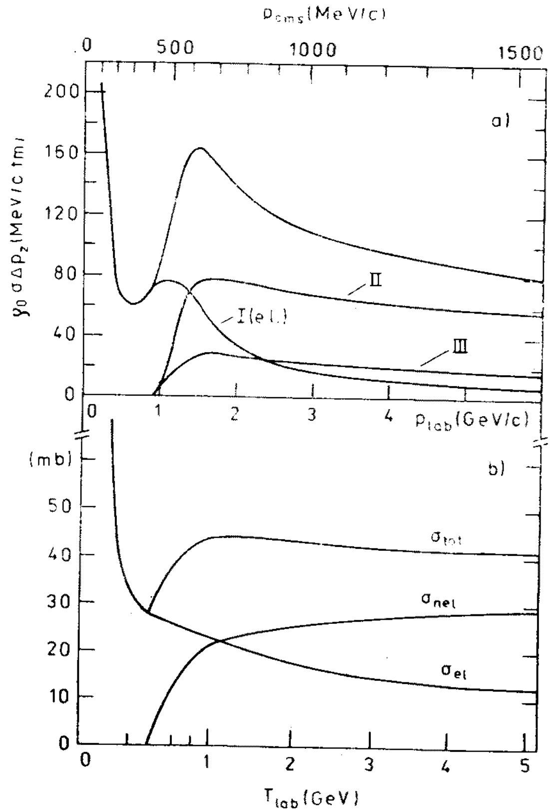

Fig. 1. a) Momentum loss of a nucleon per penetration length evaluated according to Eq. (1) with contributions from the first, second and third terms in this equation denoted by I, II and III, respectively. In calculation a Lorentzian Δ mass spectrum had been assumed. b) Spin-isospin averaged N-N cross section. Both figures had been evaluated on the basis of Refs. [8] and [9]

expect some partial transparency of the colliding nuclei. Similarly we may expect some transparency in the 1800 MeV/nucl Ar on Be and Ar on Cu collisions.

To incorporate the effect of the partial transparency we first consider the fireball model. The change in the momenta of projectile's and target's parts in the nucleon-nucleon c.m. system is, for a fixed impact parameter, evaluated as

$$\Delta P_z = \int dx \int dy \sigma \Delta p_z T_p T_t, \qquad (2)$$

where the thickness functions are defined as

$$T_{p,t} = \int dz \varrho_{p,t}(x, y, z), \qquad (3)$$

and $\varrho_{p,t}$ are the projectile and target density distributions, here taken to be sharp edged spheres. We fix the value of $\sigma \Delta p_z$ and treat it as a parameter. If ΔP_z, Eq. (2), exceeds the available momentum then we proceed like in one-fireball model. If not, then we assume that the two fireballs have equal temperatures, and this allows us to find the rest mass per baryon of fireballs M/B from the energy conservation:

$$B_p(p^2+m^2)^{1/2}+B_t(p^2+m^2)^{1/2}$$

$$= B_p\left[\left(p-\frac{\Delta P_z}{B_p}\right)^2+\left(\frac{M}{B}\right)^2\right]^{1/2}+B_t\left[\left(p-\frac{\Delta P_z}{B_t}\right)^2+\left(\frac{M}{B}\right)^2\right]^{1/2}, \tag{4}$$

where $B_{p,t}$ denote the baryon numbers in the overlapping parts of projectile and target, respectively.

Throughout the whole paper the nuclear thermodynamics of Ref. [3] is used, where besides the pions and deltas there are also included the light nuclei and the nuclear resonances. (We take a sharp Δ mass, and instead of $\eta_{\mathrm{min,max}}$ of Ref. [3], we use a cut-off for M/B equal to 955 MeV in the firestreak model, and to 939 MeV in the two other models. We also fix the value of the charge to the baryon number equal to the value of this ratio in the whole system).

The inclusive cross sections for protons and pions evaluated from the above, partially transparent, fireball model are shown in Figs. 2a and 2b for two values of $\varrho_0 \sigma \Delta p_z$ together with experimental data of Nagamiya et al. [5]. Values of $\varrho_0 \sigma \Delta p_z$ are chosen such as to demonstrate the sensitivity of the model to that parameter. A too high pion yield persists in all models studied in this paper. It should be noted that even when we have similar momentum losses as those in calculation of Ref. [4] our results are different due to the use of different thermodynamics. Part of the proton spectrum is depleted due to the composites production. The presence of composites increases the temperature and consequently the number of pions.

