MASS AND ENERGY DEPENDENCE OF THE ALPHA-PARTICLE OPTICAL MODEL POTENTIAL IN THE ENERGY RANGE 90–172 MeV AND IN THE MASS NUMBER RANGE 12–208

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The energy and mass dependent parameters of the local optical model potential for the elastic alpha particle scattering in the energy region of 90–172 MeV on target nuclei from 12 to 208 amu have been proposed and calculated. The calculated cross-sections qualitatively well fit the experimental data in the considered energy and mass region. In order to improve the fits the structure corrections that depend on the number of nucleons outside of the last closed shell could be added to the optical model potential.

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1. Introduction

Several attempts have been made to find average optical model potential (OMP) parameter sets which describe alpha particle elastic scattering over a wide range of energies [1–7]. Such average parameter sets are useful in generating the wave functions and transmission coefficients for distorted wave Born approximation, coupled channel method and statistical models of nuclear reactions involving alpha particles in exit or entrance channels. Further, the OMP is based on the idea of "gross properties" of nuclei and these "gross properties" are expected to vary smoothly with atomic mass of target nuclei and bombarding energy of alpha particles.

The present analysis is confined to the range of energies from 90 to 172 MeV. For these energies, the target nucleus structure, Pauli effect and contribution from the compound nucleus [8–10] and reaction channels to the elastic channel are much smaller than for lower energies. So we expect that potential parameters will vary smoothly with the energy and atomic mass number. For these energies of scattering, the potential can be determined unambiguously [11] if the rainbow scattering is observed.

Basing upon the above assumptions and taking into account the predictions resulting from microscopic [8, 12, 13] and phenomenological [14, 15] calculations, it is possible

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to construct a phenomenological nuclear potential which depends smoothly on mass and energy and reproduces well the experimental data on the elastic scattering of alpha particles in the considered energy range from 90 to 172 MeV and on target nuclei of the mass number $A_T = 12-208$.

2. *The universal optical model potential*

Since the spin and isospin of the alpha particles are equal to zero, the alpha-nucleus potential is purely central (target spin-orbit term can be neglected).

As an example of the local model dependent alpha particle-target nucleus OMP, a potential of the Saxon-Woods squared ((SW)^2) form can be given:

$$V(r) = -Uf^2(r, r_u, a_u) - i\left\{W_v f^2(r, r_v, a_v) - 4a_s W_s \frac{d}{dr} f^2(r, r_s, a_s)\right\}, \tag{1}$$

where

$$f(r, r_i, a_i) = \left\{1 + \exp\left(\frac{r - r_i \sqrt[3]{A_T}}{a_i}\right)\right\}^{-1} \tag{2}$$

and A_T is the mass number of target.

The parameters U, r_u, a_u, W_v, r_v, a_v, W_s, r_s, a_s are in general energy and target mass number dependent.

The (SW)^2 energy independent formfactor of the OMP has been succesfully used for the description of the elastic scattering of alpha particles on ^{40,44}Ca [6], ^{58,60}Ni [5], ^{90,92}Zr, ^{124}Sn, ^{208}Pb [7] in a wide energy range. This potential contains both volume and surface absorption terms in its imaginary part allowing the inclusion of the direct reaction channels effects localized at the nuclear surface. For lower energies at which the compound nucleus mechanism and strongly coupled reaction channels contribute significantly to the elastic scattering, the OMP parameters could be complicated functions of the alpha particle energy and target mass number.

It has been shown by Majka et al. [15] that the (SW)^2 form for the real part of the OMP is well justified in terms of the microscopic double folding calculations. The (SW)^2 formfactor was first used by Budzanowski et al. [16], and Goldberg [17].

Vinh Mau [10] has shown on the basis of the microscopic model that the form of the imaginary part of the OMP is close to the Fermi function for alpha particle scattering at high energies ($E_\alpha > 80$ MeV).

It has been also shown in the course of the phenomenological analysis that it is not necessary to use the surface term in the imaginary part of the OMP for $E_\alpha > 100$ MeV [14, 15].