In the models considered in this paper there are two possible sources of anisotropy of particle yields — the partial transparency of colliding nuclei, and the independence of separate streaks or tubes of matter. In the fireball model only the first effect is present and when adjusting the momentum loss in this model to reproduce the observed anisotropy of proton spectra one finds a lower bound for $\varrho_0 \sigma \Delta p_z$. In the analysis of the 800 MeV/nucl C on C collision one can put a lower bound for this quantity at 90 MeV/c fm.

The proton yield is systematically underestimated in the fireball model. A higher yield is provided by the firestreak model, which incorporates diffuse edged density distributions. The effect of partial transparency is in this model included as follows. The streaks contain T_p and T_t, respectively, baryon number densities per perpendicular surface unit. The change in the momenta of the streaks is now

$$\sigma \Delta p_z T_p T_t,$$

and we proceed further in the same way as for the collision of the whole overlapping parts in the two-fireball model. The critical dimension of the streaks, above which the one-

-firestreak model applies, is

$$T_p + T_t = 2p/(\sigma \Delta p_z). \tag{5}$$

Another model in which we studied the effect of transparency is the firetube model. This model is an alternative to the firestreak model, and in principle it should describe better the interaction of nucleons in the tails of the nuclear distributions. With the increasing values of the thickness functions the results of this model tend to those of the firestreak model. As in Ref. [7] we consider the tubes of section equal to the N-N total cross section σ, in the target and projectile, respectively. The probability of finding n nucleons in such a tube is

$$P_p(n) = \binom{A_p}{n}\left(\frac{\sigma T_p}{A_p}\right)^n \left(1-\frac{\sigma T_p}{A_p}\right)^{A_p-n}. \tag{6}$$

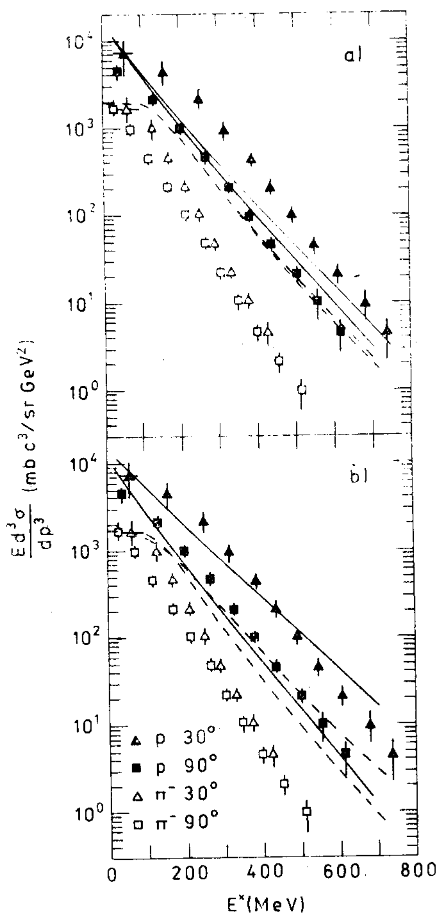

Fig. 2. Particle spectra at 30° and 90° from the two-fireball model compared with experimental results of Ref. [5] for the 800 MeV/nucl Ne on NaF collision. Solid lines — protons. Dashed lines — pions. Fig. 2a corresponds to $-\varrho_0\sigma\Delta p_z = 130$ MeV/c fm, while Fig. 2b to $\varrho_0\sigma\Delta p_z = 85$ MeV/c fm. All results are in the c.m. system

The probability that n nucleons from one tube collide with m nucleons from the opposite tube is evaluated as $P_p(n)P_t(m)$. If we consider the full thermalization then the yield function, cf. Ref. [2], is constructed as

$$Y(\eta) = \sum_{m,n\neq 0} \int 2\pi b db \int dx \int dy \frac{1}{\sigma} P_p(n)P_t(m)(n+m)\delta\left(\eta - \frac{n}{m+n}\right), \tag{7}$$

while for the case of a partial thermalization it is

$$Y\left(\frac{M}{B}, \beta\right) = \sum_{m,n\neq 0} \int 2\pi b db \int dx \int dy \frac{1}{\sigma} P_p(n)P_t(m)$$

$$\times [n\delta(\beta-\beta_p(n, m))+m\delta(\beta-\beta_t(n, m))]\delta\left(\frac{M}{B} - \frac{M}{B}(n, m)\right). \tag{8}$$