In the present paper, we propose the OMP for the elastic alpha particle scattering in the energy range of 90–172 MeV and for the target mass number 12–208 of the following form:

$$V(r) = -Uf^2(r, r_u, a_u) - iW_v f(r, r_v, a_v) + V_{\text{Coul}}. \tag{3}$$

We have assumed the Coulomb potential V_{Coul} as the potential of a homogenous charged sphere with radius $R_c = 1.34 \cdot \sqrt[3]{A_T}$ fm [5, 6].

It is possible to deduce the energy dependence of the OMP parameters from theoretical [8] or phenomenological [1–7] calculations.

The target mass number dependence of the OMP parameters is less clear. Satchler [18] has analysed the elastic alpha particle scattering up to 80° at energy 28 MeV from light (A_T = 16–28), middle weight (A_T = 48–60), and heavy (A_T = 100–200) target nuclei. In that paper, the depth U of the real part of the OMP was shown to have a tendency to decrease with increasing A_T, but the depth U at the energy 166 MeV presented in Table 7.11 [19] shows a tendency to increase with increasing A_T. This mass dependence changes the sign of the slope at the energy 104 MeV. The only clear variation of the OMP parameters was found for the half way radius R_u by Satchler, namely $R_u = 1.2 \sqrt[3]{A_T} + 1.5$ fm. Igo and Thaler [20] found for the half way radius: $R_u = 1.35 \sqrt[3]{A_T} + 1.3$ fm for medium and heavy mass nuclei. Mass dependence of the diffuseness parameter a_v of the real part of the OMP was taken from the best fit analysis.

It was shown from the model independent analysis [21] that at the given energy in the region 100–170 MeV the volume integral of the real part of the OMP is almost constant. Also no significant dependence on the energy was observed in the rms-radii. This

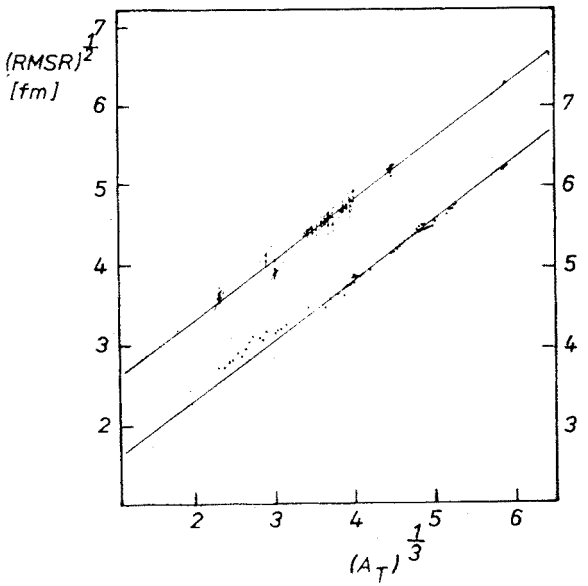

Fig. 1. The rms-radius of the real part of the model independent OMP vs. $A_T^{1/3}$ (left scale and upper part, taken from Ref. [22])

indicates a rather constant shape of the real part of the OMP for a given nucleus over the energy range from 100–170 MeV [21]. The linear (within error limits) rms-radius dependence on $A_T^{1/3}$ over the energy range 100–172.5 MeV (Fig. 1) [22] suggests the smooth variation of the shape of the real part of the OMP versus the mass number A_T.

For the parameters U, r_u, a_u, W_v, r_v, a_v in Eq. (3), we propose a simple functional form for their energy (in the center of mass system) E_{CM} and mass number A_T dependence:

$$\begin{aligned} U &= A_1 + A_2 E_{CM} + B_1 A_T, \\ W_v &= A_3 + A_4 E_{CM} + B_2 A_T, \\ Ru &= B_3 \sqrt[3]{A_T} + B_4, \quad \text{i.e.} \quad r_u = Ru/\sqrt[3]{A_T}, \\ a_u &= B_5 + B_6 A_T, \\ R_v &= B_7 \sqrt[3]{A_T}, \quad \text{i.e.} \quad r_v = B_7, \\ a_v &= B_8. \end{aligned} \tag{4}$$

3. Results

In our global analysis, we have used 28 angular distributions simultaneously consisting of about 2000 data points (Table I).