The total yield in the firetube model is necessarily lower than in the firestreak model because of the exclusion of $P_p(n = 0)$ and $P_t(m = 0)$. (It is finite independently of the applied cut--off's for the small excitation energy, in contrast to the firestreak model). The formulation of the partial transparency is analogous to the previous models, and a critical number of nucleons in the tubes above which the complete thermalization occurs is

$$n+m = 2p/\Delta p_z, \tag{9}$$

which for $E_{\text{lab}} = 800$ MeV/nucl and e.g. $\varrho_0\sigma\Delta p_z = 110$ MeV/c fm is equal to 6.

The firetube model seems to give just enough anisotropy in the proton spectra, and the firestreak model even to much, to cope with the data [5] from 800 MeV/nucl C on C, Ne on NaF, and Ar on KCl collisions. The agreement with the data does not therefore require extra anisotropy from the partial transparency. The values of the momentum loss which provide some improvement to the results of these models or do not affect the results are above 130 MeV/c fm. When compared with the measured p, d and t spectra from bombardment of Be and Cu by 1800 MeV/nucl ^{40}Ar [6] the firetube model cannot reproduce satisfactorily the 5° d and t spectra. However, a better agreement of this model with experiment is obtained for finite values of the momentum loss than with no transparency. Inclusion of transparency in the firestreak model improves the agreement with the measured data. Predictions for ^{40}Ar on Be collision of this model with $\varrho_0\sigma\Delta p_z = 110$ MeV/c fm are shown in Fig. 3. Value of $\varrho_0\sigma\Delta p_z$ can be varied in the range 100–130 MeV/c fm without losing the quality of the fit. For ^{40}Ar on Cu collision the best agreement with the data is obtained for $\varrho_0\sigma\Delta p_z$ in the range 75–100 MeV/c fm. It follows that $\varrho_0\sigma\Delta p_z$ should be probably made dependent upon the lengths of the streaks.

Summarizing, we have shown how to incorporate transparency into thermodynamic models. Inclusion of transparency in the firestreak model with momentum loss around 100 MeV/c fm improves the agreement with the data from 1800 MeV/nucl collisions (stopping length around 9 fm!). At 800 MeV/nucl lab energy higher values of the momentum loss are required in the firestreak model. For firm conclusions about transparency in the

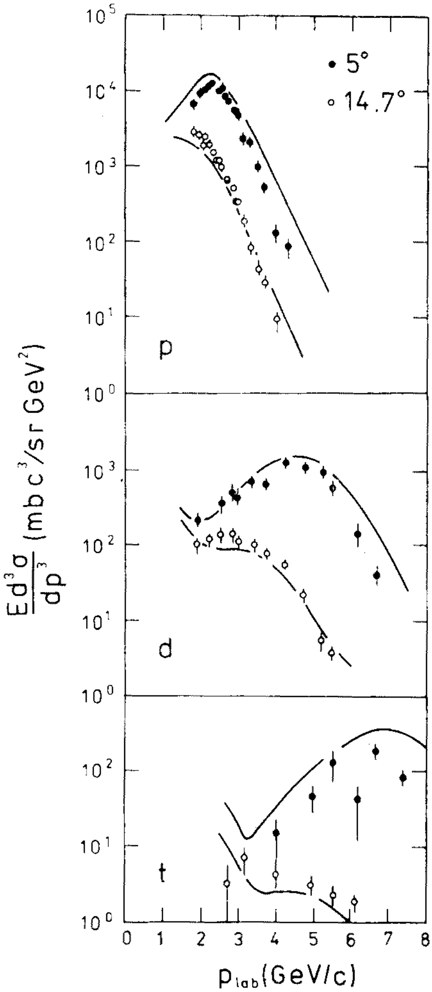

Fig. 3. Spectra of p, d and t at 5° and 14.7° from the firestreak model for the 1800 MeV/nucl ^{40}Ar on Be collision. Data is from Ref. [6]. Value of $\varrho_0\sigma\Delta p_z$ is 110 MeV/c fm. All results are in the lab system

relativistic ion collisions, one should, however, wait for analysis of further experiments. Fig. 1a and relations (5) and (9) may be helpful in estimating in which experiments the transparency may show up.

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