TABLE I

The experimental elastic alpha particles angular distributions for different target nuclei and energies of scattering

Isotope	E_{lab} [MeV]
^{12}C	120 [23], 145 [23], 172.5 [23]
^{27}Al	145 [23], 172.5 [23]
^{28}Si	104 [24], 155 [23]
^{40}Ca	104 [24], 141.7 [25]
^{58}Ni	104 [24], 139 [26], 172.5 [27]
$^{60,62,64}Ni$	104 [24], 172.5 [27]
^{90}Zr	99.5 [4], 118 [4], 166 [28]
^{92}Zr	90 [4], 120 [4]
^{124}Sn	104 [24], 166 [28]
^{208}Pb	104 [24], 139 [26], 166 [28]

In order to find the best numerical values of the global parameters, Eqs. (3) and (4), we have minimized the modified χ^2_{mod} function

$$\chi^2_{mod} = \frac{1}{M} \sum_{i=1}^{M} \frac{\chi_i^2}{\chi^2_{i,\,best}}, \tag{5}$$

where

$$\chi_i^2 = \sum_{j=1}^{N_i} \left(\frac{\sigma_{j,i}^{exp} - \sigma_{j,i}^{th}}{\Delta\sigma_{j,i}^{exp}} \right)^2, \tag{6}$$

TABLE II

Values of the universal Eqs. (3) and (4) OMP parameters and their errors

A_1 [MeV]	A_2 [MeV/MeV]	A_3 [MeV]	A_4 [MeV/MeV]
160.6 ±4.0	−0.164 ±0.04	24.7 ±2.0	0.007 ±0.005
B_1 [MeV/amu]	B_2 [MeV/amu]	B_3 [fm]	B_4 [fm]
0.0063 ±0.05	−0.0234 ±0.004	1.360 ±0.009	0.136 ±0.04
B_5 [fm]	B_6 [fm/amu]	B_7 [fm]	B_8 [fm]
1.275 ±0.03	−0.00102 ±0.0004	1.489 ±0.02	0.7494 ±0.12

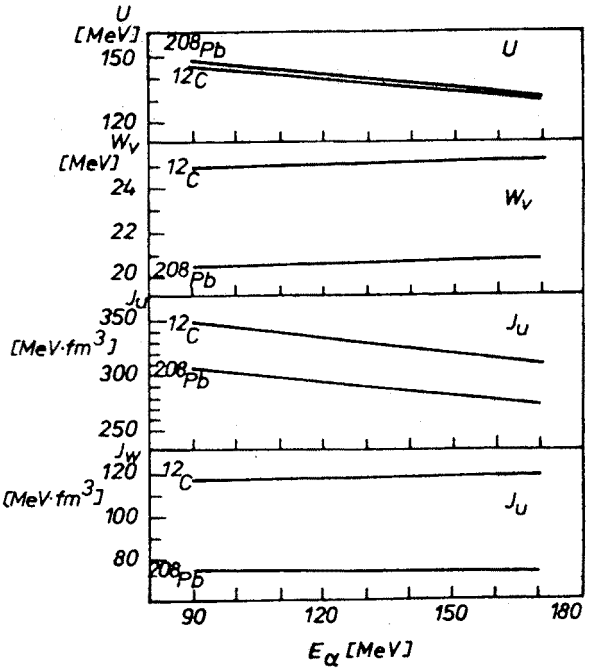

Fig. 2. Real (U) and imaginary (W_v) depths of the OMP, Eqs. (3) and (4), and their volume integrals (J_u — real, J_w — imaginary) vs energy of alpha particles (laboratory system)

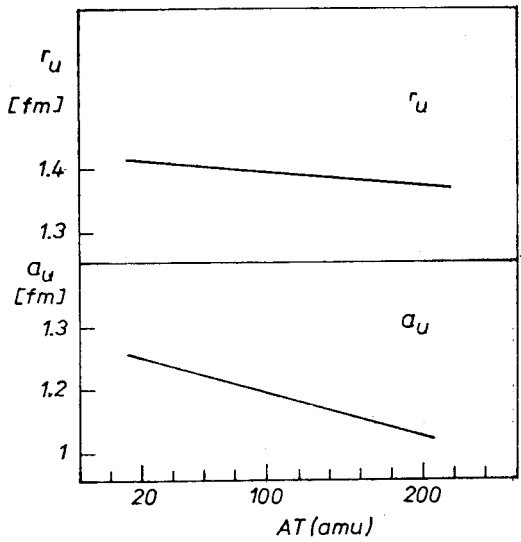

Fig. 3. Geometrical parameters r_u (reduced radius) and a_v (diffuseness) of the real part of the OMP, Eqs. (3) and (4), vs the mass number A_T

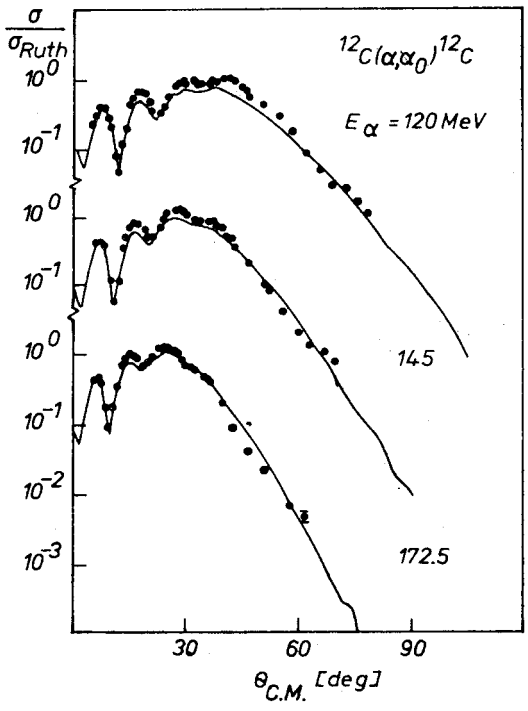

Fig. 4. $^{12}C(\alpha, \alpha_0)^{12}C$

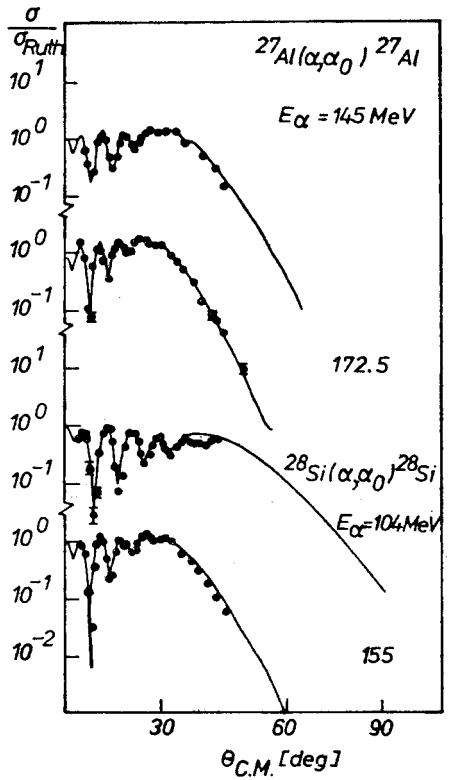

Fig. 5. $^{27}Al(\alpha, \alpha_0)^{27}Al$, $^{28}Si(\alpha, \alpha_0)^{28}Si$

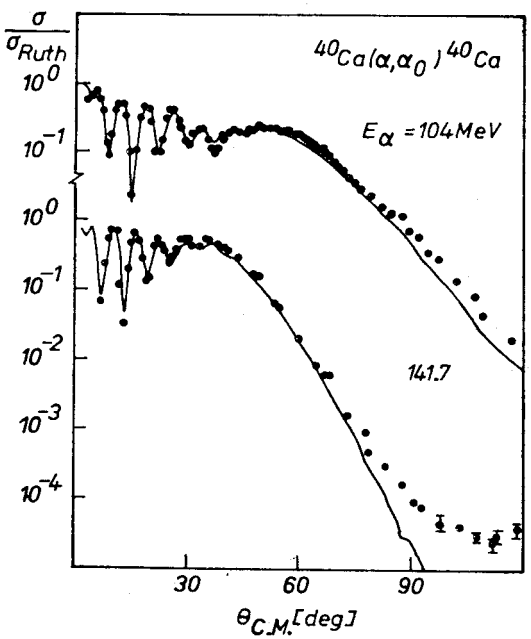

Fig. 6. $^{40}Ca(\alpha, \alpha_0)^{40}Ca$

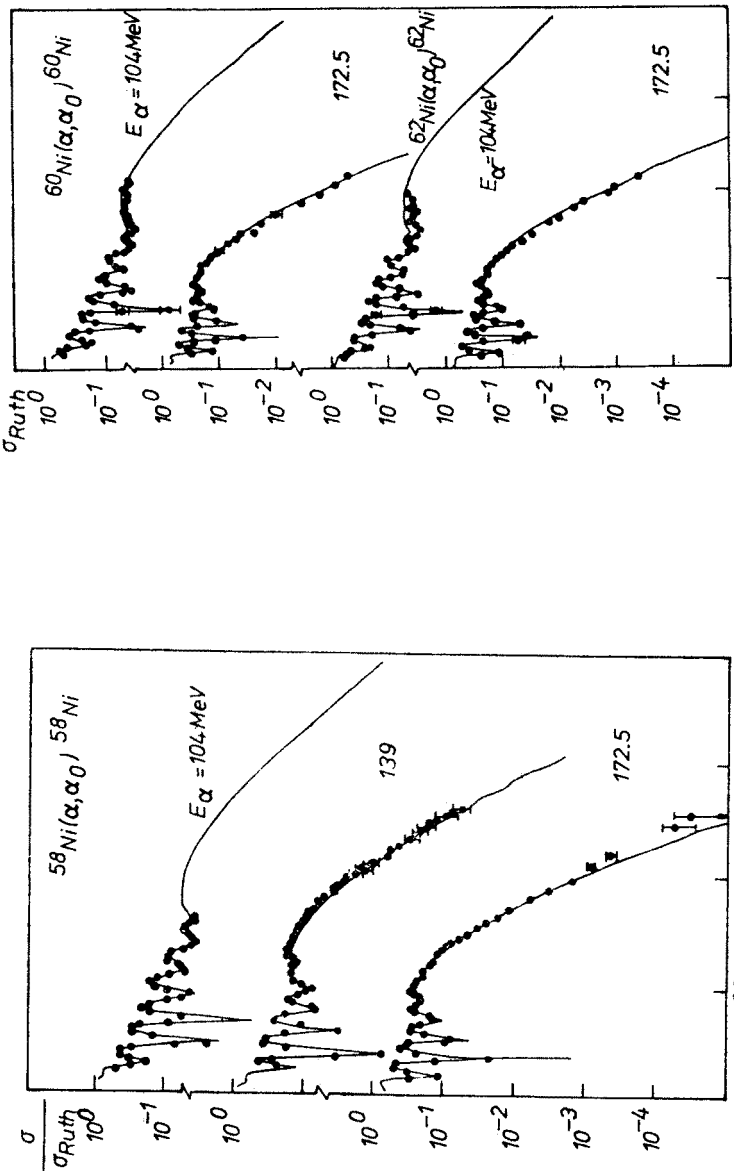

Fig. 7. $^{58}Ni(\alpha, \alpha_0)^{58}Ni$

Fig. 8. $^{60}Ni(\alpha, \alpha_0)^{60}Ni$, $^{62}Ni(\alpha, \alpha_0)^{62}Ni$

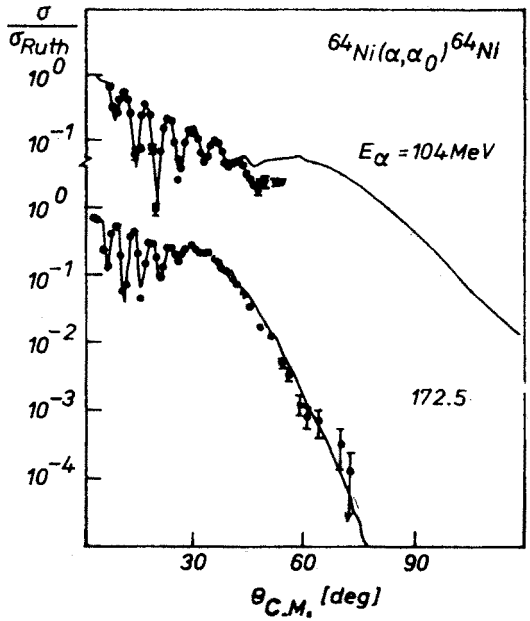

Fig. 9. $^{64}Ni(\alpha, \alpha_0)^{64}Ni$

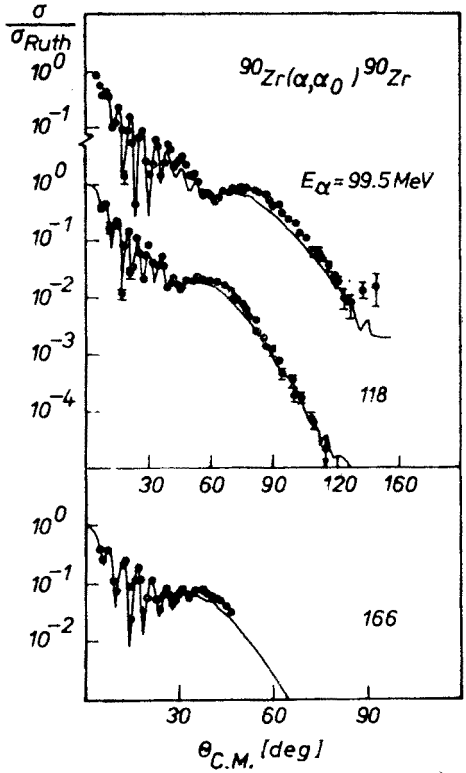

Fig. 10. $^{90}Zr(\alpha, \alpha_0)^{90}Zr$

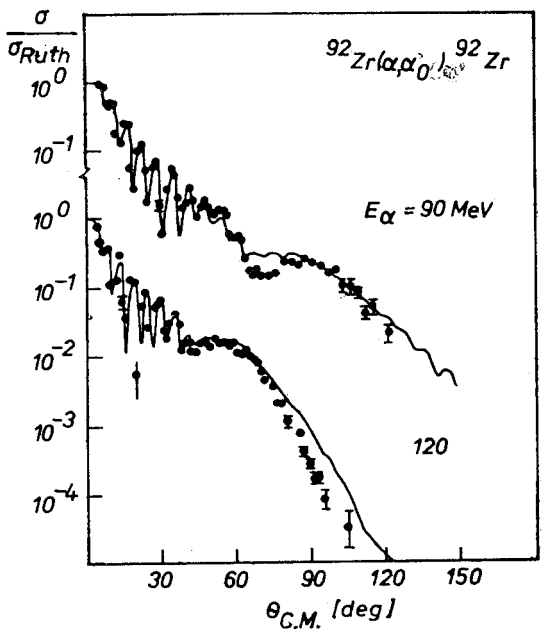

Fig. 11. $^{92}Zr(\alpha, \alpha_0)^{92}Zr$

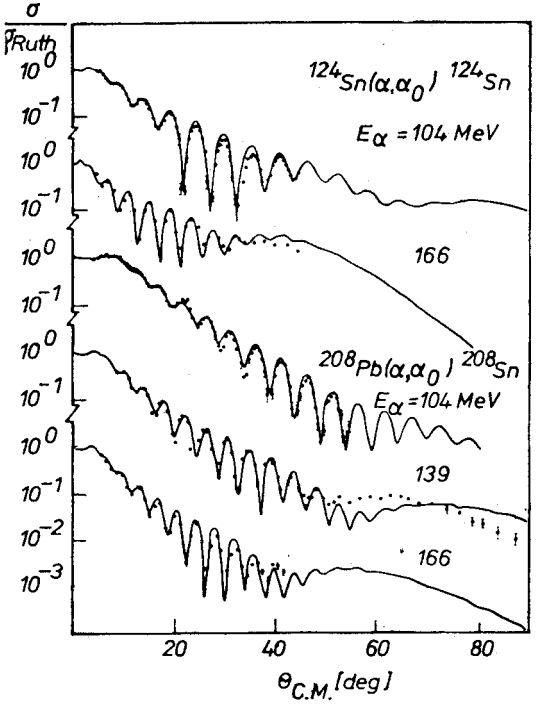

Fig. 12. $^{124}Sn(\alpha, \alpha_0)^{124}Sn$, $^{208}Pb(\alpha, \alpha_0)^{208}Pb$

M, N_i, $\chi^2_{i,\text{best}}$, $\sigma^{\text{exp}}_{j,i}$, $\sigma^{\text{th}}_{j,i}$, $\Delta\sigma^{\text{exp}}_{j,i}$, are: number of angular distributions, number of data points, the best fit χ^2_i values, experimental and theoretical cross-sections, errors of experimental data of the i-th angular distribution, respectively.

The modified normalized χ^2_{mod} value had to be used since the data from different laboratories with possibly different systematical errors were compared.

The parameters corresponding to the minimum of χ^2_{mod} value have been calculated by the modified computer code GLOB [29]. Final parameters and their correlated errors of the universal parametrization, Eqs. (3) and (4), are presented in Table II.

Fig. 2 presents the depths of the real (U) and imaginary (W_v) part of the OMP and also the volume integrals for these potentials vs. energy in the laboratory system. The geometrical parameters r_u, a_u of the real part of the OMP vs. atomic mass number A_T are presented in Fig. 3.

The resulting fits of the calculated cross-sections to experimental data (Table I) are presented in Figs. 4–12.

4. Final remarks

The calculated cross-sections follow the experimental data in the considered energy and mass region.

The depth parameters of the real part of the OMP varies very little with atomic mass number, the slope coefficient being equal to 0.0063 MeV/amu. The slope coefficient of the energy dependence is equal to 0.16 MeV/MeV. This last value is in good agreement with earlier results [5–7].

The volume integrals of the real part of the OMP are linearly depending on the energy and vary from 350 · fm^3 (^{12}C, E_{lab} = 90 MeV) to 310 MeV · fm^3 (^{12}C, E_{lab} = 170 MeV), and from 300 MeV · fm^3 (^{208}Pb, E_{lab} = 90 MeV) to 270 MeV · fm^3 (^{208}Pb, E_{lab} = 170 MeV) (Fig. 2).

From Fig. 4 we notice that the diffusenes parameter a_v of the real part of the OMP decreases with increasing mass number A_T.

The imaginary potential W_v and its volume integral depend linearly on the energy having the slope coefficient equal to 0.007 MeV/MeV (Fig. 2). W_v is a linear function of the mass number with its negative slope coefficient equal to 0.0234 MeV/amu.

The energy and atomic mass dependence of the imaginary volume integrals found by us agrees with the results presented in Ref. [21].

In order to obtain better fits of calculated cross-sections to experimental data, it is necessary to take into account the individual properties of the target nuclei under study. These structure corrections added to the OMP should take into consideration the number of nucleons outside of the last closed shell of target nuclei. For example, the structure corrections were introduced for elastic scattering of deuterons in the energy range 11.8–90 MeV and atomic mass of target nuclei from 27 to 238 amu [30].

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